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Bipartite Digraphs with Modular Concept Lattices of height 2

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This paper investigates the interaction between Formal Concept Analysis (FCA) and graph theory, with a focus on understanding the structure and representation of concept lattices derived from bipartite directed graphs. We establish connections between the complete formal contexts and their associated bipartite digraphs, providing a foundation for studying modular lattices. Particular attention is given to the structure of concept lattices arising from such contexts and their relationship to the combinatorial properties of the corresponding graphs. The results show that the concept lattice of a complete formal context is isomorphic to a modular lattice of height 2 if and only if its associated bipartite digraph is a disconnected union of bicliques. This establishes a precise correspondence between a specific class of formal contexts and well-studied objects in graph theory. Several examples are presented to illustrate these properties and demonstrate the application of the obtained results. The analysis opens the way for further exploration of lattices associated with more complex graph structures and contributes to a deeper understanding of the relationship between discrete mathematics and formal methods of knowledge representation.

Keywords: formal context, full context, formal concept, concept lattice, context graph, bipartite digraph, biclique, modular lattice.

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Introduction

Formal Concept Analysis (FCA) is a powerful mathematical framework for data analysis and knowledge representation, based on the duality between objects and attributes within a formal context. FCA was introduced in the early 1980s by Rudolf Wille as a mathematical theory [1, 2]. This framework provides a systematic method for deriving concept lattices, which capture hierarchical relationships between object-attribute pairs. These lattices have applications spanning fields such as data mining, machine learning, and ontology engineering [3, 4].

Graph theory [5], on the other hand, offers a complementary perspective by modelling relationships as vertices and edges. The interplay between FCA and graph theory has been a subject of growing interest, particularly in the study of bipartite graphs. In FCA, the incidence relation of a formal context corresponds naturally to a bipartite graph, establishing a direct link between these domains.

This paper investigates the structural properties of concept lattices derived from bipartite graphs, with an emphasis on modular lattices. By characterizing the graph-theoretic properties of bipartite digraphs corresponding to such lattices, we aim to deepen the understanding of their formation and representation.

The main contributions of this work are as follows:

1) We introduce and formalize the notion of full formal contexts, which simplify the study of concept lattices by reducing redundancy in object-attribute relations.

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2) We establish a bijective correspondence between full formal contexts and bipartite digraphs, showing that the concept lattice of a context is determined by its graph structure (Theorem 1).

3) We prove that the concept lattice of a full formal context is isomorphic to a modular lattice of height 2 if and only if the associated bipartite graph is a disjoint union of complete bipartite graphs (Theorem 2).

4) We provide examples, including the graph of a function and its context lattice, to demonstrate the practical implications of our results.

For more information on the basic notions and results of FCA, lattice theory and graph theory introduced below, and used throughout this paper, we refer the reader to [2, 5, 6].

1 Preliminaries

First, we provide the main definitions.

Definition 1. A *graph* is an algebraic structure $G = (V, E)$ where E is a binary relation on V. The set V is called a set of *vertices* (or *nodes*), and $E \subseteq V \times V$ is a set of *edges*. A graph is called *undirected* if $(a, b) \in E$ then $(b, a) \in E$, and it is called *directed* or a *digraph* if $(a, b) \in E$ then $(b, a) \notin E$.

Definition 2. A digraph $G = (V, E)$ is called *bipartite* if its vertex set V can be partitioned into two disjoint subsets V_1 and V_2 such that:

- Every edge $e \in E$ connects a vertex in V_1 to a vertex in V_2.
- No edge exists between two vertices of the same subset.

A *complete* bipartite digraph (biclique) $G = (V, E)$ is a bipartite digraph in which the vertex set V can be partitioned into two disjoint subsets V_1 and V_2 such that every vertex in V_1 is connected to every vertex in V_2 and there are no edges within V_1 or within V_2. Remark. Usually, a biclique is a complete bipartite undirected graph (see [5]).

