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The sufficient condition of embedding in the Lorentz space

In this paper considered the transformation of a series of Fourier-Price in the form k-th difference of the Fourier-Price coefficients. In addition, obtained a sufficient condition the belonging of the function f in a two-parameter Lorentz space in the terms of Fourier-Price coefficients under the conditions that the coefficients of the Fourier-Price the functions f is monotonely, $\Delta_k a_\nu \geq 0$ and a forming sequence of the Price's system $p_i, i = 1, 2, ...,$ is limited, i.e. $p_i \leq c$, $p_i \geq 2$, $i \in N$.

Key words: Price system, Lorentz space,Dirichlet kernel, Fourier-Price series, the coefficients of the Fourier-Price.

Embedding theorems for function spaces appeared very early in the 30-ies of XX- century because of the decision of mathematical physics problems. Then the theory of embedding function spaces has been used in the other areas of mathematics such as the theory of functional spaces, the approximation theory, the theory of differential equations and etc. The study of the structural properties for trigonometric series with monotonely decreasing to zero coefficients on the space L_p discussed in the works T.M.Vukolova [1; 18], S. Bitimkhan [2; 3]. In this paper we will discuss the sufficient condition the belonging of functions f to the Lorentz space in terms of the Fourier-Price coefficients under conditions that the coefficients of Fourier-Price series of f is monotonely, $\Delta_k a_\nu \geq 0$ and the forming sequence of Price's system is limited. For k=1 the sufficient condition the belonging of functions f to the Lorentz space proved in the work [3; 105].

Let $\{p_i\}_{i=1}^{+\infty}$ be any sequence of natural numbers such that $p_i \leq c$, $p_i \geq 2$, $i \in N$. Suppose $m_0 = 1, m_\mu = \prod\limits_{k=1}^{\mu} p_k$, $i \in N$. Every $x \in [0, 1]$ has an decomposition: $x = \sum\limits_{k=1}^{+\infty} \frac{x_k}{m_k}$, $0 \leq x_k \leq p_k - 1$. This representation is singular, if in the case of $x = \frac{k}{m_n}$, $0 < k < m_n$, $n \in N$, $k \in N$, to consider that the expansion with the only finitely many non-zero x_k. Any natural number n is singular represented in the form $n = \sum\limits_{k=0}^{r} n_k m_k$, where n_k – integer numbers; $0 \leq n_k \leq p_{k+1} - 1$, $k = 0, 1, 2, ...$

We define the sequence of Price functions [4; 31]:

$$\varphi_0\left(x\right) = 1, \Phi_k(x) = \varphi_{m_k}\left(x\right) = \exp\frac{2\pi i x_{k+1}}{p_{k+1}}, k = 0, 1, 2, ...,$$

where x_k from the decomposition of point x. For any natural number n suppose that

$$\varphi_n\left(x\right) = \prod_{k=0}^{r} \left[\Phi_k(x)\right]^{n_k} = \prod_{k=0}^{r} \left[\varphi_{m_k}\left(x\right)\right]^{n_k}.$$

We say that the function $f\left(x\right)$ belongs to the Lorentz space [5; 216] $L_{p\theta}[0, 1]$, if

$$\|f\|_{p\theta} = \left\{\frac{\theta}{p}\int\limits_0^1 t^{\frac{\theta}{p}-1}\left[f^*\left(t\right)\right]^\theta dt\right\}^{\frac{1}{\theta}} < +\infty, 1 \leq p < +\infty, 1 \leq \theta < +\infty,$$

and

$$\|f\|_{p\infty} = \underset{t>0}{Sup} t^{\frac{1}{p}} f^*\left(t\right) < +\infty 1 \leq p < +\infty \theta = +\infty$$

where $f^*(t)$ is non-increasing permutation of function $|f(x)|$, $x \in [0,1]$.

Fourier-Price series of function $f \in L[0,1]$ is called series $\sum\limits_{\nu=0}^{+\infty} a\nu\varphi_\nu(x)$, where $a_\nu = \int\limits_0^1 f(x)\varphi_\nu(x)dx-$ Fourier-Price coefficients of function $f(x)$ by the multiplicative Price systems.

