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Model-theoretic properties of J-non-multidimensional theories

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The issues of utilizing the central type to analyze the theoretical and model properties of the idea of heredity were examined in this research, taking into account both theories and the Jonsson spectrum. Finding solutions to issues related to the enriching language for the fixed Jonsson theory is associated with the problems of heredity of Jonsson theory. Another feature of Jonsson theories was described in the presented article. That is, the conclusion concerning J-non-multidimensional theories was presented in this study. The connection between J-P-stable theories and J-non-multidimensional theories was also characterired. In addition, the main result in the article was considered for the class of semantic pairs.

Keywords: Jonsson theory, semantic model, perfect Jonsson theory, hereditary Jonsson theory, Jonsson spectrum, permissible enrichment, central type, existentially closed model, J-stable theory, semantic pairs, existentially finite cover property, J-non-multidimensional theory.

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Introduction

Although the Model Theory is the youngest science in terms of its development, it has spread its roots and is growing in all directions. It can be seen from the new concepts and findings that are being studied and defined year after year, and from the new methods of researching these findings. Model theorists use various limiting conditions to obtain results concerning incomplete theories. One of the relevant directions in this sense is studying of Jonsson theories. The number and variety of methodologies under development [1–7] indicates the tremendous expansion of the apparatus for analyzing Jonsson theories in recent years.

The difficulty of defining the notion of heredity in Jonsson theory remains unresolved. This problem's relevance is supported by the following significant counterexample: the elementary theory of an algebraically closed field loses its Jonsson character when it is enhanced with a unary predicate. Accordingly, one key model-theoretic challenge for characterizing the hereditary Jonsson theories is the study of model-theoretic features of central types in predicate enrichment.

We will further study the J-non-multidimensional theories. The study of non-multidimensional theories in general begins with the work of S. Shelah [8]. The theory T called non-multidimensional, if there is a bound to the size of families of pairwise orthogonal types. A. Pillay developed a classification of models for ω-stable non-multidimensional theories [9]. And T. Mustafin and T. Nurmagambetov obtained the main results of non-multidimensional theories for superstable theories [10].

The purpose of the article is to show the connection between J-non-multidimensional theories and J-P-stable theories. In general, the scope of study of P-stable theory is wide. For the first time, French mathematician B. Poizat began to study in [11], i.e. he found the conditions for the completeness of elementary pairs. E. Bouscaren [12, 13] further argued that a different class of stable

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theories should be used to address these questions. She showed that elementary pairs theories are complete for stable and superstable theories. In the work of D. Laskar [14] it is said that those theories should be uncountable categorical theories. And in work of T. Nurtazin [15] on the proof for the class of uncountable categorical theories. In [16] T. Mustafin introduces the concept of T^*-stability, which generalizes the well-known fact of λ-stability. In a separate case, the concept of P-stability related to the concept of elementary pairs is studied. In [10; 88–100] obtained a description of P-stability for any superstable theories. E^*-stable theories were introduced by E. A. Palyutin in [17]. The discontinuity requirement, which states that stability, P-stability, and other significant independent conditions are satisfied in the trivial situation, distinguishes this idea from T^*-stability. The types in P-sets of P-stable theories constructed by T. Nurmagambetov and B. Poizat for types in the context of P-models are defined [18], in addition to the definition of types in stable theories. As of right now, A.R. Yeshkeyev's work [19] has yielded several innovations in P-stability for the Jonsson theories, or perfect Jonsson theories.

This paper consists of two sections. In Section 1, we give some basic information on Jonsson theories. In Section 2, we present our results obtained for cosemanticness classes of Jonsson spectrum in permissible enrichment, so-called J-non-multidimensional theories.

1 Basic information concerning Jonsson theories

To set the stage for the major result, let us define several terms and results associated with Jonsson theories that are well known.

Definition 1. [20] A theory $\mathbb{T}$ is called a Jonsson theory, if
1. $\mathbb{T}$ has at least one infinite mode;
2. $\mathbb{T}$ is an inductive theory;
3. $\mathbb{T}$ has the joint embedding property (JEP);
4. $\mathbb{T}$ has the amalgam property (AP).

The main properties and theorems related to Jonsson theory can be found in work [20].

