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On a boundary value problem for a parabolic-hyperbolic equation of the fourth order

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In this paper a boundary value problem for a fourth-order equation of parabolic-hyperbolic type within a pentagonal domain was investigated. In the equation under consideration, one characteristic aligns with the Ox axis while the other aligns with the Oy axis. Initially, the problem was examined within the lower triangle of the specified domain. Utilizing a differential equation solution construction method, a solution to the formulated problem was derived. Subsequently, within the rectangles of the domain, employing the continuation method, two relationships between the solution's traces were established. Moreover, from the parabolic segment of the domain, two additional relationships between unknown traces will be derived. Solving this system of four equations enables determination of these traces, thereby resolving the problem.

Keywords: characteristic of the equation, differential and integral equations, method of constructing a solution, boundary value problem, equations of parabolic-hyperbolic type, the line of type changing.

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Introduction

Intensive research into equations of mixed elliptic-parabolic and parabolic-hyperbolic types is motivated by the fact that, on one hand, these new types of mixed equations have been little studied theoretically, and on the other hand, they are widely used in important issues of mechanics, physics, and technology.

The necessity of considering conjugation problems arises when a parabolic equation is defined in one part of the domain and a hyperbolic equation in another, as emphasized by I.M. Gelfand in 1959 [1]. He provides an example concerning the movement of gas in a channel surrounded by a porous medium: within the channel, the gas movement is described by the wave equation, while outside it, it is governed by the diffusion equation.

One of the earliest works dedicated to the study of boundary value problems for parabolic-hyperbolic equations was conducted by G.M. Struchina [2]. Subsequently, Y.S. Uflyand [3] further explored the problem of electrical oscillation propagation in composite lines, where losses are neglected in the semi-infinite section of the line, treating the remainder of the line as a cable without leakage, through the solution of boundary value problems for parabolic-hyperbolic equations.

Since the 1970s, research on boundary value problems for equations of third, fourth, and higher orders of the parabolic-hyperbolic type has seen intensive development. These boundary value problems were initially explored by T.D. Dzhuraev and his students [4, 5].

Subsequently, research on boundary value problems for third and fourth-order equations [6] and those of higher orders in the parabolic-hyperbolic type has significantly broadened. Currently, it is expanding into areas concerning the complexity of equations, the breadth of their application, and the generalization of problems related to these equations. The investigation has extended to numerous

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boundary value problems for such equations across various domains, involving two or three lines of change in type [7–9]. In the works [10, 11], some boundary value problems for fourth-order equations of parabolic-hyperbolic type, similar to equations of type (1) (see below) in a pentagonal region with two lines of type change, were studied. In the works [12, 13], considered boundary value problems for a mixed parabolic-hyperbolic equation with known and unknown dividing lines, as well as nonlocal boundary value problems and problems with a free boundary for parabolic, hyperbolic and mixed parabolic-hyperbolic equations. Work [14] is devoted to the study of boundary value problems and their spectral properties for equations of mixed parabolic-hyperbolic and mixed-composite types. In [15], boundary value problems for linear loaded differential and third-order integro-differential equations of mixed type were posed and studied. Then the study began of a number of different boundary value problems for mixed parabolic-hyperbolic equations of the third and fourth orders in various domains with two and three lines of change of type see, for example, [16–28].

1 Formulation of the problem

In the work [29], an equation of the form

$$\left(a_1\frac{\partial}{\partial x}+b_1\frac{\partial}{\partial y}\right)\left(a_2\frac{\partial}{\partial x}+b_2\frac{\partial}{\partial y}\right)(Lu)=0 \tag{1}$$

in the pentagonal area G, indicated below, where $a_1, b_1, a_2, b_2 \in R$, and $a_i^2+b_i^2 \neq 0 \; (i=1,2)$. Depending on the value of the coefficients a_1, b_1, a_2, b_2, a number of boundary value problems are posed for equation (1). In this work, boundary value problems are posed for 21 cases separately. In this present work, we consider the case of 3° ($a_1 = 1$, $b_1 = 0$, $a_2 = 0$, $b_2 = 1$). Then equation (1) has the form

$$\frac{\partial^2}{\partial x \partial y}(Lu)=0, \tag{2}$$

where $G = G_1 \cup G_2 \cup G_3 \cup G_4 \cup J_1 \cup J_2 \cup J_3$; G_1, G_3, G_4 are rectangles with vertices at points $A\,(0,0)$, $A_0\,(0,1)$, $B_0\,(1,1)$, $B\,(1,0)$; A, A_0, $D_0\,(-1,1)$, $D\,(-1,0)$; B, B_0, $C_0\,(2,1)$, $C\,(2,0)$ respectively, G_2 is a triangle with vertices at points C, D, $E\,(1/2\,, -3/2\,)$; J_1, J_2, J_3 are open segments with vertices at points C, D; A, A_0; B, B_0 respectively, which are lines of change like equation (2); $u = u\,(x, y)$ is an unknown function,

$$Lu=\begin{cases} u_{1xx}-u_{1y}, \; (x,y)\in G_1, \\ u_{ixx}-u_{iyy}, \; (x,y)\in G_i \;\; (i=2,3,4)\,. \end{cases}$$