Let $\{a_\nu\}_{\nu=0}^{+\infty}$, is non-negative monotone sequence of numbers. The first difference of the numerical sequence $\{a_\nu\}_{\nu=0}^{+\infty}$ we shall call the value $\Delta_1 a_\nu = a_\nu - a_{\nu+1}$, and the k-th difference value $\Delta_k a_\nu = = \Delta_1(\Delta_{k-1}a_\nu)$. We will consider $\Delta_0 a_\nu = a_\nu$.

If $\{a_\nu\}_{\nu=0}^{+\infty}$ is non-negative sequence of numbers, then on the k-th difference value $\Delta_k a_\nu$ by applying the method of Pascal's triangle we can write it in the form

$$\Delta_k a_\nu = \sum_{j=0}^{k}(-1)^j C_k^j a_{\nu+j},$$

where $C_k^j = \frac{k!}{j!(k-j)!}$, $0! = 1$ is called binomial coefficient.

Let $D_n(x) = \sum\limits_{k=0}^{n-1} \varphi_k(x)$ is the Dirichlet kernel. We introduce the following notations

$$D_n^{(0)}(x) = \varphi_n(x), D_n^{(1)}(x) = D_n(x) = \sum_{\nu=0}^{n-1}\varphi_\nu(x), D_n^{(k)}(x) = \sum_{\nu=0}^{n-1} D_\nu^{(k-1)}(x), k \geq 1, n \in N.$$

Lemma 1 [6]. Let the sequence $\{\mu(l)\}_{l=0}^{+\infty}$ such that $\mu(0) = 1$, $\frac{\mu(l+1)}{\mu(l)} \geq \alpha > 1$, $\forall l \in Z^+$. Then for number $q > 0$ and the sequence $\{a_k\}_{k=0}^{+\infty}$, $a_k \geq 0$, inequality holds

$$\sum_{l=0}^{+\infty} \mu^r(l) \left(\sum_{k=0}^{l} a_k\right)^q \leq c_1 \sum_{l=0}^{+\infty} \mu^r(l) a_l^q, r < 0,$$

where the constant $c_1 > 0$ depends only on the parameters α, r, q.

Statement 1 [7]. Let $D_n(x) = \sum\limits_{k=0}^{n-1}\varphi_k(x)$ is Price system's Dirichlet kernel and $1 < p < +\infty$, $1 < \theta < +\infty$. Then $c'_{p\theta} n^{1-\frac{1}{p}} \leq \|D_n\|_{p\theta} \leq c_{p\theta} n^{1-\frac{1}{p}}$, $\forall n \in N$, where $const(c'_{p\theta}, c_{p\theta}) > 0$ does not depend on the $n \in N$.

Let's present the main results of this work.

Statement 2. For $D_n^{(k)}(x)$, $k \geq 1$, $n \in N$, $x \in [0,1]$:

1. $|D_n^{(k)}(x)| \leq \frac{1}{k!} n^k$.

2.$\|D_n^{(k)}\|_{p\theta} \leq c_{p\theta} n^{k-\frac{1}{p}}$, $1 < p < +\infty$, $1 < \theta < +\infty$.

Proof. To the proof the statements we use the method of mathematical induction. Let's make sure that the first inequality is true. When $k = 1$ due to the properties of the system Price we have

$$\left|D_n^{(1)}(x)\right| = \left|\sum_{\nu=0}^{n-1}\varphi_k(x)\right| \leq \sum_{\nu=0}^{n-1} \mid \varphi_k(x) \mid \leq \sum_{\nu=0}^{n-1} 1 = n.$$

Now we show the validity for $k = 2$

$$\left|D_n^{(2)}(x)\right| \leq \sum_{\nu=0}^{n-1} \mid D_\nu^{(1)}(x) \mid \leq \sum_{\nu=0}^{n-1}\sum_{s=0}^{\nu} \mid \varphi_s(x) \mid \leq \sum_{\nu=0}^{n-1} \nu \leq \frac{1}{2}n^2.$$

When $k=3$ by applying the previous inequality we obtain that

$$\left|D_n^{(3)}(x)\right| \leq \sum_{\nu=0}^{n-1} \mid D_\nu^{(2)}(x) \mid \leq \frac{1}{2}\sum_{\nu=0}^{n-1}\nu^2 = \frac{1}{2}\int\limits_0^n x^2 dx \leq \frac{1}{6}n^3.$$