Any inductive theory has a nonempty class of existential closed models, and the class of Jonsson theories is a subset of inductive theories. Consequently, by including more Jonsson theory properties, the description of the class of existential closed models above can be strengthened. One of these characteristics is the power saturation of the semantic model. Such theories are called perfect Jonsson theories. Let's become acquainted with the features of these theories.

Definition 2. [20; 162] A Jonsson theory $\mathbb{T}$ is called a perfect theory, if its semantic model is saturated.

As the examples below (Fig.) make abundantly evident, any Jonsson theory can be perfect; not all Jonssons can be perfect.

One of the striking examples of such a phenomenon is the example of the theory of fields of a fixed characteristic. In this example, the interpretation of an one-place predicate is realized by an existentially closed submodel of the semantic model of this theory. It is well known that an algebraic closed field with the same characteristic will serve as this theory's semantic model. The notion of a hereditary Jonsson theory was defined with the knowledge of such cases. Hereditary Jonsson theories are those in which the qualities of jonssonness are retained, that is, the enriched theory stays Jonsson despite any permissible enrichment of the theory's language. The term "admissibility of enrichment" refers to the preservation of the definability of any new language type with respect to the stability of the enriched theory, where stability is considered in relation to the enrichment framework. The idea of a Jonsson theory's central type was established in order to research hereditary hypotheses.

Concepts of "hereditary" and "permissible enrichment" belongs to professor Yeshkeyev A.R.

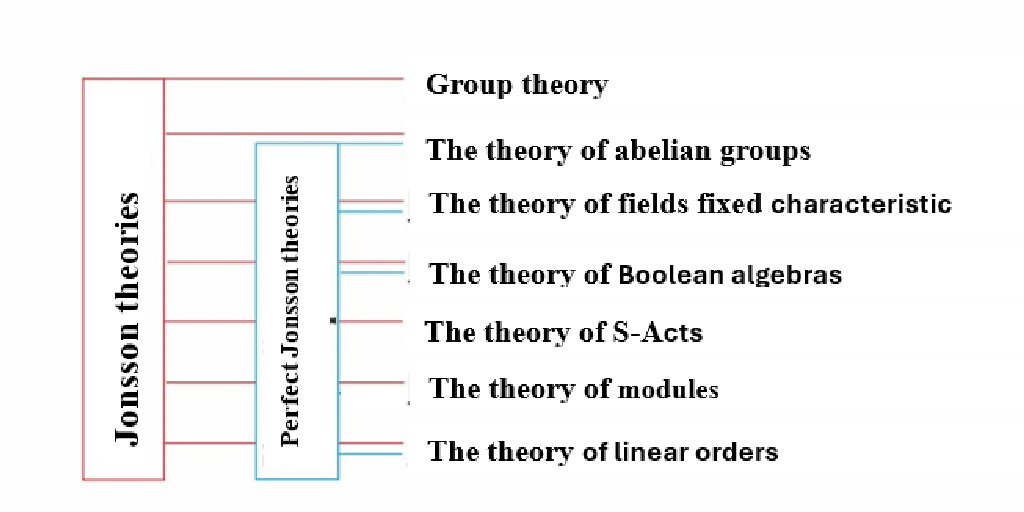

Figure. Examples of Jonsson theories

Definition 3. Let $\overline{\mathbb{T}}$ be an enrichment of Jonsson theory $\mathbb{T}$, $\Gamma = \{P\} \cup \{c\}$, P be some unary predicate symbol and c be a constant symbol. $\overline{\mathbb{T}}$ is called permissible, if any Δ-type in this enrichment is definable within $\mathbb{T}_\Gamma$-stability, where Δ is a subset of L_σ, a Δ-type means that any formula of this type belongs to Δ.

The following definition shows only the necessity of the concept, and the sufficient condition has not yet been defined. Let's give the definition.

Definition 4. A Jonsson theory is called hereditary, if with any of its permissible enrichments, any expansion of it in this enrichment will be Jonsson theory.

The subsequent discussion will solely focus on permissible enrichments, specifically examining those for the Jonsson spectrum that comprise solely of hereditary Jonsson theories [21–26].