Definition 3. A *formal context* $\mathbb{K} = (G, M, I)$ consists of the set of objects G, the set of attributes M, and the incidence relation $I \subseteq G \times M$.

For a formal context $\mathbb{K} = (G, M, I)$ and $A \subseteq G$, $B \subseteq M$ we put $\alpha_{\mathbb{K}}(\varnothing) = M$, $\beta_{\mathbb{K}}(\varnothing) = G$ and

$$\alpha_{\mathbb{K}}(A) = \{m \in M \mid (\forall g \in A)\,[\,(g, m) \in I\,]\},$$

$$\beta_{\mathbb{K}}(B) = \{g \in G \mid (\forall m \in B)\,[\,(g, m) \in I\,]\}.$$

The mappings $\beta_{\mathbb{K}} \circ \alpha_{\mathbb{K}} : \mathcal{P}(G) \to \mathcal{P}(G)$ and $\alpha_{\mathbb{K}} \circ \beta_{\mathbb{K}} : \mathcal{P}(M) \to \mathcal{P}(M)$ are closure operators. The set $\mathcal{L}_{\mathbb{K}}(G)$ ($\mathcal{L}_{\mathbb{K}}(M)$) of the closed subsets of G (M) with respect to $\beta_{\mathbb{K}} \circ \alpha_{\mathbb{K}}$ ($\alpha_{\mathbb{K}} \circ \beta_{\mathbb{K}}$) forms a lattice under inclusion $\subseteq$ (conclusion $\supseteq$). And $\mathcal{L}_{\mathbb{K}}(G)$ is dually isomorphic to $\mathcal{L}_{\mathbb{K}}(M)$.

If $\mathbb{K}$ is clear from the discussion then we omit the subscript $\mathbb{K}$ — e.g., for example, we write $\alpha(A)$ instead of $\alpha_{\mathbb{K}}(A)$.

Definition 4. A *formal concept* of the context $\mathbb{K}$ is a pair (A, B) such that $A \subseteq G$, $B \subseteq M$, $B = \alpha_{\mathbb{K}}(A)$, and $A = \beta_{\mathbb{K}}(B)$. For a formal concept $\Delta = (A, B)$, A is called the *extent* of Δ, and B is the *intent* of Δ.

The ordering $\preceq$ of the concepts of $\mathbb{K}$ is defined as follows:

$$(A_0, B_0) \preceq (A_1, B_1) \;\Leftrightarrow\; A_0 \subseteq A_1 \;\Leftrightarrow\; B_0 \supseteq B_1.$$

The Basic Theorem on Concept Lattices (see [1]) establishes that ordering $\preceq$ on the set of all concepts of $\mathbb{K}$ induces a complete lattice which is called the *concept lattice* of $\mathbb{K}$, and we denote it by $\mathcal{L}(\mathbb{K})$.

From the definition of the partial order $\preceq$, one can see that for a formal context $\mathbb{K} = (G, M, I)$ the mapping $\varphi : \mathcal{L}(\mathbb{K}) \to \mathcal{L}_{\mathbb{K}}(G)$ defined by $\varphi((A, B)) = A$, establishes an isomorphism between $\mathcal{L}(\mathbb{K})$ and $\mathcal{L}_{\mathbb{K}}(G)$.

For the sets A, B and a binary relation $R \subseteq A \times B$, we put

$$\pi_A(R) = \{a \in A \mid \exists b\, [\, (a,b) \in R\,]\}, \quad \pi_B(R) = \{b \in B \mid \exists a\, [\, (a,b) \in R\,]\}.$$

A formal context $\mathbb{K} = (A, B, I)$ is called *full* if $\pi_A(I) = A$, $\pi_B(I) = B$ and $\alpha_{\mathbb{K}}(A) = \beta_{\mathbb{K}}(B) = \varnothing$.