Suppose that for $k-1$ is performed $\left|D_n^{(k-1)}(x)\right| \leq \frac{1}{(k-1)!}n^{k-1}$. Then

$$\left|D_n^{(k)}(x)\right| = \sum_{\nu=0}^{n-1} \mid D_\nu^{(k-1)}(x) \mid \leq \frac{1}{(k-1)!}\sum_{\nu=0}^{n-1}\nu^{k-1} = \frac{1}{(k-1)!}\int\limits_0^n x^{k-1} dx \leq \frac{1}{k!}n^k.$$

Now we show that correctness of the second inequality. When $k=1$ is obviously. Let $k=2$. Using the proposition 1 we can to prove the relation easily

$$\left\|D_n^{(2)}\right\|_{p\theta} = \left\|\sum_{\nu=0}^{n-1} D_\nu^{(1)}\right\|_{p\theta} \leq \sum_{\nu=0}^{n-1}\left\|D_\nu^{(1)}\right\|_{p\theta} = \sum_{\nu=0}^{n-1}\|D_\nu\|_{p\theta} \leq c_{p\theta}\sum_{\nu=0}^{n-1}\nu^{1-\frac{1}{p}} \leq c'_{p\theta}n^{2-\frac{1}{p}}.$$

When $k=3$ by applying the previous inequality we obtain that

$$\left\|D_n^{(3)}\right\|_{p\theta} = \left\|\sum_{\nu=0}^{n-1} D_\nu^{(2)}\right\|_{p\theta} \leq \sum_{\nu=0}^{n-1}\left\|D_\nu^{(2)}\right\|_{p\theta} \leq c'_{p\theta}\sum_{\nu=0}^{n-1}\nu^{2-\frac{1}{p}} \leq c''_{p\theta}n^{3-\frac{1}{p}}.$$

Let's suppose that when $k-1$ is performed $\left\|D_n^{(k-1)}\right\|_{p\theta} \leq \tilde{c}_{p\theta}n^{k-1-\frac{1}{p}}$, $1<p<+\infty$, $1<\theta<+\infty$. Then

$$\left\|D_n^{(k)}\right\|_{p\theta} = \left\|\sum_{\nu=0}^{n-1} D_\nu^{(k-1)}\right\|_{p\theta} \leq \sum_{\nu=0}^{n-1}\left\|D_\nu^{(k-1)}\right\|_{p\theta} \leq \tilde{c}_{p\theta}\sum_{\nu=0}^{n-1}\nu^{k-1-\frac{1}{p}} \leq \bar{c}_{p\theta}n^{k-\frac{1}{p}}.$$

The statement is proved.

Lemma 2. Let $\{a_\nu\}_{\nu=0}^{+\infty}$ the sequence of positive numbers and $\Delta_{k-1}a_\nu = \sum\limits_{j=0}^{k-1}(-1)^j C_{k-1}^j a_{\nu+j}$, where $C_{k-1}^j = \frac{(k-1)!}{j!(k-1-j)!}$. Then the Fourier-Price series $\sum\limits_{\nu=0}^{+\infty} a_\nu\varphi_\nu(x)$ of the function $f \in L[0,1]$ can be represented as $\sum\limits_{\nu=0}^{+\infty}\Delta_{k-1}a_\nu D_{\nu+1}^{(k-1)}(x), \forall k \in N$.

If for each element of the sequence $\{a_\nu\}_{\nu=0}^{+\infty}$ satisfies the conditions $a_\nu > 0$ and $a_\nu \downarrow 0$, $\nu \to +\infty$ then also $\Delta_k a_\nu \downarrow 0$, $\nu \to +\infty$ for $\forall k \in Z^+$. We could see it easily in the following

$$\lim_{\nu\to+\infty}\Delta_k a_\nu = \lim_{\nu\to+\infty}\sum_{j=0}^{k}(-1)^j C_k^j a_{\nu+j} = \sum_{j=0}^{k}(-1)^j C_k^j \lim_{\nu\to+\infty} a_{\nu+j} = 0.$$

