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The Properties of Similarity for Jonsson's Theories and Their Models

Actually, we study the connections of the Δ -PJ-theories with their centers. The properties of various companions of some Δ -PJ-theories and their connection with this theory are considered. Also the similarity of the central types of Δ -PJ-theories in the enriched language is considered. In the class of perfect Δ-PJ-theories the conditions of coincidence of algebraic primeness with some sort of atomic models are found. In the class of Δ -PJ-theories, the concepts of syntactic and semantic similarities are introduced and the results on the relationship of these similarities in this class with their centers are obtained.

Key words: Jonsson's theory, semantic model, companion, model completeness, syntactic and semantic similarities, atomic and algebraically prime models, existentially closed model, lattice of existential formulas.

Introduction

In [1] some of the model-theoretic properties for Jonsson's theories have been considered. Jonsson's conditions are natural algebraic requirements that arise in studying a wide class of algebras. Jonsson's properties are satisfied by theories such as group theory, the theory of Abelian groups, the theory of fields of fixed characteristic, the theory of Boolean algebra, the theory of ordered sets, the theory of polygons, and many others. As it is evident from the list, the obtaining of technical apparatus of results for Jonsson's theories, with respect to applications, would be wide enough. In this paper, the object of our research will focus on a new class of theories related to the notion jonssonness and positivity. A subject of the research are theories related to the so-called "Eastern" model theory. This conventional definition and division of the general model theory into two main areas: "Western" and "Eastern" were identified by the well-known expert in model theory H.J. Keisler in his survey article [2]. However, he notes that the Western model theory studies the complete theories, and the Eastern model theory- Jonsson's theories. This work is a review of results concerning researches of the notion of positive Jonsson's theories and its class of models. The given ones are devoted to research of properties of similarity of Jonsson's theories and its positive generalizations and also to studying of model-theoretic properties of a class of existentially closed models of such theories.

In the model theory, some of the important concepts are prime and algebraically prime model. These concepts respectively appear in the works [3, 4]. In the beginning we will do some review of the results [5] of the author of this article regarding the atomic and algebraically prime models of positive Jonsson's theories. And in the other part of this work we study the syntactic and semantic similarity of the positive Jonsson's theories. The main idea of obtaining results is transfer of the properties of syntactical and semantic similarity of centers of given Jonsson's theories in the sense of [1] on themselves. In the paper [4] it was proved that syntactic and semantic similarity between two Jonsson's theories is equivalent to that of their centers. In the work [6, 7] of I. Ben-Yaacov, a positive model theory was introduced, and within it the so-called CATs were considered. One can find as a syntactic feature of this work the elimination of symbols of a universal quantifier and a negation on the basic formulas. It is a semantic feature to consider extensions and immersions, instead of standard morphisms (whereas in model theory isomorphic embeddings and elementary embeddings are generally considered). It is easy to note that the problematic of positive Jonsson's and CATs are very closely connected. We should say that presenting results about models of Δ-PJ-theories which are positive generalization of Jonsson's theories, if they are such, in general. But we will not go beyond the first order. Even in the case where Δ-PJ-theory is not Jonsson's, the idea of the generalization of a semantic method [1] for Jonsson's theories is used. The essence of this generalization is that properties of Δ-PJ-central completion will be translated on Δ-PJ-preimage.

If a Δ-PJ-theory is Jonsson's, we will work with $\mathrm{Mod}\,T$ as with the class of models of a Jonsson's theory. If a Δ-PJ-theory is not Jonsson's, then as with $\mathrm{Mod}\,T$, we consider E_T^+ - a positive class of existentially closed models of this theory. This approach for the class E_T - a class of existentially closed models of any universal theory T has been studied in [9]. Since for relatively Jonsson's theories there are two possibilities:

the perfect and imperfect cases, we will adhere to the following. It is known [10] that if the Jonsson's theory T is perfect, the class of its existentially closed models E_T is elementary and coincides with $\mathrm{Mod}\,T^*$ where T^* is its center. In the opposite case, i.e., if the theory T is not perfect, we do as in [9], i.e., instead of Mod T we work with the class E_T^+. When we consider an arbitrary Δ-PJ-theory T, the class E_T^+ is considered as an extension of the class E_T (both classes are always available), and depending on the perfectness and incompleteness of the model-theoretic properties of a class E_T^+ represent interest. In this article usually we consider that a Δ-PJ-theory is Δ-PJ-perfect, and it is a natural generalization of perfectness in Jonsson's case. And now we shall give some important definitions and statements from the paper [4, 6, 7] in order to show results from [5] regarding algebraically prime and other types of atomic models of the above mentioned theories.

The Main results. We shall use the following definitions:

Agreements about positiveness and Δ.

Let L be the language of the first order.

At is a set of all atomic formulas of this language.

$B^+(At)$ contains all atomic formulas closed under positive Boolean combination and for subformulas and substitution of variables.