For equation (2), the following problem is formulated:

Problem M. It is required to find a function $u\,(x, y)$, satisfying the following conditions:

1) it is continuous in $\overline{G}$ and in the domain $G\backslash J_1\backslash J_2\backslash J_3$ has continuous derivatives involved in equation (2), and u_x, u_y, u_{xx}, u_{xy} and u_{yy} are continuous up to part of the boundary of the domain G, indicated in the boundary conditions;

2) it satisfies the equation (2) in the domain $G\backslash J_1\backslash J_2\backslash J_3$;

3) it satisfies the following boundary conditions:

$$u(2,\,y)=\varphi_1\,(y)\,, \quad 0\le y\le 1, \tag{3}$$

$$u(-1,\,y)=\varphi_2\,(y)\,, \;\; 0\le y\le 1, \tag{4}$$

$$u_x\,(-1,\,y)=\varphi_4\,(y)\,, \;\; 0\le y\le 1, \tag{5}$$

$$u|_{CE}=\psi_1\,(x)\,, \;\; 1/2\;\le x\le 2, \tag{6}$$

$$u|_{DP} = \psi_2(x), \quad -1 \le x \le -1/2, \tag{7}$$

$$u|_{QE} = \psi_3(x), \quad 0 \le x \le 1/2, \tag{8}$$

$$\left.\frac{\partial u}{\partial n}\right|_{DE} = \psi_4(x), \quad -1 \le x \le 1/2, \tag{9}$$

$$\left.\frac{\partial^2 u}{\partial n^2}\right|_{DE} = \psi_5(x), \quad -1 \le x \le 1/2, \tag{10}$$

$$\left.\frac{\partial u}{\partial n}\right|_{CE} = \psi_6(x), \quad 1/2 \le x \le 2; \tag{11}$$

and

4) the following continuous gluing conditions:

$$u(x, +0) = u(x, -0) = T(x), \quad -1 \le x \le 2, \tag{12}$$

$$u_y(x, +0) = u_y(x, -0) = N(x), \quad -1 \le x \le 2, \tag{13}$$

$$u_{yy}(x, +0) = u_{yy}(x, -0) = M(x), \quad x \in (-1, 0) \cup (0, 1) \cup (1, 2), \tag{14}$$

$$u(+0, y) = u(-0, y) = \tau_4(y), \quad 0 \le y \le 1, \tag{15}$$

$$u_x(+0, y) = u_x(-0, y) = \nu_4(y), \quad 0 \le y \le 1, \tag{16}$$

$$u_{xx}(+0, y) = u_{xx}(-0, y) = \mu_4(y), \quad 0 < y < 1, \tag{17}$$

$$u(1+0, y) = u(1-0, y) = \tau_5(y), \quad 0 \le y \le 1, \tag{18}$$

$$u_x(1+0, y) = u_x(1-0, y) = \nu_5(y), \quad 0 \le y \le 1, \tag{19}$$

$$u_{xx}(1+0, y) = u_{xx}(1-0, y) = \mu_5(y), \quad 0 < y < 1. \tag{20}$$

Here φ_1, φ_2, φ_4 and $\psi_j \left(j = \overline{1,6}\right)$ are given sufficiently smooth functions, n is the internal normal to the line $x - y = 2$ (CE) or $x + y = -1$ (DE), and $P(-1/2, -1/2)$, $Q(0, -1)$. Besides,

$$T(x) = \begin{cases} \tau_2(x), & -1 \le x \le 0, \\ \tau_1(x), & 0 \le x \le 1, \\ \tau_3(x), & 1 \le x \le 2, \end{cases} \quad N(x) = \begin{cases} \nu_2(x), & -1 \le x \le 0, \\ \nu_1(x), & 0 \le x \le 1, \\ \nu_3(x), & 1 \le x \le 2, \end{cases} \quad M(x) = \begin{cases} \mu_2(x), & -1 < x < 0, \\ \mu_1(x), & 0 < x < 1, \\ \mu_3(x), & 1 < x < 2, \end{cases}$$

where τ_i, ν_i, μ_i $\left(i = \overline{1,5}\right)$ are temporarily unknown sufficiently smooth functions.

2 The solution of the problem

The following theorem holds:

Theorem 1. Let $\varphi_1 \in C^4[0, 1]$, $\varphi_2 \in C^4[0, 1]$, $\varphi_4 \in C^3[0, 1]$, $\psi_1 \in C^4[1/2, 2]$, $\psi_2 \in C^4[-1, -1/2]$, $\psi_3 \in C^4[0, 1/2]$, $\psi_4 \in C^3[-1, 1/2]$, $\psi_5 \in C^2[-1, 1/2]$, $\psi_6 \in C^3[1/2, 2]$, and the matching conditions $\varphi_1(0) = \psi_1(2)$, $\varphi_2(0) = \psi_2(-1)$, $\tau_1(0) = \tau_2(0) = \tau_4(0)$, $\tau_1(1) = \tau_3(1) = \tau_5(0)$, $\nu_1(0) = \nu_2(0) = \tau_4'(0)$, $\nu_1(1) = \nu_3(1) = \tau_5'(0)$, $\tau_1'(0) = \nu_4(0)$, $\tau_1'(1) = \nu_5(0)$ are satisfied, then the problem M has a unique solution.