Theorem 1. Let the sequence of Fourier-Price coefficients of the function $f(x)$ by the multiplicative Price system $\{a_\nu\}_{\nu=0}^{+\infty}$ is monotonically tends to zero and $\Delta_k a_\nu \geq 0$ for some natural number $k \geq 1$ and $\forall \nu \in Z^+$. Besides of it the forming sequence of the Price system $\{p_i\}_{i=1}^{+\infty}$ is limited, i.e $\exists c \in N$: $p_i \leq c$, $p_i \geq 2$, $i \geq 1$, $m_0 = 1, m_\mu = \prod\limits_{i=1}^{\mu} p_i$ and $f \approx \sum\limits_{\nu=0}^{+\infty} a_\nu\varphi_\nu(x)$. If the series

$$\sum_{\mu=0}^{+\infty} m_{\mu+1}^{k\theta-\frac{\theta}{p}}\left(\Delta_{k-1}a_{m_\mu}\right)^\theta, \Delta_0 a_{m_\mu} = a_{m_\mu},$$

converges, then the function $f \in L_{p\theta}[0,1]$, $1<p<+\infty$, $1<\theta<+\infty$.

Proof. Let the series $\sum\limits_{\mu=0}^{+\infty} m_{\mu+1}^{k\theta-\frac{\theta}{p}} \left(\Delta_{k-1}a_{m_\mu}\right)^\theta$ converges. Suppose $m_0=1, m_\mu=\prod\limits_{k=1}^{\mu} p_k$.

$$\frac{1}{t} \sup_{E\subset[0,1],|E|=\left[\frac{1}{m_{\mu+1}},\frac{1}{m_\mu}\right]} \int\limits_E \mid f(x) \mid dx = \sup_{E\subset[0,1],|E|=\left[\frac{1}{m_{\mu+1}},\frac{1}{m_\mu}\right]} \int\limits_E \left| \sum_{n=0}^{+\infty} \Delta_{k-1}a_n D_{n+1}^{(k-1)}(x) \right| dx \le$$

$$\le \sup_{E\subset[0,1],|E|=\left[\frac{1}{m_{\mu+1}},\frac{1}{m_\mu}\right]} \int\limits_E \sum_{n=0}^{+\infty} \Delta_{k-1}a_n \left| D_{n+1}^{(k-1)}(x) \right| dx =$$

$$= \sup_{E\subset[0,1],|E|=\left[\frac{1}{m_{\mu+1}},\frac{1}{m_\mu}\right]} \int\limits_E \sum_{n=0}^{m_{\mu+1}-1} \Delta_{k-1}a_n \left| D_{n+1}^{(k-1)}(x) \right| dx +$$

$$+ \sup_{E\subset[0,1],|E|=\left[\frac{1}{m_{\mu+1}},\frac{1}{m_\mu}\right]} \int\limits_E \sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1}a_n \left| D_{n+1}^{(k-1)}(x) \right| dx = I_1 + I_2.$$

Applying the proposition 2 we obtain that

$$I_1 = \sup_{E\subset[0,1],|E|=\left[\frac{1}{m_{\mu+1}},\frac{1}{m_\mu}\right]} \int\limits_E \sum_{n=0}^{m_{\mu+1}-1} \Delta_{k-1}a_n \left| D_{n+1}^{(k-1)}(x) \right| dx \le$$

$$\le \frac{1}{k!} \sum_{n=0}^{m_{\mu+1}-1} \Delta_{k-1}a_n (n+1)^{k-1} \left[\frac{1}{m_\mu} - \frac{1}{m_{\mu+1}} \right] \le$$

$$\le \frac{1}{k!} \frac{p_{\mu+1}-1}{m_{\mu+1}} \sum_{n=0}^{m_{\mu+1}-1} \Delta_{k-1}a_n (n+1)^{k-1} \le c_k m_{\mu+1}^{-1} \sum_{n=0}^{m_{\mu+1}-1} \Delta_{k-1}a_n (n+1)^{k-1}. \tag{1}$$

$$I_2 = \sup_{E\subset[0,1],|E|=\left[\frac{1}{m_{\mu+1}},\frac{1}{m_\mu}\right]} \int\limits_E \sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1}a_n \left| D_{n+1}^{(k-1)}(x) \right| dx =$$

$$= \sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1}a_n \sup_{E\subset[0,1],|E|=\left[\frac{1}{m_{\mu+1}},\frac{1}{m_\mu}\right]} \int\limits_E \left| D_{n+1}^{(k-1)}(x) \right| dx =$$