For a formal context $\mathbb{K} = (A, B, I)$ we define the graph $\mathbf{G}_K = (A \cup B; I)$ that consists of the set of vertices $A \cup B$ and the set of edges $I \subseteq A \times B$. The graph $\mathbf{G}_K = (A \cup B; I)$ is called *a context graph* if $A \cap B = \varnothing$. Such a graph we call a *context graph.* It is easy to see that $\mathbf{G}_K$ is a bipartite digraph. We also note that any bipartite digraph $\mathbf{G} = (A \cup B; I)$ with $I \subseteq A \times B$ defines the formal context $\mathbb{K}_G = (A, B, I)$. Similar constructions occur in many papers (see for example [7, 8]).

For any graph $\mathbf{G} = (G, R)$ we define the formal context $\mathbb{K}_G = (G, G, R)$ and the concept lattice $\mathcal{L}(\mathbb{K}_G)$, respectively.

The next theorem, as the reviewer noted: "Theorem 1 is a simple observation which, seemingly, is a "folklore" assertion. For example, in [7], the definition of a formal context is followed by the remark that "The correspondence to a bipartite graph (network) is at hand", brief description of this correspondence, and the conclusion that "In the following we use the terms network, (bipartite) graph, and formal context interchangeably in the sense above". However, I have not found a published formal proof of the assertion". For convenience we provide the formal proof.

Theorem 1. Let $\mathbb{K} = (A, B, I)$ be a full formal context in which $A \cap B = \varnothing$. And let $\mathbf{G}$ be the corresponding context graph $(A \cup B; I)$. Then $\mathcal{L}(\mathbb{K}) \cong \mathcal{L}(\mathbb{K}_G)$.

Proof. By definition, $\mathbb{K}_G = \{A \cup B, A \cup B, I\}$ and

$$\alpha_{\mathbb{K}_G}(X) = \{m \in A \cup B \mid (\forall g \in X)\, [\, (g, m) \in I\,]\},$$

$$\beta_{\mathbb{K}_G}(Y) = \{g \in A \cup B \mid (\forall m \in Y)\, [\, (g, m) \in I\,]\}.$$

By $\alpha_{\mathbb{K}}(\pi_A(I)) = \varnothing$ and $\beta_{\mathbb{K}}(\pi_B(I)) = \varnothing$, one can see that

$$\alpha_{\mathbb{K}_G}(X) = \begin{cases} \alpha_{\mathbb{K}}(X), & \text{if} \quad X \subseteq A, \\ \varnothing, & \text{otherwise}, \end{cases}$$

$$\beta_{\mathbb{K}_G}(Y) = \begin{cases} \beta_{\mathbb{K}}(Y), & \text{if} \quad Y \subseteq B, \\ \varnothing, & \text{otherwise}. \end{cases}$$

Therefore,

$$\beta_{\mathbb{K}_{\mathbb{G}}} \circ \alpha_{\mathbb{K}_G}(X) = \begin{cases} \beta_{\mathbb{K}} \circ \alpha_{\mathbb{K}}(X), & \text{if} \quad \alpha_{\mathbb{K}_G}(X) \neq \varnothing, \\ A \cup B, & \text{otherwise}, \end{cases}$$

$$\alpha_{\mathbb{K}_{\mathbb{G}}} \circ \beta_{\mathbb{K}_G}(Y) = \begin{cases} \alpha_{\mathbb{K}} \circ \beta_{\mathbb{K}}(Y), & \text{if} \quad \beta_{\mathbb{K}_G}(Y) \neq \varnothing, \\ A \cup B, & \text{otherwise}. \end{cases}$$

It means that a pair (X, Y) is a concept of $\mathbb{K}_G$ if and only if (X, Y) is a concept of $\mathbb{K}$ for all $X, Y \neq \varnothing$. Also $(A \cup B, \varnothing)$ and $(\varnothing, A \cup B)$ are the concepts of $\mathbb{K}_G$. Since the context $\mathbb{K}$ is full, $(A, \varnothing)$ and $(\varnothing, B)$ are the concepts of $\mathbb{K}$.