Proof. We shall prove the theorem by the method of constructing a solution. To do this, we rewrite equation (2) as

$$u_{1xx} - u_{1y} = \omega_{11}(x) + \omega_{12}(y), \quad (x, y) \in G_1, \tag{21}$$

$$u_{ixx} - u_{iyy} = \omega_{i1}(x) + \omega_{i2}(y), \quad (x, y) \in G_i \ \ (i = 2, 3, 4), \tag{22}$$

where the notation $u(x, y) = u_i(x, y)$, $(x, y) \in G_i$ $\left(i = \overline{1, 4}\right)$ is introduced, and $\omega_{i1}(x)$, $\omega_{i2}(y)$ $(i = \overline{1,4})$ are unknown sufficiently smooth functions.

First, consider problem M in the domain G_2. The solution to equation (22) $(i = 2)$, satisfying conditions (12), (13), is represented in the form

$$u_2(x, y) = \frac{1}{2}\left[T(x+y) + T(x-y)\right] + \frac{1}{2}\int\limits_{x-y}^{x+y} N(t)\,dt - \frac{1}{2}\int\limits_0^y d\eta \int\limits_{x-y+\eta}^{x+y-\eta} \omega_{21}(\xi)\,d\xi - \int\limits_0^y (y-\eta)\,\omega_{22}(\eta)\,d\eta. \tag{23}$$

Substituting (23) into conditions (9) and (10) after some calculations, we obtain the following system of equations

$$\omega_{21}(x) + \omega_{22}(-1-x) = \sqrt{2}\psi_4'(x), \ -1 \le x \le 1/2,$$

$$\omega_{21}(x) - \omega_{22}(-1-x) = 2\psi_5(x) - 2T''(-1) - 2N'(-1) - 2\omega_{21}(-1), \ -1 \le x \le 1/2.$$

From this system after some transformations, we find

$$\omega_{21}(x) = \psi_5(x) + \frac{\sqrt{2}}{2}\psi_4'(x) - T''(-1) - N'(-1) - \omega_{21}(-1), \ -1 \le x \le 1/2, \tag{24}$$

$$\omega_{22}(y) = -\psi_5(-1-y) + \frac{\sqrt{2}}{2}\psi_4'(-1-y) + T''(-1) + N'(-1) + \omega_{21}(-1), \ -3/2 \le y \le 0. \tag{25}$$

Adding (24) and (25), we find

$$\begin{gathered} \omega_{21}(x) + \omega_{22}(y) = \left[\psi_5(x) - \psi_5(-1-y)\right] + \tfrac{\sqrt{2}}{2}\left[\psi_4'(x) + \psi_4'(-1-y)\right], \\ -1 \le x \le 1/2, \ -3/2 \le y \le 0. \end{gathered} \tag{26}$$

Now substituting (23) into condition (11), we have

$$\omega_{21}(x) + \omega_{22}(x-2) = -\sqrt{2}\psi_6'(x), \ 1/2 \le x \le 2.$$

Setting in (25) $y = x - 2$ and substituting the obtained equality into the last equality, we find

$$\omega_{21}(x) = -\sqrt{2}\psi_6'(x) + \psi_5(1-x) - \frac{\sqrt{2}}{2}\psi_4'(1-x) - T''(-1) - N'(-1) - \omega_{21}(-1), \ 1/2 \le x \le 2. \tag{27}$$

Hence, adding (25) and (27), we get

$$\begin{gathered} \omega_{21}(x) + \omega_{22}(y) = -\sqrt{2}{\psi'}_6(x) + \left[\psi_5(1-x) - \psi_5(-1-y)\right] \\ -\tfrac{\sqrt{2}}{2}\left[\psi_4'(1-x) - \psi_4'(-1-y)\right], \quad 1/2 \le x \le 2, \ -3/2 \le y \le 0. \end{gathered} \tag{28}$$

From (26) and (28), it follows that $\psi_4'(1/2) = -{\psi'}_6(1/2)$.

Thus, we have found the function $\omega_{21}(x) + \omega_{22}(y)$ for $-1 \le x \le 2$, $-3/2 \le y \le 0$ completely. It is determined by formulas (26), (28).