$$= \sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1}a_n \int\limits_{\frac{1}{m_{\mu+1}}}^{\frac{1}{m_\mu}} x^{\frac{1}{p}-\frac{1}{\theta}} \left[D_{n+1}^{(k-1)}(x) \right]^* x^{\frac{1}{p'}-\frac{1}{\theta'}} dx \le$$

$$\le \sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1}a_n \left\{ \int\limits_0^1 x^{\frac{\theta}{p}-1} \left(\left[D_{n+1}^{(k-1)}(x) \right]^* \right)^\theta dx \right\}^{\frac{1}{\theta}} \left\{ \int\limits_{\frac{1}{m_{\mu+1}}}^{\frac{1}{m_\mu}} x^{\frac{\theta'}{p'}-1} dx \right\}^{\frac{1}{\theta'}} =$$

$$= \sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1} a_n \left\| D_{n+1}^{(k-1)} \right\|_{p\theta} \left\{ \frac{p'}{\theta'} \, x^{\frac{\theta'}{p'}} \Big|_{\frac{1}{m_{\mu+1}}}^{\frac{1}{m_\mu}} \right\}^{\frac{1}{\theta'}} =$$

$$= \left\{ \frac{p'}{\theta'} \right\}^{\frac{1}{\theta'}} \sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1} a_n \left\| D_{n+1}^{(k-1)} \right\|_{p\theta} \left\{ \left(\frac{1}{m_\mu} \right)^{\frac{\theta'}{p'}} - \left(\frac{1}{m_{\mu+1}} \right)^{\frac{\theta'}{p'}} \right\}^{\frac{1}{\theta'}} =$$

$$= \left\{ \frac{p'}{\theta'} \right\}^{\frac{1}{\theta'}} \left(p_{\mu+1}^{\frac{\theta'}{p'}} - 1 \right)^{\frac{1}{\theta'}} \sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1} a_n \left\| D_{n+1}^{(k-1)} \right\|_{p\theta} m_{\mu+1}^{-\frac{1}{p'}} \leq$$

$$\leq c'_{p\theta} m_{\mu+1}^{\frac{1}{p}-1} \sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1} a_n (n+1)^{k-\frac{1}{p}-1}. \qquad (2)$$

Considering $\exists c \in N : p_k \leq c$, $p_k \geq 2$, $k \geq 1$, inequalities (1), (2) and Lemma 2 we get

$$\|f\|_{p\theta}^{\theta} = \frac{\theta}{p} \int_0^1 t^{\frac{\theta}{p}-1} [f^*(t)]^{\theta} dt = \frac{\theta}{p} \sum_{\mu=0}^{+\infty} \int_{\frac{1}{m_{\mu+1}}}^{\frac{1}{m_\mu}} t^{\frac{\theta}{p}-1} [f^*(t)]^{\theta} dt =$$

$$= \frac{\theta}{p} \sum_{\mu=0}^{+\infty} \int_{\frac{1}{m_{\mu+1}}}^{\frac{1}{m_\mu}} t^{\frac{\theta}{p}-1} \left(\frac{1}{t} \sup_{E \subset [0,1], |E| = \left[\frac{1}{m_{\mu+1}}, \frac{1}{m_\mu}\right]} \int_E | \, f(x) \, | \, dx \right)^{\theta} dt =$$

$$= \frac{\theta}{p} \sum_{\mu=0}^{+\infty} \int_{\frac{1}{m_{\mu+1}}}^{\frac{1}{m_\mu}} t^{\frac{\theta}{p}-\theta-1} \left(\sup_{E \subset [0,1], |E| = \left[\frac{1}{m_{\mu+1}}, \frac{1}{m_\mu}\right]} \int_E \left| \sum_{n=0}^{+\infty} \Delta_{k-1} a_n D_{n+1}^{(k-1)}(x) \right| dx \right)^{\theta} dt \leq$$

$$\leq \frac{\theta}{p} \sum_{\mu=0}^{+\infty} \int_{\frac{1}{m_{\mu+1}}}^{\frac{1}{m_\mu}} t^{\frac{\theta}{p}-\theta-1} \left(\sup_{E \subset [0,1], |E| = \left[\frac{1}{m_{\mu+1}}, \frac{1}{m_\mu}\right]} \int_E \sum_{n=0}^{+\infty} \Delta_{k-1} a_n \left| D_{n+1}^{(k-1)}(x) \right| dx \right)^{\theta} dt =$$