Hence the mapping $\varphi\colon \mathcal{L}(\mathbb{K}_G) \to \mathcal{L}(\mathbb{K})$, defined by

$$\varphi((X, Y)) = \begin{cases} (X, Y), & X, Y \neq \varnothing, \\ (A, \varnothing), & Y = \varnothing, \\ (\varnothing, B), & X = \varnothing, \end{cases}$$

is one to one and onto. It is easy to see that φ preserves partial order $\preceq$. Therefore, φ is an isomorphism.

These allow us to study the concept lattices through the bipartite digraphs. We demonstrate this approach in the next section.

2 *Representation of M_n*

For any $n > 2$, by M_n (M_ω) we denote a modular lattice of height 2 with n (ω) atoms.

Theorem 2. Let $\mathbb{K} = (A, B, I)$ be a full formal context in which $A \cap B = \varnothing$. Then the concept lattice $\mathcal{L}(\mathbb{K})$ is isomorphic to M_n for some $n \leq \omega$ if and only if the context graph $\mathbf{G}_K = (A \cup B; I)$ is a disjoint union of n complete bipartite digraphs.

Proof. $\Rightarrow$ By $\mathcal{L}(\mathbb{K}) \cong \mathcal{L}(A)$, we have $\mathcal{L}(A) \cong M_n$. Since $\beta(B) = \varnothing$, $\varnothing = 0_L$ is the least element of $\mathcal{L}(A)$. By $\alpha(A) = \varnothing$ and $\beta(\varnothing) = A \cup B$, we get that $A \cup B = 1_L$ is the greatest element of $\mathcal{L}(A)$. Let S be the set of all non-empty proper closed subsets of A. Since $\mathcal{L}(A) \cong M_n$, $A_0 \cap A_1 = \varnothing$ and $A_0 \vee A_1 = A \cup B$ for any $A_0, A_1 \in S$ with $A_0 \neq A_1$. Since $\pi_A(I) = A$, $\cup\{C \mid C \in S\} = A$. Hence S is a partition of A.

Let $\alpha(S) = \{\alpha(C) \mid C \in S\}$. By definition, $\alpha(C)$ is a closed subset of B. Since $\beta(B) = \varnothing$, $\beta(\varnothing) = A$, as $\mathbb{K}$ is full, and $\beta\alpha(C) = C$, then $\alpha(C)$ is non-empty proper subset of B for all $C \in S$, as well as $\alpha(C_0) \neq \alpha(C_1)$ for all $C_0, C_1 \in S$ and $C_0 \neq C_1$. Since $\mathcal{L}(B)$ is dual isomorphic to $\mathcal{L}(A)$, $\mathcal{L}(B) \cong M_n$. Let $D = \alpha(C_0) \cap \alpha(C_1)$, $C_0 \neq C_1$. Then, by definition, $\beta(D) \supset C_0$ and is a closed subset of A. Since the height of $\mathcal{L}(A)$ is equal to 2, $\beta(D) = A$. It implies $D = \varnothing$, that is, $\alpha(C_0) \cap \alpha(C_1) = \varnothing$. Thus, $\alpha(C_0) \cap \alpha(C_1) = \varnothing$ for all $C_0, C_1 \in S$, $C_0 \neq C_1$.

Let $D = B \backslash \cup \{\alpha(C) \mid C \in S\}$. By definition of D, $\beta(D) \notin S$. Also $\beta(D) \neq \varnothing$ because in this case $\cup\{\alpha(C) \mid C \in S\}$ is empty. Thus $\beta(D) = A$. It implies D is empty. Hence $B = \cup\{\alpha(C) \mid C \in S\}$. Thus, we establish that $\{\alpha(C) \mid C \in S\}$ is a partition of the set B.