Now, substituting (23) into the condition (6), we arrive at the relation

$$T'(x) + N(x) = \alpha_1(x), \ -1 \le x \le 2, \tag{29}$$

where

$$\alpha_1(x) = \psi_1'\left(\frac{x+2}{2}\right) + \int_0^{\frac{x-2}{2}} \omega_{21}(x-\eta)\, d\eta + \int_0^{\frac{x-2}{2}} \omega_{22}(\eta)\, d\eta.$$

And substituting (23) into condition (7), we get

$$\tau_2'(x) - \nu_2(x) = \delta_1(x), \ -1 \le x \le 0, \tag{30}$$

where

$$\delta_1(x) = \psi_2'\left(\frac{x-1}{2}\right) - \int_0^{-\frac{x+1}{2}} \omega_{21}(x+\eta)\, d\eta - \int_0^{-\frac{x+1}{2}} \omega_{22}(\eta)\, d\eta.$$

Next, substituting (23) into condition (8), we have

$$\tau_3'(x) - \nu_3(x) = \delta_2(x), \ 1 \le x \le 2, \tag{31}$$

where

$$\delta_2(x) = \psi_3'\left(\frac{x-1}{2}\right) - \int_0^{-\frac{x+1}{2}} \omega_{21}(x+\eta)\, d\eta - \int_0^{-\frac{x+1}{2}} \omega_{22}(\eta)\, d\eta.$$

a) For $0 \le x \le 1$ the relation (29) has the form

$$\tau_1'(x) + \nu_1(x) = \alpha_1(x), \ 0 \le x \le 1; \tag{32}$$

b) for $-1 \le x \le 0$,

$$\tau_2'(x) + \nu_2(x) = \alpha_1(x), \ -1 \le x \le 0; \tag{33}$$

c) and when $1 \le x \le 2$,

$$\tau_3'(x) + \nu_3(x) = \alpha_1(x), \ 1 \le x \le 2. \tag{34}$$

Solving the system {(30), (33)}, we find

$$\tau_2'(x) = \frac{1}{2}\left[\alpha_1(x) + \delta_1(x)\right], \quad \nu_2(x) = \frac{1}{2}\left[\alpha_1(x) - \delta_1(x)\right]. \tag{35}$$

Integrating the first of equalities (35) from -1 to x, we obtain

$$\tau_2(x) = \frac{1}{2}\int_{-1}^{x} \left[\alpha_1(t) + \delta_1(t)\right] dt + \psi_2(-1), \ -1 \le x \le 0. \tag{36}$$

And solving system (31), (34), we get

$$\tau_3'(x) = \frac{1}{2}\left[\alpha_1(x) + \delta_2(x)\right], \quad \nu_3(x) = \frac{1}{2}\left[\alpha_1(x) - \delta_2(x)\right]. \tag{37}$$

Integrating the first of equalities (37) from 2 to x, we have

$$\tau_3(x) = \frac{1}{2}\int_{2}^{x} \left[\alpha_1(t) + \delta_2(t)\right] dt + \psi_1(2), \ 1 \le x \le 2. \tag{38}$$

Now, by differentiating the equation (21) with respect to y passing to the limit at $y \to 0$ in the resulting equation and in equation (22) ($i = 2$), we obtain

$$\nu_1''(x) - \mu_1(x) = \omega_{12}'(0), \quad \tau_1''(x) - \mu_1(x) = \omega_{21}(x) + \omega_{22}(0).$$

Eliminating the function $\mu_1(x)$ from these relations and integrating the resulting equation twice from 0 to x, we arrive at the relation

$$\nu_1(x) - \tau_1(x) = -\int_0^x (x-t)\left[\omega_{21}(t) + \omega_{22}(0)\right] dt + \frac{1}{2}\omega_{12}'(0)\, x^2 + k_1 x + k_2,$$

where $\omega_{12}'(0)$, k_1 and k_2 are unknown constants.

Excluding the function $\nu_1(x)$ from the last relation and from (32), we have

$$\tau_1'(x) + \tau_1(x) = \alpha_2(x) + \frac{1}{2}\omega_{12}'(0)\, x^2 + k_1 x + k_2, \ 0 \le x \le 1, \tag{39}$$

where

$$\alpha_2(x) = \alpha_1(x) + \int_0^x (x-t)\left[\omega_{21}(t) + \omega_{22}(0)\right] dt.$$

Solving equation (39) under the conditions (see (35), (36), (37), (38))

$$\tau_1(0) = \psi_2(-1) + \frac{1}{2}\int_{-1}^0 \left[\alpha_1(t) + \delta_1(t)\right] dt, \quad \tau_1'(0) = \frac{1}{2}\left[\alpha_1(0) + \delta_1(0)\right],$$

$$\tau_1'(1) = \frac{1}{2}\left[\alpha_1(1) + \delta_2(1)\right], \quad \tau_1(1) = \psi_1(2) - \frac{1}{2}\int_1^2 \left[\alpha_1(t) + \delta_2(t)\right] dt,$$

we find the function $\tau_1(x)$ as

$$\tau_1(x) = \int_0^x e^{t-x}\alpha_2(t)\, dt + \frac{\omega_{12}'(0)}{2}\left(x^2 - 2x + 2 - 2e^{-x}\right) + k_1\left(x - 1 + e^{-x}\right) + k_2\left(1 - e^{-x}\right) + k_3 e^{-x},$$

where

$$k_3 = \frac{1}{2}\int_{-1}^0 \left[\alpha_1(t) + \delta_1(t)\right] dt + \psi_2(-1),$$

$$k_2 = \frac{1}{2}\left[\alpha_1(0) + \delta_1(0)\right] - \alpha_2(0) + k_3,$$

$$k_1 = \frac{e-2}{2(e-3)}\left[\alpha_1(1) + \delta_2(1)\right] + \frac{e-2}{3-e}\alpha_2(1) + \frac{2}{3-e}\psi_1(2) +$$

$$+\frac{3e-4}{e(3-e)}k_2 + \frac{4-e}{e(3-e)}k_3 - \frac{1}{3-e}\int_0^1 e^t\alpha_2(t)\, dt - \frac{1}{3-e}\int_1^2 \left[\alpha_1(t) + \delta_2(t)\right] dt,$$

$$\omega'_{12}(0) = \frac{e}{2}\left[\alpha_1(1) + \delta_2(1)\right] - \alpha_2(1)\, e + \int_0^1 e^t\alpha_2(t)\, dt - k_2 + k_3 - k_1(e-1).$$