$$= \frac{\theta}{p} \sum_{\mu=0}^{+\infty} \int_{\frac{1}{m_{\mu+1}}}^{\frac{1}{m_\mu}} t^{\frac{\theta}{p}-\theta-1} \left(\sup_{E \subset [0,1], |E| = \left[\frac{1}{m_{\mu+1}}, \frac{1}{m_\mu}\right]} \int_E \sum_{n=0}^{m_{\mu+1}-1} \Delta_{k-1} a_n \left| D_{n+1}^{(k-1)}(x) \right| dx + \right.$$

$$\left. + \sup_{E \subset [0,1], |E| = \left[\frac{1}{m_{\mu+1}}, \frac{1}{m_\mu}\right]} \int_E \sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1} a_n \left| D_{n+1}^{(k-1)}(x) \right| dx \right)^{\theta} dt \leq$$

$$\leq c''_{p\theta k} \sum_{\mu=0}^{+\infty} \left(m_{\mu+1}^{-\theta} \left[\sum_{n=0}^{m_{\mu+1}-1} \Delta_{k-1} a_n (n+1)^{k-1} \right]^{\theta} + m_{\mu+1}^{\frac{\theta}{p}-\theta} \left[\sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1} a_n (n+1)^{k-\frac{1}{p}-1} \right]^{\theta} \right) \times$$

$$\times \int\limits_{\frac{1}{m_{\mu+1}}}^{\frac{1}{m_\mu}} t^{\frac{\theta}{p}-\theta-1}dt = c''_{p\theta k}\frac{p}{\theta(p-1)}\left[p_{\mu+1}^{\theta\left(1-\frac{1}{p}\right)}-1\right]=$$

$$=\sum_{\mu=0}^{+\infty}\left(m_{\mu+1}^{-\theta}\left[\sum_{n=0}^{m_{\mu+1}-1}\Delta_{k-1}a_n(n+1)^{k-1}\right]^\theta + m_{\mu+1}^{\frac{\theta}{p}-\theta}\left[\sum_{n=m_{\mu+1}}^{+\infty}\Delta_{k-1}a_n(n+1)^{k-\frac{1}{p}-1}\right]^\theta\right)m_\mu^{\theta\left(1-\frac{1}{p}\right)} \le$$

$$\le c'''_{p\theta k}\left\{\sum_{\mu=0}^{+\infty}m_{\mu+1}^{-\frac{\theta}{p}}\left[\sum_{n=0}^{m_{\mu+1}-1}\Delta_{k-1}a_n(n+1)^{k-1}\right]^\theta + \sum_{\mu=0}^{+\infty}\left[\sum_{n=m_{\mu+1}}^{+\infty}\Delta_{k-1}a_n(n+1)^{k-\frac{1}{p}-1}\right]^\theta\right\}.$$

Using the Lemma 1 we get the following

$$\sum_{\mu=0}^{+\infty}m_{\mu+1}^{-\frac{\theta}{p}}\left(\sum_{n=0}^{m_{\mu+1}-1}\Delta_{k-1}a_n(n+1)^{k-1}\right)^\theta = \sum_{\mu=0}^{+\infty}m_{\mu+1}^{-\frac{\theta}{p}}\left(\Delta_{k-1}a_0+\sum_{n=1}^{m_{\mu+1}-1}\Delta_{k-1}a_n(n+1)^{k-1}\right)^\theta=$$

$$=\sum_{\mu=0}^{+\infty}m_{\mu+1}^{-\frac{\theta}{p}}\left(\Delta_{k-1}a_0+\sum_{n=0}^{\mu}\sum_{j=m_n}^{m_{n+1}-1}\Delta_{k-1}a_j(j+1)^{k-1}\right)^\theta = (\Delta_{k-1}a_n\downarrow 0,\ \ n\to+\infty)\le$$

$$\le\sum_{\mu=0}^{+\infty}m_{\mu+1}^{-\frac{\theta}{p}}\left(\Delta_{k-1}a_0+\sum_{n=0}^{\mu}\Delta_{k-1}a_{m_n}\sum_{j=m_n}^{m_{n+1}-1}(j+1)^{k-1}\right)^\theta\le$$

$$\le\sum_{\mu=0}^{+\infty}m_{\mu+1}^{-\frac{\theta}{p}}\left(\Delta_{k-1}a_0+\sum_{n=0}^{\mu}\Delta_{k-1}a_{m_n}\int\limits_{m_n}^{m_{n+1}}(x+1)^{k-1}dx\right)^\theta=$$