Then the $JSp(A) = \{\mathbb{T}|\ A \in Mod\mathbb{T},\ \mathbb{T} -$ Jonsson theory of the signature $\sigma\}$ is the Jonsson spectrum of the model A.

Definition 5. If two models A, B have the same semantic model, then we will state that they are cosemantic among themselves. Symbolically $A \bowtie B$.

The relation $\bowtie$ is the equivalence relation between models, which generalizes the concept of elementary equivalence. Therefore we consider the factor set $JSp(A)/_{\bowtie}$ for the model A.

The task of describing the properties of heredity of Jonsson theory is significantly complicated by the transition of the theory to a spectrum. We set ourselves the task of studying the cosemanticity classes of a fixed Jonsson spectrum with respect to all the above-mentioned arising questions, their consequences and all possible combinations. In this case, the cosemanticity classes from the considered Jonsson spectrum will, as a rule, be convex, that is, the theories from this class will be convex. Accordingly, the central types will reflect the essence of the permissible enrichment of the considered hereditary Jonsson theory along with the convexity of the obtained theories.

Definition 6. [12] A class $JSp(A)/_{\bowtie}$ is called perfect (hereinafter, $PJSp(A)/_{\bowtie}$), if each class $[\mathbb{T}] \in JSp(A)/_{\bowtie}$ is perfect, $[\mathbb{T}]$ is called perfect, if $C_{[\mathbb{T}]}$ is a saturated model.

Note that for perfect Jonsson theories $\mathbb{T} \in [\mathbb{T}]$ their central types are equal.

The study in article heavily relies on the central type. The law of first-order logic, which states that a constant can be substituted by a variable when the constant was not belong of the theory's language prior to enrichment, is used in its construction. Following this substitution, the formal expression that was formerly a sentence becomes a formula. The central type is a complete 1-type that results, if this formal expression represents complete theory.

The concept of "central type" was first introduced in 2008 by professor A.R. Yeshkeyev. We enter the central type according to the following algorithm:

1. We enrich the σ signature, i.e. $\sigma_\Gamma = \sigma \bigcup \Gamma$, where $\Gamma = \{P\} \bigcup \{c\}$.

2. We write the enriched theory accordingly $\overline{\mathbb{T}} = Th_{\forall\exists}\left(\mathcal{C}, c_a\right)_{a \in C} \bigcup Th_{\forall\exists}(E_{\mathbb{T}}) \bigcup \{P\left(c\right)\} \bigcup \{"P \subseteq "\}$, in the language of the signature σ_Γ, the interpretation of the symbol P is an existentially closed submodel, as expressed by the infinite set of sentences $\{"P \subseteq "\}$.

3. The solution to the equation $P\left(C\right) = M \in E_{\mathbb{T}}$ in the signature $\bar{\sigma}$ is the interpretation of the symbol P.

4. We take into account all complements in the $\bar{\sigma}$ signature of the $\mathbb{T}$ theory. The center $\bar{\mathbb{T}}^*$ of the theory $\bar{\mathbb{T}}$ is one of the complements of the theory $\bar{\mathbb{T}}$ exhibited since the theory $\mathbb{T}$ is a Jonsson theory.

5. We restrict the $\bar{\sigma}$ signature to the signature $\sigma \cup \{P\}$.

6. The constant c is not included in the new signature because of constraints.

7. We substitute any variable, such x, for the constant symbol in accordance with the law of first-order logic.

8. Accordingly, we designate the theory $\bar{\mathbb{T}}^*$ by p^c, which is a complete type 1 in the admissible enrichment.

This enrichment is denoted by $\odot$.

The concept of the Jonsson set, which is a definable set with the aid of an existential formula and whose definable closure defines some existentially closed submodel of the semantic model under consideration, is useful for manipulating the properties of elements and subsets of a semantic model.

Definition 7. [20]. In the theory $\mathbb{T}$, a set X is referred to as a Jonsson set, if it meets the following criteria:

1) X is a definable subset of $C_{\mathbb{T}}$, where $C_{\mathbb{T}}$ is a semantic model of the theory $\mathbb{T}$;

2) $dcl(X)$ is a universe of existentially closed submodel C_T, where $dcl(X)$ is definable closure of X.