Thus, we have defined the functions $\nu_1(x)$, $\mu_1(x)$ and $u_2(x, y)$.

Now, let us consider the problem in the domain G_3. Passing to the limit at $y \to 0$ in equations (22) $(i = 2)$ and (22) $(i = 3)$, we find

$$\omega_{31}(x) = \omega_{21}(x), \ -1 \le x \le 0,$$

where it should be $\omega_{32}(0) = \omega_{22}(0)$.

Consider the following problem:

$$\begin{cases} u_{3xx} - u_{3yy} = \Omega_{31}(x) + \omega_{32}(y), \\ u_3(x, 0) = T_2(x), \ u_{3y}(x, 0) = N_2(x), \quad -2 \le x \le 1, \\ u_3(-1, y) = \varphi_2(y), \ u_{3x}(-1, y) = \varphi_4(y), \ u_3(0, y) = \tau_4(y), \ 0 \le y \le 1. \end{cases}$$

Here we used (4), (5) in the form $u_3(-1, y) = \phi_2(y)$, $u_{3x}(-1, y) = \phi_4(y)$.

We will seek a solution to this problem satisfying all conditions except the condition $u_{3x}(-1, y) = \varphi_4(y)$, in the form

$$u_3(x, y) = u_{31}(x, y) + u_{32}(x, y) + u_{33}(x, y), \tag{40}$$

where $u_{31}(x, y)$ is the solution of the problem

$$\begin{cases} u_{31xx} - u_{31yy} = 0, \\ u_{31}(x, 0) = T_2(x), \ u_{31y}(x, 0) = 0, \ -2 \le x \le 1, \\ u_{31}(-1, y) = \varphi_2(y), \ u_{31}(0, y) = \tau_4(y), \ 0 \le y \le 1, \end{cases} \tag{41}$$

$u_{32}(x, y)$ is the solution of the problem

$$\begin{cases} u_{32xx} - u_{32yy} = \omega_{32}(y), \\ u_{32}(x, 0) = 0, \ u_{32y}(x, 0) = N_2(x), \ -2 \le x \le 1, \\ u_{32}(-1, y) = 0, \ u_{32}(0, y) = 0, \ 0 \le y \le 1, \end{cases} \tag{42}$$

$u_{33}(x, y)$ is the solution of the problem

$$\begin{cases} u_{33xx} - u_{33yy} = \Omega_{31}(x), \\ u_{33}(x, 0) = 0, \ u_{33y}(x, 0) = 0, \ -2 \le x \le 1, \\ u_{33}(-1, y) = 0, \ u_{33}(0, y) = 0, \ 0 \le y \le 1. \end{cases} \tag{43}$$

Using the continuation method, we find solutions to problems (41), (42) and (43). They have the following forms

$$u_{31}(x, y) = \frac{1}{2}\left[T_2(x + y) + T_2(x - y)\right], \tag{44}$$

$$u_{32}(x, y) = \frac{1}{2}\int\limits_{x-y}^{x+y} N_2(t)\,dt - \int\limits_0^y (y - \eta)\,\omega_{32}(\eta)\,d\eta, \tag{45}$$

$$u_{33}(x, y) = -\frac{1}{2}\int\limits_0^y d\eta \int\limits_{x-y+\eta}^{x+y-\eta} \Omega_{31}(\xi)\,d\xi, \tag{46}$$

where

$$T_2(x) = \begin{cases} 2\varphi_2(-1-x) - \tau_2(-2-x), \ -2 \le x \le -1, \\ \tau_2(x), \ -1 \le x \le 0, \\ 2\tau_4(x) - \tau_2(-x), \ 0 \le x \le 1, \end{cases}$$

$$\Omega_{31}(x) = \begin{cases} -\omega_{31}(-2-x), \ -2 \le x \le -1, \\ \omega_{31}(x), \ -1 \le x \le 0, \\ -\omega_{31}(-x), \ 0 \le x \le 1, \end{cases}$$

$$N_2(x) = \begin{cases} -\nu_2(-2-x) + 2\int\limits_0^{-1-x} \omega_{32}(\eta)\,d\eta, \ -2 \le x \le -1, \\ \nu_2(x), \ -1 \le x \le 0, \\ -\nu_2(-x) + 2\int\limits_0^{x} \omega_{32}(\eta)\,d\eta, \ 0 \le x \le 1. \end{cases}$$