$$=\sum_{\mu=0}^{+\infty}m_{\mu+1}^{-\frac{\theta}{p}}\left(\Delta_{k-1}a_0+\sum_{n=0}^{\mu}\Delta_{k-1}a_{m_n}\frac{(x+1)^{k-1}}{k-1}\Big|_{m_n}^{m_{n+1}}\right)^\theta=$$

$$=\sum_{\mu=0}^{+\infty}m_{\mu+1}^{-\frac{\theta}{p}}\left(\Delta_{k-1}a_0+\frac{1}{k-1}\sum_{n=0}^{\mu}\Delta_{k-1}a_{m_n}\left[(m_{n+1}+1)^k-m_n^k\right]\right)^\theta\le$$

$$\le\sum_{\mu=0}^{+\infty}m_{\mu+1}^{-\frac{\theta}{p}}\left(\Delta_{k-1}a_0+c_k\sum_{n=0}^{\mu}\Delta_{k-1}a_{m_n}m_{n+1}^k\right)^\theta\le c'_k2^{\theta-1}\sum_{\mu=0}^{+\infty}m_{\mu+1}^{-\frac{\theta}{p}+k\theta}\left(\Delta_{k-1}a_{m_\mu}\right)^\theta.$$

Applying Holder's inequality, when $\frac{1}{\theta}+\frac{1}{\theta'}=1$ and

$$\sum_{n=m_{\mu+1}}^{+\infty}(n+1)^{\frac{\theta'}{\theta}-\theta'}\le\int\limits_{m_{\mu+1}-1}^{+\infty}(x+1)^{-\theta'\left(1-\frac{1}{\theta}\right)}d(x+1)=c_\theta m_{\mu+1}^{-\theta'\left(1-\frac{1}{\theta}\right)};$$

$$\sum_{n=m_{\mu+1}}^{+\infty}\left(\Delta_{k-1}a_n\right)^\theta(n+1)^{k\theta-\frac{\theta}{p}-1}=\sum_{n=\mu}^{+\infty}\sum_{j=m_{n+1}}^{m_{n+2}-1}\left(\Delta_{k-1}a_j\right)^\theta(j+1)^{k\theta-\frac{\theta}{p}-1}\le$$

$$\leq \sum_{n=\mu}^{+\infty} \left(\Delta_{k-1} a_{m_{n+1}}\right)^{\theta} \sum_{j=m_{n+1}}^{m_{n+2}-1} (j+1)^{k\theta-\frac{\theta}{p}-1} \leq$$

$$\leq \sum_{n=\mu}^{+\infty} \left(\Delta_{k-1} a_{m_{n+1}}\right)^{\theta} \int\limits_{m_{n+1}}^{m_{n+2}} (x+1)^{k\theta-\frac{\theta}{p}-1} d(x+1) \leq c_{k\theta p} \sum_{n=\mu}^{+\infty} \left(\Delta_{k-1} a_{m_{n+1}}\right)^{\theta} m_{n+1}^{k\theta-\frac{\theta}{p}}$$

we obtain that

$$\sum_{\mu=0}^{+\infty} \left(\sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1} a_n (n+1)^{k-\frac{1}{p}-1} \right)^{\theta} = \sum_{\mu=0}^{+\infty} \left(\sum_{n=m_{\mu+1}}^{+\infty} \Delta_{k-1} a_n (n+1)^{k-\frac{1}{p}-\frac{1}{\theta}+\frac{1}{\theta}-1} \right)^{\theta} \leq$$

$$\leq \sum_{\mu=0}^{+\infty} \left(\sum_{n=m_{\mu+1}}^{+\infty} \left(\Delta_{k-1} a_n\right)^{\theta} (n+1)^{k\theta-\frac{\theta}{p}-1} \right)^{\theta\frac{1}{\theta}} \left(\sum_{n=m_{\mu+1}}^{+\infty} (n+1)^{\frac{\theta'}{\theta}-\theta'} \right)^{\frac{1}{\theta'}\theta} \leq$$