2 *The connection between J-P-stable theory and J-non-multidimensional theory*

From the main result in paper [10], for superstable theories, the notions of P-stability and P-superstability and non-multidimensional theories for complete theories coincide.

Theorem 1. [10; 90] Let T be a superstable theory. Then the following conditions are equivalent:

1) the theory T is non-multidimensional;

2) the theory T is P-superstable;

3) the theory T is P-stable.

Now a stable theory is superstable iff every type does not fork over a finite set. A generalization of stability for Jonsson theories is proved in the work [19].

We'll also talk about the idea that "type p does not fork over" in relation to Theorem 8 from [5].

Definition 8. Let p be complete $\exists$-type over A, A is a Jonsson subset of C. Then p is J-stationary over A, if

1) p does not fork over A;

2) p has a unique consistent extension that does not fork over A.

Definition 9. 1) A is a Jonsson subset of C, if $p(\bar{x}_1), q(\bar{x}_2)$ are complete $\exists$-types over A. If and only if $p(\bar{x}_1) \cup q(\bar{x}_2)$ is a $\exists$-complete type (over A), then p is J-weakly orthogonal to q.

2) For each p_1 and p_2, let us consider two $\exists$-complete or J-stationary types, respectively. If A is the universe of a $\exists_1$-saturated model, then p_1 is J-orthogonal to p_2, and q_1 is weakly J-orthogonal to q_2, where q_1, q_2 are any J-nonforking extensions of p_1 and p_2 over A, respectively.

Definition 10. Given a $C_{\mathbb{T}}$ semantic model and a Jonsson theory $\mathbb{T}$, let A be a Jonsson subset of the model. If p is orthogonal to any complete $\exists$-type over A, then p is considered J-multidimensional and a $\exists$-complete type. If $\mathbb{T}$ has a J -multidimensional type, then it is called a J-multidimensional theory. In the absence of this, the theory $\mathbb{T}$ is called the J-non-multidimensional theory or the J-restricted dimension theory.

Definition 11. A class $[\mathbb{T}]$ is called J-non-multidimensional, if every theory in this class does not have a J-multidimensional type.

We consider the class of semantic pairs as the main result [27].

Definition 12. [27; 188]. An existentially closed pair $(C_{\mathbb{T}}, M)$ is a semantic pair, if the following conditions hold:

1) M is $|\mathbb{T}|^+$-$\exists$-saturated (it means that it is $|\mathbb{T}|^+$-saturated restricted up to existential types);

2) for any tuple $\bar{a} \in C$ each its $\exists$-type in sense of $\mathbb{T}$ over $M \cup \{\bar{a}\}$ is satisfiable in C.

Let $[\nabla]$ be $\exists$-complete and J-λ-stable class of Jonsson theories, $\mathcal{C}_{[\nabla]}$ be a semantic model of the theory $[\nabla]$, $\overline{[\nabla]} = [\nabla]$ in the enrichment of $\odot$, $\overline{[\nabla]}^*$ is the center of the $\overline{[\nabla]}$, $p, q \in S(\overline{[\nabla]}^*)$, $\nabla' = Th_{\forall\exists}(\mathcal{C}, \mathcal{M})$.

Theorem 2. [27; 189]. $(\mathcal{C}_{[\nabla]}, M_1)$ and $(\mathcal{C}_{[\nabla]}, M_2)$ are two semantic pairs, $\bar{a}$ and $\bar{b}$ tuples taken from each of them, M_1, $M_2 \in E_{[\nabla]}$. Then $(\mathcal{C}_{[\nabla]}, M_1) \equiv_{\forall\exists} (\mathcal{C}_{[\nabla]}, M_2)$, if their central types are equivalent by the fundamental order $\overline{\nabla}^*$.

Definition 13. [27; 187] Let T be the Jonsson L-theory and $f(\bar{x}, \bar{y})$ be an $\exists$ formula of L language. If for any arbitrary large n exists $\bar{a}^0, \ldots, \bar{a}^{n-1}$ in some existentially closed model of T and $\bar{a}^0, \ldots, \bar{a}^{n-1}$ satisfies $\neg(\exists\bar{x}) \bigwedge_{k<n} f(\bar{x}, \bar{a}^k)$ and for any $l < n$ $\neg(\exists\bar{x}) \bigwedge_{k<n} f(\bar{x}, \bar{a}^k)$, then $f(\bar{x}, \bar{y})$ is said to have e.f.c.p. (existentially finite cover property).