Substituting (44), (45) and (46) into (40), we have

$$u_3(x, y) = \frac{1}{2}\left[T_2(x+y) + T_2(x-y)\right] + \frac{1}{2}\int\limits_{x-y}^{x+y} N_2(t)\,dt$$

$$-\frac{1}{2}\int\limits_0^y d\eta \int\limits_{x-y+\eta}^{x+y-\eta} \Omega_{31}(\xi)\,d\xi - \int\limits_0^y (y-\eta)\,\omega_{32}(\eta)\,d\eta.$$

Differentiating this solution with respect to x, we obtain

$$u_{3x}(x, y) = \frac{1}{2}\left[T_2'(x+y) + T_2'(x-y)\right] + \frac{1}{2}\left[N_2(x+y) - N_2(x-y)\right]$$
$$-\frac{1}{2}\int\limits_0^y \left[\Omega_{31}(x+y-\eta) - \Omega_{31}(x-y+\eta)\right]d\eta. \quad (47)$$

Passing to the limit in (47) at $x \to -1$ and considering the condition $u_{3x}(-1, y) = \varphi_4(y)$, we find

$$\omega_{32}(y) = \tau_2''(y-1) + \nu_2'(y-1) - \varphi_2''(y) - \varphi_4'(y) - \omega_{31}(y-1).$$

Similarly, from (47) using the conditions (15), (16), we have

$$\nu_4(y) = \tau_4'(y) + \beta_1(y), \quad (48)$$

where

$$\beta_1(y) = \tau_2'(-y) - \nu_2(-y) + \int\limits_0^y \omega_{31}(\eta - y)\,d\eta + \int\limits_0^y \omega_{32}(\eta)\,d\eta.$$

Now let us move to the domain G_4. Passing in equations (22) $(i = 4)$, (22) $(i = 2)$ to the limit at $y \to 0$ and considering (12) and (14) for $0 \le x \le 1$, we have $\omega_{41}(x) + \omega_{42}(0) = \omega_{21}(x) + \omega_{22}(0)$. Let's assume that $\omega_{42}(0) = \omega_{22}(0)$. Then, we have $\omega_{41}(x) = \omega_{21}(x)$.

Next, passing in equations (22) ($i = 4$), (22) ($i = 2$) to the limit at $x \to 1$ due to (18) and (20), we find

$$\omega_{12}(y) = \tau_5''(y) - \tau_5'(y) + \omega_{42}(y) + \omega_{41}(1) - \omega_{11}(1). \tag{49}$$

Passing in equations (22) ($i = 3$) and (21) to the limit at $x \to 0$ due to (15) and (17), we obtain

$$\omega_{12}(y) = \tau_4''(y) - \tau_4'(y) + \omega_{32}(y) + \omega_{31}(0) - \omega_{11}(0). \tag{50}$$

Eliminating function $\omega_{12}(y)$ from (49) and (50), we find

$$\omega_{42}(y) = \left[\tau_4''(y) - \tau_4'(y)\right] - \left[\tau_5''(y) - \tau_5'(y)\right] + \omega_{32}(y) + \omega_{31}(0) - \omega_{41}(1) - \omega_{11}(0) + \omega_{11}(1).$$

Consider the following problem:

$$\begin{cases} u_{4xx} - u_{4yy} = \Omega_{41}(x) + \omega_{42}(y), \\ u_4(x, 0) = T_3(x), \ u_{4y}(x, 0) = N_3(x), \quad 0 \le x \le 3, \\ u_4(2, y) = \varphi_1(y), \ u_4(1, y) = \tau_5(y), \ 0 \le y \le 1. \end{cases}$$

Here we used condition (3) in the form $u_4(2, y) = \phi_1(y)$.

We will seek a solution to the last problem in the form

$$u_4(x, y) = u_{41}(x, y) + u_{42}(x, y) + u_{43}(x, y), \tag{51}$$

where $u_{41}(x, y)$ is the solution of the problem

$$\begin{cases} u_{41xx} - u_{41yy} = 0, \\ u_{41}(x, 0) = T_3(x), \ u_{41y}(x, 0) = 0, \ 0 \le x \le 3, \\ u_{41}(2, y) = \varphi_1(y), \ u_{41}(1, y) = \tau_5(y), \ 0 \le y \le 1, \end{cases} \tag{52}$$

and $u_{42}(x, y)$ and $u_{43}(x, y)$ are the solution of the problems respectively

$$\begin{cases} u_{42xx} - u_{42yy} = \omega_{42}(y), \\ u_{42}(x, 0) = 0, \ u_{42y}(x, 0) = N_3(x), \ 0 \le x \le 3, \\ u_{42}(2, y) = 0, \ u_{42}(1, y) = 0, \ 0 \le y \le 1, \end{cases} \tag{53}$$

$$\begin{cases} u_{43xx} - u_{43yy} = \Omega_{41}(x), \\ u_{43}(x, 0) = 0, \ u_{43y}(x, 0) = 0, \ 0 \le x \le 3, \\ u_{43}(2, y) = 0, \ u_{43}(1, y) = 0, \ 0 \le y \le 1. \end{cases} \tag{54}$$

