

Normality and superperfection of Jonsson spectra

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This paper investigates the structural properties of Jonsson spectra with respect to the notions of K_T -normality, cl -normality, and superperfection. Particular attention is paid to the role of the Kaiser class K_T as an internal structural layer of a Jonsson spectrum. In contrast to the classical approach based exclusively on the class of existentially closed models, a more general normalization framework for Jonsson spectra is developed through the consistency of Jonsson pregeometry with the Kaiser class. The study is carried out within the framework of Jonsson model theory and relies on methods of pregeometry and spectral analysis. The characteristic features of Jonsson spectra are examined from the standpoint of their interaction with structural closure operators and the Kaiser class. The notions of a K_T -normal Jonsson spectrum, a cl -normal Jonsson spectrum, an ideal Jonsson spectrum, and a K_T -beautiful pair are introduced and formally characterized. Criteria for the stability of Jonsson spectra in the superperfect case are obtained. Connections between spectrum normality, Jonsson pregeometry, central types, the heredity of Jonsson theories, and P -stability are established. The results contribute to the further development of the structural theory of Jonsson spectra and provide new tools for their investigation.

Keywords: Jonsson spectrum, Jonsson theory, Kaiser class, superperfection, K_T -normality, cl -normality, Jonsson pregeometry, central types, P -stability, beautiful pairs.

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Introduction

Jonsson theory is one of the most natural generalizations of the theory of complete first-order theories in modern model theory. It provides a framework for studying broad classes of incomplete theories while preserving many important semantic and geometric properties. During the last decades, methods of classical model theory have been successfully adapted to Jonsson theories, leading to the development of a rich theory of existentially closed models, semantic invariants, and geometric structures.

The study of Jonsson theories is based both on classical results in model theory obtained by B. Poizat [1], E. Bouscaren [2], E. Palyutin [3], and T.G. Mustafin [4], in which model-theoretic aspects of Jonsson theories were developed. Their investigations demonstrated that many classical notions of stability theory can be transferred to the context of Jonsson theories and provided the basis for studying Jonsson spectra, semantic models, and related geometric constructions.

A substantial contribution to the development of modern Jonsson theory has been made by A.R. Yeshkeyev. A systematic exposition of the theory, including semantic models, Jonsson spectra, central types, and positive Jonsson theories, was presented in the monograph [5]. This work summarizes the principal directions of the Karaganda school of model theory and serves as a foundation for further investigations.

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The study of companions of Jonsson sets was further developed in [6], where model-theoretic properties of the $\sharp$ -companion of a Jonsson set were investigated. These results provided new methods for analyzing semantic relations between Jonsson structures and their associated theories.

Further development of spectral methods was obtained in [7], where several classes of algebras of Jonsson spectra were introduced and studied. These constructions established new connections between algebraic and geometric properties of Jonsson spectra.

The structure of perfect classes associated with Jonsson spectra was investigated in [8], where the authors determined the number of fragments of a perfect class and described their fundamental properties. This direction was continued in [9], where the notion of 1-perfectness for fragments of normal Jonsson theories was studied.

An important role in modern Jonsson theory is played by positive Jonsson theories. Their amalgamation and joint embedding properties were analyzed in [10], providing additional tools for the study of existentially closed models and hereditary subclasses.

The interaction between positive logic and Jonsson theories was investigated by B. Poizat and A.R. Yeshkeyev in [11]. Their results established new back-and-forth techniques in positive logic and contributed to the understanding of semantic equivalence in positive Jonsson frameworks.

Recent investigations have focused on the Kaiser class and its role as a semantic invariant of Jonsson theories. In [12, 13], the Kaiser class K_T was studied as a fundamental component of the semantic structure of a Jonsson theory. The obtained results demonstrated that the Kaiser class provides a natural tool for analyzing geometric properties of Jonsson spectra and their associated semantic models.

Several related directions have also been developed within the Karaganda model-theoretic school. Semantic pairs and existentially fragmentally complete predicates in Jonsson spectra were investigated in [14]. Jonsson existentially closed unars of expanded signatures were studied in [15]. The properties of 1-model companions of Jonsson sets in normal Jonsson theories were analyzed in [16], while companions of hybrids of fragments of theoretical subsets were considered in [17]. More recently, syntactic similarity of hybrids of classes of normal Jonsson theories was investigated in [18].

These results collectively demonstrate the formation of a coherent geometric approach to the study of Jonsson theories, their spectra, semantic models, central types, and Kaiser classes. The present paper continues this line of research and is devoted to the investigation of normality and superperfection of Jonsson spectra.