Theorem 3. [27; 189]. Let $[\nabla]$ be a hereditary, $\exists$-complete perfect, and J-λ-stable class of Jonsson theories. Then the following conditions are equivalent:

1) $\overline{[\nabla]}^*$ does not have e.f.c.p.

2) Any $|\mathbb{T}|^+$-saturated model from ∇' is a semantic pair.

3) Two tuples $\bar{a}$ and $\bar{b}$ from the models of $\overline{[\nabla]}^*$ have the same type if and only if their central types in sense of $\overline{[\nabla]}^*$ over $\mathcal{M}$ are equivalent by fundamental order $\overline{[\nabla]}^*$.

4) Two tuples $\bar{a}$ and $\bar{b}$ from models of ∇' and that are in $\mathcal{C}_{[\nabla]} \setminus M$ have the same central types in the sense of $\overline{[\nabla]}$ if and only if they have the same central types in the sense of $\overline{[\nabla]}^*$.

Theorem 4. Let $[\nabla]$ be a hereditary, $\exists$-complete perfect, and J-λ-stable class of Jonsson theories. If $\overline{[\nabla]}^*$ does not have e.f.c.p. and λ-stable class, then $[\nabla]'$ is J-λ-stable and does not have e.f.c.p.

Let be $K = \{(C, M) | M \preceq_{\exists_1} C, (C, M)$is semantic pair$\}$, $JSp(K) = \{\Delta | \Delta$ is Jonsson theory, $\Delta = Th_{\forall\exists}(\mathcal{C}, \mathcal{M})$, where $(\mathcal{C}, \mathcal{M}) \in K\}$, let $[\Delta] \in JSp(K)/_{\bowtie}$. Let $[\Delta]$ be a $\exists$-complete and J-λ-stable class of Jonsson theories, the class $\overline{[\Delta]}$ be $[\Delta]$ in an permissible enrichment $\odot$, let $\overline{[\Delta]}^*$ be the center of the class $\overline{[\Delta]}$.

Theorem 5. Let $\mathbb{T}$ be a perfect, J-λ-stable $\exists$-complete Jonsson theory, K be the class of J-beautiful pairs of T. Let $[\Delta] \in JSpK/_{\bowtie}$ be a complete for $\exists$-sentences. Then the following conditions are equivalent:

1) the class $\overline{[\Delta]}^*$ is non-multidimensional (in the classical sense);

2) the class $\overline{[\Delta]}$ is J -non-multidimensional.

Proof. Let us prove implication 2)$\Rightarrow$ 1). Suppose $\overline{[\Delta]}$ is a J-non-multidimensional class. That is, not every theory in this class is J-multidimensional. $p \perp A$, where A is the φ-Jonsson set of the semantic model $C_{[\Delta]}$, is a J-multidimensional orthogonal $\exists$-type, if the theory is not J-multidimensional. $[\Delta]$ is a

J-stable and $\exists$ -complete class according to the theorem's condition, which is equivalent to the Morley rank condition. Furthermore, a measure of forking is Morley rank. Furthermore, the Lindenbaum theorem states that each theory in the class $\overline{[\Delta]}$ can be extended to the maximum, or to complete theories, because the theories in this class are incomplete.

The proof of 1)$\Rightarrow$ 2) is trivial.

Theorem 6. If the class $[\Delta]$ is J-P-λ-stable, then it is J-non-multidimensional and does not have e.f.c.p.

Proof. If $[\Delta]$ is J-non-multidimensional, then every elementary extension of a semantic pair is a semantic pair. Indeed, let (C, M) be a semantic pair, that is, for each dimension the cardinality of M is at least $|\mathbb{T}|^+$. Then, by Theorems 3 and 4, $[\Delta]$ does not have e.f.c.p.

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Author Contributions

All authors contributed equally to this work.

Conflict of Interest

The authors declare no conflict of interest.

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