$$\leq \sum_{\mu=0}^{+\infty} c_{\theta} m_{\mu+1}^{-\theta'\left(1-\frac{1}{\theta}\right)\frac{\theta}{\theta'}} \sum_{n=m_{\mu+1}}^{+\infty} \left(\Delta_{k-1} a_n\right)^{\theta} (n+1)^{k\theta-\frac{\theta}{p}-1} =$$

$$= \sum_{\mu=0}^{+\infty} c_{\theta} m_{\mu+1}^{-(\theta-1)} \sum_{n=m_{\mu+1}}^{+\infty} \left(\Delta_{k-1} a_n\right)^{\theta} (n+1)^{k\theta-\frac{\theta}{p}-1} \leq c_{\theta k p} \sum_{\mu=0}^{+\infty} m_{\mu+1}^{-(\theta-1)} \sum_{n=\mu}^{+\infty} \left(\Delta_{k-1} a_{m_{n+1}}\right)^{\theta} m_{n+1}^{k\theta-\frac{\theta}{p}} =$$

$$= c_{\theta k p} \sum_{n=0}^{+\infty} \left(\Delta_{k-1} a_{m_{n+1}}\right)^{\theta} m_{n+1}^{k\theta-\frac{\theta}{p}} \sum_{\mu=0}^{n} m_{\mu+1}^{-(\theta-1)} \leq (\theta > 1) \leq \overline{c}_{\theta k p} \sum_{n=0}^{+\infty} \left(\Delta_{k-1} a_{m_{n+1}}\right)^{\theta} m_{n+1}^{k\theta-\frac{\theta}{p}}.$$

So considering the ratio $\Delta_{k-1} a_{m_{\mu+1}} \leq \Delta_{k-1} a_{m_{\mu}}, k \in N$ we finally obtain

$$\|f\|_{p\theta} \leq c'''_{p\theta k} \left\{ 2^{\theta-1} \sum_{\mu=0}^{+\infty} m_{\mu+1}^{-\frac{\theta}{p}+k\theta} \left(\Delta_{k-1} a_{m_{\mu}}\right)^{\theta} + \overline{c}_{\theta k p} \sum_{n=0}^{+\infty} \left(\Delta_{k-1} a_{m_{n+1}}\right)^{\theta} m_{n+1}^{k\theta-\frac{\theta}{p}} \right\}^{\frac{1}{\theta}} \leq$$

$$\leq C_{pk\theta} \left\{ \sum_{\mu=0}^{+\infty} m_{\mu+1}^{-\frac{\theta}{p}+k\theta} \left(\Delta_{k-1} a_{m_{\mu}}\right)^{\theta} \right\}^{\frac{1}{\theta}}.$$

Hence if the series $\sum\limits_{\mu=0}^{+\infty} m_{\mu+1}^{-\frac{\theta}{p}+k\theta} \left(\Delta_{k-1} a_{m_{\mu}}\right)^{\theta}$ converges, then the function $f \in L_{p\theta}[0,1], 1 < p < +\infty$, $1 < \theta < +\infty$. The theorem is proved.

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А.Ө. Бимендина

Лоренц кеңістігінде енгізудің жеткілікті шарты

Мақалада Фурье-Прайс коэффициенттерінің k-ші ретті айырмалары тұрғысында Фурье-Прайс қатарының түрлендіруі қарастырылды. Сонымен қатар f функциясы үшін Фурье-Прайс қатарының коэффициенттері монотонды, $\Delta_k a_\nu \geq 0$ және Прайс жүйесінің құрушы тізбегі $p_i, i = 1, 2, ...$ ақырлы, яғни, $p_i \leq c$, $p_i \geq 2$, $i \in N$ болған жағдайда f функциясының екі параметрлі Лоренц кеңістігіне енгізілуінің жеткілікті шарты талқыланды.

А.У. Бимендина

Достаточное условие вложения в пространство Лоренца

В статье рассмотрены преобразования ряда Фурье-Прайса в виде k-ой разности коэффициентов Фурье-Прайса. Кроме того, установлено достаточное условие принадлежности функции f в двухпараметрическое пространство Лоренца в терминах коэффициентов Фурье-Прайса. При этом коэффициенты ряда Фурье-Прайса для функции f монотонны, $\Delta_k a_\nu \geq 0$ и образующая последовательность системы Прайса $p_i, i = 1, 2, ...$, ограничена, то есть $p_i \leq c$, $p_i \geq 2$, $i \in N$.

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