Now we need to show that $\cup\{C \times \alpha(C) \mid C \in S\} = I$. First we note that $\alpha(c) = \alpha(C)$ for any $c \in C$. Indeed, assume that $\alpha(c) \supset \alpha(C)$ for some $c \in C$ and $C \in S$. Since $\alpha(c)$ is a closed subset in B and $\mathcal{L}(B) \cong M_n$ (because $\mathcal{L}(B)$ is dually isomorphic to $\mathcal{L}(A)$), $\alpha(c) = B$. Therefore, $c \in \beta(B)$. Since $\mathbb{K}$ is full, $\beta(B) = \varnothing$. Contradiction. Thus, $\alpha(c) = \alpha(C)$ for any $c \in C$. Hence $\cup\{C \times \alpha(C) \mid C \in S\} \subseteq I$. Let $(a, b) \in I$. Then $a \in \beta(b)$ whence $(a, b) \in C \times \alpha(C)$. Thus $\cup\{C \times \alpha(C) \mid C \in S\} = I$.

$\Leftarrow$ Since the graph $G_K = (A \cup B; I)$ is a disjoint union of n complete bipartite digraphs, $G_K = (\cup_{i \leq n} A_i, \cup_{i \leq n} B_i; \cup_{i \leq n} I_i)$ for some partitions $\{A_i \mid i \leq n\}$, $\{B_i \mid i \leq n\}$ of the sets A and B respectively, and $I = \cup_{i \leq n} I_i$ where $I_i = A_i \times B_i$.

The condition $I = \cup_{i \leq n} I_i = \cup_{i \leq n} A_i \times B_i$ give us that $\pi_A(I) = A$, $\pi_B(I) = B$ and the sets $\{b \in B \mid (a, b) \in I \text{ for all } a \in A\}$ and $\{a \in A \mid (a, b) \in I \text{ for all } b \in B\}$ are empty. These mean that $\alpha_{\mathbb{K}}(\pi_A(I)) = \varnothing$ and $\beta_{\mathbb{K}}(\pi_B(I)) = \varnothing$. Therefore, by Theorem 1, we get $\mathcal{L}(\mathbb{K}) \cong \mathcal{L}(\mathbb{K}_G)$. Thus we need to show that $\mathcal{L}(\mathbb{K}_G) \cong M_n$.

For the formal context $\mathbb{K}_G$ we have

$$\alpha_{\mathbb{K}_G}(X) = \begin{cases} B_i, & \text{if} \quad X \subseteq A_i, \\ \varnothing, & \text{otherwise,} \end{cases}$$

$$\beta_{\mathbb{K}_G}(X) = \begin{cases} A_i, & \text{if} \quad X \subseteq B_i, \\ \varnothing, & \text{otherwise.} \end{cases}$$

Thus, for any A_i and $X \supset A_i$, we have

$$\beta_{\mathbb{K}_{\mathbb{G}}} \circ \alpha_{\mathbb{K}_G}(A_i) = A_i,$$

$$\beta_{\mathbb{K}_{\mathbb{G}}} \circ \alpha_{\mathbb{K}_G}(X) = A \cup B.$$

That is, A_i, $1 \leq i \leq n$, and $A \cup B$ are the closed subsets in $A \cup B$ with respect to closure operator $\beta_{\mathbb{K}_{\mathbb{G}}} \circ \alpha_{\mathbb{K}_G}$. Therefore, $A_i \vee A_j = A \cup B$. Since $\{A_i \mid i \leq n\}$ forms a partition of A then $A_i \cap A_j = \varnothing$ for all $i \neq j \leq n$. It means that $\mathcal{L}(A) \cong M_n$. Hence $\mathcal{L}(\mathbb{K}_G) \cong M_n$ because $\mathcal{L}(\mathbb{K}_G) \cong \mathcal{L}(A)$.