By using the continuation method, it is easy to see that the solutions to the problems (52), (53) and (54) have the forms

$$u_{41}(x, y) = \frac{1}{2}\left[T_3(x + y) + T_3(x - y)\right], \tag{55}$$

$$u_{42}(x, y) = \frac{1}{2}\int\limits_{x-y}^{x+y} N_3(t)\, dt - \int\limits_0^y (y - \eta)\,\omega_{42}(\eta)\, d\eta, \tag{56}$$

$$u_{43}(x, y) = -\frac{1}{2}\int\limits_0^y d\eta \int\limits_{x-y+\eta}^{x+y-\eta} \Omega_{41}(\xi)\, d\xi, \tag{57}$$

where

$$T_3(x) = \begin{cases} 2\varphi_1(x-2) - \tau_3(4-x), \ 2 \le x \le 3, \\ \tau_3(x), \ 1 \le x \le 2, \\ 2\tau_5(1-x) - \tau_3(2-x), \ 0 \le x \le 1, \end{cases}$$

$$\Omega_{41}(x) = \begin{cases} -\omega_{41}(4-x), \ 2 \le x \le 3, \\ \omega_{41}(x), \ 1 \le x \le 2, \\ -\omega_{41}(2-x), \ 0 \le x \le 1, \end{cases}$$

$$N_3(x) = \begin{cases} -\nu_3(4-x) + 2\int\limits_0^{x-2} \omega_{42}(\eta)\, d\eta, \ 2 \le x \le 3, \\ \nu_3(x), \ 1 \le x \le 2, \\ -\nu_3(2-x) + 2\int\limits_0^{1-x} \omega_{42}(\eta)\, d\eta, \ 0 \le x \le 1. \end{cases}$$

Substituting (55), (56) and (57) into (51), we have

$$u_4(x, y) = \frac{1}{2}\left[T_3(x+y) + T_3(x-y)\right] + \frac{1}{2}\int\limits_{x-y}^{x+y} N_3(t)\, dt$$
$$- \frac{1}{2}\int\limits_0^y d\eta \int\limits_{x-y+\eta}^{x+y-\eta} \Omega_{41}(\xi)\, d\xi - \int\limits_0^y (y-\eta)\, \omega_{42}(\eta)\, d\eta.$$

Differentiating this solution with respect to x, we obtain

$$u_{41x}(x, y) = \frac{1}{2}\left[T_3'(x+y) + T_3'(x-y)\right] + \frac{1}{2}\left[N_3(x+y) - N_3(x-y)\right]$$
$$- \frac{1}{2}\int\limits_0^y \Omega_{41}(x+y-\eta)\, d\eta + \frac{1}{2}\int\limits_0^y \Omega_{41}(x-y+\eta)\, d\eta. \quad (58)$$

Passing to the limit at $x \to 1$ in (58) after some calculations and transformations by virtue of (18), (19), we obtain the relation

$$\nu_5(y) = -\tau_4'(y) + \tau_4(y) - \tau_5(y) + \beta_2(y), \quad (59)$$

where

$$\beta_2(y) = \nu_1(0) - \tau_1(0) - \nu_1(1) + \tau_1(1) + \gamma_1(y),$$

$$\gamma_1(y) = \tau_3'(1+y) + \nu_3(1+y) - \int\limits_0^y \omega_{41}(1+y-\eta)\, d\eta +$$

$$+ \int\limits_0^y \omega_{32}(\eta)\, d\eta + \left[\omega_{31}(0) - \omega_{41}(1) + \tau_1''(1) - \nu_1(1) - \mu_1(0) + \nu_1(0) - \omega_{31}(0) - \omega_{32}(0)\right] y.$$

Finally, we consider the problem in the domain G_1. Passing to the limit in equation (21) at $y \to 0$, we find

$$\omega_{11}(x) = \tau''_1(x) - \nu_1(x) - \omega_{12}(0).$$

Next, we write down the solution to equation (21), satisfying conditions (12) for $0 \le x \le 1$, (15) and (18), differentiating this solution with respect to x after some calculations and transformations, we obtain

$$\begin{aligned} u_{1x}(x, y) = &-\int_0^y \tau'_4(\eta) N(x, y; 0, \eta)\, d\eta + \int_0^y \tau'_5(\eta) N(x, y; 1, \eta)\, d\eta \\ &+ \int_0^1 \tau'_1(\xi) N(x, y; \xi, 0)\, d\xi + \int_0^y [\omega_{11}(1) + \omega_{12}(\eta)] N(x, y; 1, \eta)\, d\eta \\ &+ \int_0^y [\omega_{11}(0) + \omega_{12}(\eta)] N(x, y; 0, \eta)\, d\eta - \int_0^y d\eta \int_0^1 \omega'_{11}(\xi) N(x, y; \xi, \eta)\, d\xi, \end{aligned} \tag{60}$$

where

$$\left.\begin{matrix} G(x, y; \xi, \eta) \\ N(x, y; \xi, \eta) \end{matrix}\right\} = \frac{1}{2\sqrt{\pi(y-\eta)}} \sum_{n=-\infty}^{+\infty} \left\{ \exp\left[-\frac{(x-\xi-2n)^2}{4(y-\eta)} \right] \mp \exp\left[-\frac{(x+\xi-2n)^2}{4(y-\eta)} \right] \right\}$$

are the Green's functions of the first and second boundary value problems for the heat equation.