Our main objective is to study the interaction between the Kaiser class, Jonsson pregeometry, and central types, and to describe conditions under which these structures become compatible. We introduce the notions of K_T -normality, cl -normality, ideal Jonsson spectra, and K_T -beautiful pairs. Particular attention is paid to superperfect Jonsson spectra characterized by the equality $K_T = E_T$, which establishes a strong connection between the Kaiser class and the internal geometry of the spectrum.

The obtained results contribute to the further development of geometric methods in Jonsson model theory and provide new tools for investigating hereditary structures, central types, semantic pairs, and spectral invariants associated with Kaiser classes.

1 Basic definitions and preliminary results

In this section we introduce the main notions and recall the basic structural framework used throughout the paper.

Let T be a Jonsson theory in a language L .

Recall that a Jonsson theory is defined as follows.

Definition 1. [19, p. 80] A theory T is called a *Jonsson theory* if the following conditions are satisfied:

1. The theory T has at least one infinite model;
2. T is *inductive*, that is, it is equivalent to a set of sentences of the form $\forall\exists$;
3. T has the *joint embedding property (JEP)*, meaning that for any two models $A, B \models T$, there exists a model $C \models T$ into which both A and B can be embedded isomorphically;
4. T has the *amalgamation property (AP)*: for any models $A, B_1, B_2 \models T$ and isomorphic embeddings $f_1: A \rightarrow B_1$ and $f_2: A \rightarrow B_2$, there exists a model $C \models T$ together with isomorphic embeddings $g_1: B_1 \rightarrow C$ and $g_2: B_2 \rightarrow C$ such that $g_1 f_1 = g_2 f_2$.

The semantic model plays a fundamental role in the study of Jonsson theories and serves as their principal semantic invariant.

Definition 2 (Ye.T. Mustafin [20]). A model C_T of a Jonsson theory T is called a semantic model of T if

$$|C_T| = 2^\omega$$

and C_T is an ω^+ -universal and ω^+ -homogeneous model of the theory T .

The semantic model determines the complete theory that serves as the main syntactic invariant of a Jonsson theory.

Definition 3 (T.G. Mustafin [5]). The theory

$$T^* = Th(C_T)$$

is called the center of the Jonsson theory T .

Since the center is a complete theory, many methods of classical model theory can be transferred to the study of Jonsson theories through their centers. An important subclass of Jonsson theories is formed by perfect theories.

Definition 4. [5] A Jonsson theory T is called perfect if its semantic model C_T is ω^+ -saturated.

We now define the Jonsson spectrum of a model, which describes the collection of Jonsson theories realized by a given structure. Previously, a number of results concerning Jonsson spectra, which generalize the concept of Jonsson theories, had already been obtained. The notion of a Jonsson spectrum was first introduced by A.R. Yeshkeyev [5].

Throughout the paper, let L be a first-order language, let σ be its signature, and let K be a class of σ -structures. We use the notation $A \models T$ to indicate that a structure A is a model of the theory T .

Definition 5 (A.R. Yeshkeyev [5]). The following set of Jonsson theories is called the *Jonsson spectrum* of the class K :

$$JSp(K) = \{T \mid T \text{ is a Jonsson theory and } A \models T \text{ for every } A \in K\}.$$

The notion of a Jonsson spectrum has been extensively studied in connection with fragments, companions, and algebraic structures associated with classes of Jonsson theories [6–8].

According to the semantic approach introduced by T.G. Mustafin, the properties of a Jonsson theory are determined by its semantic model. Therefore, Jonsson theories having the same semantic model are naturally considered as belonging to the same cosemantic class.

The relation of cosemanticity is an equivalence relation on $JSp(K)$. Hence, the quotient set

$$JSp(K)/\simeq$$

consists of the cosemantic classes of Jonsson theories.

Let $[T]$ denote the cosemantic class containing the theory T , where $T \in JSp(K)$. Since all theories in the class $[T]$ share the same semantic model, they also have the same center. We denote the common semantic model of the class by $C_{[T]}$, and its complete theory by

$$[T]^* = Th(C_{[T]}),$$

which is called the center of the Jonsson class.

Next, we recall the standard notion of existentially closed models associated with a Jonsson theory. By E_T we denote the class of all existentially closed models of the theory T .

We now introduce the structural notion of the Kaiser class, which plays a central role in the present work.

Definition 6 (A.R. Yeshkeyev [13]). Let T be a Jonsson theory. A class K_T of L -structures is called a *Kaiser class* of T if

$$K_T = \{M \mid M \models T \text{ and } Th_{\forall\exists}(M) \text{ is a Jonsson theory}\}.$$

We next introduce the operator of Jonsson pregeometry, which provides the geometric structure on the semantic model.