Recall that a *bipartite dimension* of a graph is the minimum number of complete bipartite graphs whose union is the given graph. Thus

Corollary 1. Let $\mathbb{K}$ be a formal context and $\mathcal{L}(\mathbb{K}) \cong M_n$. Then the bipartite dimension of the graph $G_{\mathbb{K}}$ is equal to n.

3 Examples

Here we provide some examples.

Example 1. (cf. [9,10]) Let $f : A \to B$ be a function from A onto B and

$$gr(f) = \{(x, y) \mid f(x) = y \text{ for all } x \in A\ ,\ y \in B\}$$

the graph of function f. Consider a formal context $\mathbb{K} = (A, B, gr(f))$, where A represents objects, B represents attributes, and the incident relation is $gr(f)$. Then the concept $\mathbb{K} = (A, B, gr(f))$ satisfies Theorem 2. Hence $\mathcal{L}(\mathbb{K}) \cong M_{|B|}$ where $|B|$ is the size of B, and the bipartite dimension of the graph $G_{\mathbb{K}}$ is equal to $|B|$.

Indeed, let, for any $b \in B$,

$$A_b = \{x \in A \mid f(x) = b\} \subseteq A.$$

Since f is a function and maps A onto B,

$$A = \bigcup_{b \in B} A_b, \quad A_b \cap A_c = \emptyset,$$

for any $b, c \in B$, $b \neq c$. Moreover, $(A_b \cup \{b\}, gr(f_{|A_b}))$ forms a complete digraph (biclique) (Fig. 1). Thus, $\mathbb{K} = (A, B, gr(f))$ is a disjoint union of the bicliques $(A_b \cup \{b\}, gr(f_{|A_b}))$, $b \in B$. By Theorem 2, $\mathcal{L}(\mathbb{K}) \cong M_{|B|}$.

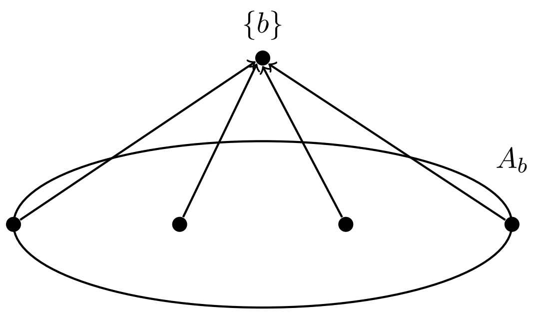

Figure 1. Bipartite digraph $(A_b \cup \{b\}, gr(f_{|A_b}))$

More general

Example 2. Let $f : A \to B$ be a many-valued function from A onto B and $gr(f) = \{(x, y) \mid y \in f(x) \text{ for all } x \in A\}$ the graph of the many-valued function f. And let the set of images of points of A forms a partition of B, that is the set of all proper subsets of B of the form $\{f(a) \subset B \mid a \in A\}$ is a partition of B. Then the concept $\mathbb{K} = (A, B, gr(f))$ satisfies Theorem 2. Hence $\mathcal{L}(\mathbb{K}) \cong M_n$, where n is the bipartite dimension of the graph $G_{\mathbb{K}}$.

Conclusion

In this paper, we explored the interplay between Formal Concept Analysis and graph theory, focusing on the structural representation of concept lattices through bipartite digraphs. The introduction of full formal contexts allowed us to establish a bijective correspondence between these contexts and bipartite digraphs, providing a framework for studying modular lattices. We demonstrated that the concept lattice of a full formal context is isomorphic to a modular lattice of height 2 if and only if its corresponding bipartite digraph is a disjoint union of complete bipartite graphs. This result not only advances the theoretical understanding of FCA but also provides practical tools for analyzing data structures in diverse applications. Future research may investigate the extension of these results to other types of lattices and exploring their computational implications.

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Author Contributions

All authors contributed equally to this work.

Conflict of Interest

The authors declare no conflict of interest.

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