Passing to the limits $x \to 0$ and $x \to 1$ in (60) by virtue of (48) and (59), we obtain two Abel's type integral equations for the unknowns $\tau''_4(y)$ and $\tau'_5(y)$. Applying the Abel inversion to these equations, we obtain a system of Volterra integral equations of the second kind for $\tau''_4(y)$ and $\tau'_5(y)$:

$$\tau''_4(y) + \int_0^y K_1(y, \eta)\, \tau''_4(\eta)\, d\eta + \int_0^y K_2(y, \eta)\, \tau'_5(\eta)\, d\eta = g_1(y), \tag{61}$$

$$\tau'_5(y) + \int_0^y K_3(y, \eta)\, \tau'_5(\eta)\, d\eta + \int_0^y K_4(y, \eta)\, \tau''_4(\eta)\, d\eta = g_2(y), \tag{62}$$

where $K_1(y, \eta)$, $K_2(y, \eta)$, $K_3(y, \eta)$, $K_4(y, \eta)$, $g_1(y)$, $g_2(y)$ are known functions, $K_1(y, \eta)$ and $K_3(y, \eta)$ have a weak singularity (1/2), and the remaining functions are continuous. Therefore, system $\{(61),(62)\}$ admits a unique solution in the class of continuous functions. Solving the system $\{(61),(62)\}$, we find functions $\tau''_4(y)$, $\tau'_5(y)$, and thus functions $\tau_4(y)$, $\tau_5(y)$, $\nu_4(y)$, $\nu_5(y)$. Then all the functions $u_3(x, y)$, $u_4(x, y)$ and $u_1(x, y)$, will be known. So, we have found a solution to the considered problem M in a unique way.

Conclusion

In this paper, we consider a new correct boundary value problem for a third-order parabolic-hyperbolic equation in a pentagonal domain consisting of three rectangles and one triangle. In the central rectangular domain the equation is parabolic, and in the two side rectangles and in the lower triangle it belongs to hyperbolic type. Straight lines $x = 0$, $x = 1$ and $y = 0$ are lines of change in the type of equation.

When constructing a solution in the lower characteristic triangle, writing a solution to equation (22) $(i = 2)$ that satisfies conditions (12), (13) and substituting this solution into conditions (4) and (5), we find the function $\omega_2(x)$.

Then, substituting this solution into conditions (6), we obtain the first functional relation between the unknown functions $T(x)$ and $N(x)$ for $-1 \le x \le 2$. Next, substituting this solution into condition (7), we obtain another relation between $\tau_2(x)$ and $\nu_2(x)$ for $-1 \le x \le 0$. From these two relations we find the functions $\tau_2(x)$ and $\nu_2(x)$ for $-1 \le x \le 0$. Similarly, satisfying condition (8), we find the functions $\tau_3(x)$ and $\nu_3(x)$ for $1 \le x \le 2$.

Then, differentiating equation (21) with respect to y, in the resulting equation and in equation (22) $(i = 2)$ assuming $y = 0$, we obtain two more relations between the unknown functions $\tau_1(x)$, $\nu_1(x)$ and $\mu_1(x)$ for $1 \le x \le 2$. Eliminating from these two relations and from the relation between the functions $T(x)$ and $N(x)$ for $0 \le x \le 1$ the function $\nu_1(x)$ and $\mu_1(x)$, we arrive at an ordinary differential equation of the second order with respect to $\tau_1(x)$, on the right side of which one unknown constant is involved. Solving this equation under the known three conditions, we find the function $\tau_1(x)$, and thus the functions $\nu_1(x)$, $u_2(x, y)$.

Further, by the method of continuation in the left and right rectangular regions using the written solutions, directing x to zero and to one, we obtain two relations between the unknown functions $\tau_4(y)$, $\nu_4(y)$ and $\tau_5(y)$, $\nu_5(y)$ respectively.

Next, passing to the limit in equation (21) at $y \to 0$, we find the unknown function $\omega_{11}(x)$. In the parabolic part of the rectangular region, writing the representation of the solution in terms of the known Green's function of the first boundary value problem and differentiating this solution with respect to x and assuming $x \to 0$ and $x \to 1$, we obtain two more relations between the unknown functions $\tau_4(y)$, $\nu_4(y)$ and $\tau_5(y)$, $\nu_5(y)$ respectively. Excluding the functions $\nu_4(y)$ and $\nu_5(y)$ from these four relations, we arrive at a system of Volterra integral equations of the second kind with respect to ${\tau''}_4(y)$ and ${\tau'}_5(y)$. The unique solvability of this system follows from the theory of integral equations. Solving this system, we find traces of the solution ${\tau''}_4(y)$ and ${\tau'}_5(y)$. Thus, we have proven the unique solvability of the considered problem.

Conflict of Interest

The author declares no conflict of interest.

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