Let $cl : \mathcal{P}(C_T) \rightarrow \mathcal{P}(C_T)$ be the operator of Jonsson pregeometry.

Definition 7. [5] Let T be a Jonsson theory, and let $X \subseteq C_T$. The pair (X, cl) is called a Jonsson pregeometry τ , if there exists a closure operator $cl : \mathcal{P}(C_T) \rightarrow \mathcal{P}(C_T)$ such that the following conditions hold:

- (1) If $A \subseteq X$, then $A \subseteq cl(A)$ and $cl(cl(A)) = cl(A)$;
- (2) If $A \subseteq B \subseteq X$, then $cl(A) \subseteq cl(B)$;
- (3) (Exchange) If $A \subseteq X$, $a, b \in X$, and $a \in cl(A \cup \{b\}) \setminus cl(A)$, then $b \in cl(A \cup \{a\})$;
- (4) (Finite character) If $A \subseteq X$ and $a \in cl(A)$, then there exists a finite subset $A_0 \subseteq A$ such that $a \in cl(A_0)$.

Using this operator, we define several notions of normality for classes of Jonsson theories.

Definition 8 (A.R. Yeshkeyev). A class of Jonsson theories $[T]$ is called K_T -normal if for any models $M_1, M_2 \in K_T$ there exists a model $N \in K_T$, into which M_1 and M_2 embed isomorphically, and the Jonsson- cl -closures are preserved under these embeddings.

Definition 9 (A.R. Yeshkeyev). A class of Jonsson theories $[T]$ is called cl -normal if for any Jonsson subsets $A, B \subseteq C_T$ the following condition holds: $cl(A \cap B) = cl(A) \cap cl(B)$.

cl -normality is an internal consistency condition of a Jonsson pregeometry. It guarantees the consistent behavior of the Jonsson closure operator under intersections of Jonsson sets.

We now introduce the notion of an ideal class of Jonsson theories, which captures the compatibility between the two types of normality.

Definition 10 (A.R. Yeshkeyev). A class of Jonsson theories $[T]$ is called ideal if K_T -normality $\iff cl$ -normality.

Thus, ideality means the coincidence of two distinct geometries of the Jonsson spectrum:

- * the geometry induced by the Kaiser class;
- * the geometry induced by the Jonsson pregeometry.

Finally, we define the important special case of superperfect Jonsson theories.

Definition 11 (A.R. Yeshkeyev). A Jonsson theory T is called superperfect if $K_T = E_T$.

Note that superperfection does not imply the trivialization of the Jonsson spectrum.

On the contrary, the coincidence of K_T and E_T stabilizes the Jonsson spectrum and makes the Jonsson pregeometry consistent with the Kaiser class.

In the imperfect case, the inclusion

$$K_T \subseteq E_T$$

may be proper, and then the Jonsson spectrum loses its internal geometric consistency.

2 Superperfect Jonsson spectra

The notion of superperfection occupies a special place in the study of Jonsson spectra. The importance of this concept is closely related to the Kaiser class, which serves as a refined semantic invariant of a Jonsson theory. In contrast to the class of existentially closed models, the Kaiser class reflects a broader range of structural properties and allows one to investigate the interaction between semantic and geometric aspects of Jonsson spectra.

In the superperfect case, the distinction between the Kaiser class and the class of existentially closed models disappears. This fact significantly simplifies the internal structure of the spectrum and creates a natural connection between the center of a Jonsson theory, its semantic model, and the corresponding closure structures. As a consequence, several normality conditions admit a more transparent description and can be studied within a common geometric framework.

Throughout this section, we investigate the relationship between superperfection, normality conditions, and Jonsson pregeometry. The results obtained below demonstrate that superperfect spectra possess a particularly stable structure and provide a natural setting for the study of hereditary properties and stability-theoretic phenomena.

Recall that a Jonsson theory is called perfect if its semantic model is saturated.

Theorem 1. Let $[T]$ be a superperfect class of Jonsson theories. Then $\text{Mod}(T^*) = K_T = E_T$.

Proof. Since $[T]$ is superperfect, by definition of superperfection, $K_T = E_T$.

On the other hand, for a perfect Jonsson theory it is known that the class of existentially closed models coincides with the class of models of the center. Therefore, $E_T = \text{Mod}(T^*)$.

Combining these equalities, we obtain $\text{Mod}(T^*) = K_T = E_T$.

Thus the center, the Kaiser class, and the class of existentially closed models determine the same semantic layer of the spectrum. $\square$

Recall that a central type is a complete 1-type obtained from an admissible enrichment of a Jonsson theory. Central types were introduced by A.R. Yeshkeyev and play an important role in the study of hereditary Jonsson theories, semantic pairs, and admissible enrichments [5]. Since central types are closely related to the center T^* of a Jonsson theory, their realizability reflects essential structural properties of the corresponding Jonsson spectrum.

Corollary 1. In a superperfect class of Jonsson theories, every central type is realized inside K_T .

Proof. Since $K_T = E_T$, every existentially closed model belongs to the Kaiser class.

Central types are realized in existentially closed models. Therefore all realizations of central types belong to K_T . $\square$

The obtained result shows that, in the superperfect case, the Kaiser class becomes the natural carrier of central types.

In particular, central types cease to depend on the choice of a particular existentially closed model and become stabilized with respect to the entire K_T -structure.

Theorem 2. Let $[T]$ be a superperfect class of Jonsson theories. Then the following conditions are equivalent:

1. $[T]$ is ideal;
2. $[T]$ is both K_T -normal and cl -normal;
3. Jonsson pregeometry is consistent with K_T .

Proof. $(1 \Rightarrow 2)$. Assume that $[T]$ is ideal. By the definition of an ideal class of Jonsson theories, K_T -normality $\iff cl$ -normality.

Hence, whenever one of these properties holds, the other also holds. Therefore the spectrum is simultaneously K_T -normal and cl -normal.

$(2 \Rightarrow 3)$. Assume that $[T]$ is both K_T -normal and cl -normal.

It follows from K_T -normality that any Jonsson embeddings within K_T preserve the Jonsson- cl structure.

Indeed, if

$$M_1, M_2 \in K_T,$$

then there exists a model

$$N \in K_T,$$

into which M_1 and M_2 Jonsson-embed with preservation of Jonsson- cl closures.

Furthermore, cl -normality guarantees stability of the closure operator under intersections of Jonsson sets:

$$cl(A \cap B) = cl(A) \cap cl(B).$$

Therefore the closure structure behaves coherently with respect to the basic operations of Jonsson pregeometry. Hence Jonsson pregeometry is consistent with the Kaiser class.

$(3 \Rightarrow 1)$. Suppose that Jonsson pregeometry is consistent with K_T .

Consistency implies that embeddings inside the Kaiser class preserve the closure structure induced by the operator cl . As a consequence, K_T -normality yields cl -normality.

Conversely, preservation of the closure structure guarantees that the geometric behavior of the spectrum determines the corresponding closure-preserving embeddings in K_T . Therefore cl -normality implies K_T -normality.

Hence, K_T -normality $\iff cl$ -normality.

Thus $[T]$ is an ideal class of Jonsson theories. □

3 Structural properties of normal Jonsson spectra

The notion of a beautiful pair plays an important role in the study of stability-theoretic properties of model-theoretic structures. In the context of Jonsson theories, the Kaiser class provides a natural environment for the construction of beautiful pairs compatible with Jonsson pregeometry and central types. The following definition adapts the classical concept to the framework of superperfect Jonsson spectra.

Definition 12 (A.R. Yeshkeyev). A pair (C_T, M) is called a K_T -beautiful pair if:

1. $M \in K_T$;
2. $cl(M) = M$;
3. every central type over M is realized in C_T .

Theorem 3. Let $[T]$ be a superperfect ideal Jonsson class. Then every K_T -beautiful pair is hereditary.

Proof. Since $[T]$ is superperfect, by Theorem 1 we have $K_T = E_T$. Hence every model belonging to the Kaiser class is existentially closed.

Furthermore, since $[T]$ is ideal, K_T -normality $\iff cl$ -normality.

Therefore the closure structure induced by Jonsson pregeometry is preserved inside the Kaiser class.

Let (C_T, M) be a K_T -beautiful pair. By definition, M belongs to K_T , is cl -closed, and every central type over M is realized in the semantic model C_T .

Consider an admissible enrichment of the language. Since admissible enrichments preserve the Jonsson nature of the theory, the corresponding enriched theory remains within the same semantic framework. Moreover, the closure operator associated with Jonsson pregeometry remains compatible with the Kaiser class.

Since $cl(M) = M$, the model M remains cl -closed after enrichment. Realizability of central types is also preserved because all realizations remain inside the semantic model.

Consequently:

- M remains in K_T ;
- the Jonsson- cl structure is preserved;
- realizability of central types is preserved.

Therefore all defining properties of a K_T -beautiful pair are preserved under admissible enrichments. Hence the enriched theory remains a Jonsson theory and the pair remains hereditary. $\square$

The notion of P -stability extends classical stability-theoretic ideas to the setting of Jonsson spectra. In the superperfect case, the coincidence of the Kaiser class, the class of existentially closed models, and the class of models of the center allows one to describe P -stability in terms of the center of the corresponding Jonsson theory.

The definition of P -stability is based on an admissible enrichment of the language. Let

$$\sigma_\Gamma = \sigma \cup \{P\} \cup \{c\},$$

where P is a unary predicate symbol and c is a constant symbol. The interpretation of P is assumed to be an existentially closed submodel of the semantic model C_T . We denote by $S_P(A)$ the set of complete P -types over a Jonsson set A .

Definition 13. A class of Jonsson theories $[T]$ is called P -stable if for any Jonsson set $A \subseteq C_T$ the following holds:

$$|S_P(A)| \leq |A| + \omega.$$

Theorem 4. Let $[T]$ be a superperfect ideal class of Jonsson theories. Then the P -stability of the spectrum is determined by the center T^* .

Proof. Since $[T]$ is superperfect, we have

$$K_T = E_T = \text{Mod}(T^*).$$

Hence, all central types are realized within a single K_T -layer. Since $[T]$ is ideal, the Jonsson pregeometry is consistent with the Kaiser class. Therefore, any P -types are determined solely by the structure of the center T^* .

Now assume that the center T^* is stable. Then the number of types over any set is bounded by the cardinality of that set.

Since all Jonsson- P -types are induced by types of the center, we obtain:

$$|S_P(A)| \leq |A| + \omega.$$

Hence, $[T]$ is P -stable. Conversely, if $[T]$ is P -stable, then the number of central types is also bounded. Therefore, the center T^* is stable.

Thus,

$$[T] \text{ is } P\text{-stable} \iff T^* \text{ is stable.}$$

$\square$

Recall that a central type arises from an admissible enrichment of a Jonsson theory. It is precisely central types that allow one to control the behavior of the Jonsson spectrum with respect to admissible enrichments.

Definition 14 (A.R. Yeshkeyev). A central type p_c is called K_T -normal if all its realizations belong to models of the Kaiser class.

Definition 15 (A.R. Yeshkeyev). A central type p_c is called cl -normal if for any Jonsson set A the following holds:

$$cl(A \cup p_c) = cl(A) \cup cl(p_c).$$

Theorem 5. Let $[T]$ be an ideal class of Jonsson theories. Then every K_T -normal central type is cl -normal.

Proof. Since $[T]$ is ideal, K_T -normality $\iff cl$ -normality.

Let p_c be a K_T -normal central type. Then all of its realizations belong to models of the Kaiser class.

Since the Jonsson pregeometry is consistent with K_T , the Jonsson- cl closure is preserved with respect to all realizations of p_c .

Therefore,

$$cl(A \cup p_c) = cl(A) \cup cl(p_c).$$

Hence, p_c is cl -normal. □

Conclusion

In this paper, we investigated normality conditions for Jonsson spectra in the framework of the Kaiser class and Jonsson pregeometry.

The main objective of the study was to analyze the interaction between the Kaiser class, closure operators, central types, and stability-theoretic properties of Jonsson spectra. Particular attention was devoted to superperfect spectra, where the coincidence of the Kaiser class and the class of existentially closed models provides a natural semantic stabilization.

We showed that the Kaiser class constitutes a distinguished structural layer of a Jonsson spectrum and serves as a convenient environment for the study of normality conditions. The relationship between K_T -normality and cl -normality was investigated, and it was established that ideal Jonsson spectra are characterized by the compatibility of these two notions.

Furthermore, we demonstrated that superperfection leads to a strong connection between the center T^* , the Kaiser class, and existentially closed models. This connection allows one to describe stability-theoretic properties of the spectrum through the center of the corresponding Jonsson theory.

The notions of K_T -beautiful pairs and normal central types were studied within this framework. It was shown that hereditary properties of K_T -beautiful pairs are preserved under admissible enrichments, while normal central types remain compatible with the closure structure induced by Jonsson pregeometry.

The obtained results indicate that Jonsson spectra possess a rich internal geometry determined by the interaction between the Kaiser class, central types, and closure operators. These methods may be further applied to the study of hereditary Jonsson theories, admissible enrichments, and stability phenomena in classes of cosemantic Jonsson theories.

Author contributions

All authors contributed equally to the development of the study, including the formulation of main results, analysis, and writing of the manuscript. All authors have read and approved the final version.

Conflict of interest

The authors declare no conflict of interest.

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