$L^+ = Q(B^+(At))$ is the set of formulas in quantifier prefix normal type obtained by application of quantifiers ($\forall$and $\exists$) to $B^+(At)$.

$B(L^+)$ is any Boolean combination of formulas from L^+.

$\Delta \subseteq B(L^+)$.

Let M and N be the structure of language, $\Delta \subseteq B(L^+)$.

The map $h: M \to N$ is a Δ- homomorphism (symbolically $h: M \leftrightarrow_\Delta N$) if for any $\phi(\bar{x}) \in \Delta, \forall \bar{a} \in M$ such that $M \models \phi(\bar{a})$ we have that $N \models \phi(h(\bar{a}))$.

The model M is said to begin in N and we say that M continues to N, with $h(M)$ a continuation of M. If the map h is injective, we say that h is an immersion of M into N (symbolically $h: M \leftrightarrow_\Delta N$). In the future we will use the terms Δ-continuation and Δ-immersion.

The theory T admits Δ-joint embedding property ($\Delta-$JEP) if for any $A, B \in ModT$ there exists $C \in ModT$ and Δ-homomorphisms $h_1: A \to_\Delta C, h_2: B \to_\Delta C$.

The theory T admits Δ-amalgamation property ($\Delta-$AP) if for any $A, B, C \in ModT$ with $h_1: A \to_\Delta C, g_1: A \to_\Delta B$, where h_1, g_1 are Δ-homomorphisms, there exists $D \in ModT$ and $\mathrm{h}_2: C \to_\Delta D, g_2: B \to_\Delta D$ where h_2, g_2 are Δ-homomorphisms such that $h_2 \circ h_1 = g_2 \circ g_1$.

The notion of Δ-PJ theory.

The theory T is called Δ-positive Jonsson's (Δ-PJ) theory if it satisfies the following conditions:

1. T has an infinite model.
2. T is positive $\forall\exists$-axiomatizable.
3. T admits Δ-JEP.
4. T admits Δ-AP.

When $\Delta = \mathrm{B(At)}$, one can note that we can obtain the usual Jonsson's theory, the only difference is that it has only positive $\forall\exists$-axiom.

Later Σ^+ is a set of positive existential formulas.

Property (ES) [4].

If $\phi(\bar{x}), \psi(\bar{x})$ are existential formulas such that $T \models \neg\exists\bar{x}(\phi \wedge \psi)$, then there exist Δ-formulas $\theta(\bar{x})$ such that $T \models (\phi \to \theta)$ and $T \models (\phi \to \neg\theta)$.

Property (SA) [4].

If B_1, B_2 are models of theory T and $X \subseteq B_1 \cap B_2$ such that $(B_1, a)_{a \in X} \equiv_\Delta (B_2, a)_{a \in X}$, then there exists a model B_* of theory T and attachments f_1, f_2 of models B_1, B_2 in B_* such that $f_1(a) = f_2(a)$ for $a \in X$.

Theorem 1.1 [4]. For any theory T, (ES) is right if and only if (SA) is right.

Let $\Delta = B^{+}(At)$.

Theorem 1.2 [5]. Let T be a Δ-PJ-perfect Δ-PJ-theory, complete for Σ^{+}-sentences.

Then the following conditions are equivalent to each other:

1) T has an $h-\Delta$ algebraically prime model;

2) T^{*} has a (Σ^{+},Σ^{+})-atomic model.

Theorem 1.3 [14]. Let T be a Δ-PJ-perfect Δ-PR-theory, complete for $\forall\exists^{+}$-statements.

Then the following conditions are equivalent to each other:

1) A is a countable and (Σ^{+},Σ^{+}) atomic model of T^{*};

2) A is a countable and Δ-positively existentially closed and Σ^{+}- nice model of T

Theorem 1.4 [4]. Suppose T is a universal theory, complete for existential sentences.

Then the following conditions are equivalent:

(i) T has a minimal algebraically simple model;

(ii) T has a (Δ,Σ)-atomic model;

(iii) T has only one algebraically prime model and it is (Σ,Σ)- atomic;

(iv) There is some open formula $\alpha(x)$ complete for existential formulas.

Theorem 1.5 [5]. Let T be a Δ-PJ-perfect Δ-PR-theory, complete for Σ^{+}-sentences.

Then the following conditions are equivalent:

1) T has a minimal $h-\Delta$-algebraically prime model;

2) T^{*} has exactly one $h-\Delta$ algebraically prime model, which is (Σ^{+},Σ^{+}) - atomic.

If Γ is a set of formulas, then $t_{\Gamma}^{A}(\overline{a}) = \{\phi(\overline{x}) \mid \Gamma, A \models \phi(a)\}$ is called Γ-type of from A.

Lemma 1.6 [14]. Let A be a countable model of Δ-PJ-theory $T, a^{-\omega} = \langle a_1,...,a_n,...\rangle$ realized Σ^{+}-main $\Sigma^{+}-\omega$-type. $B \models T, B\Delta$ - continues in A. Then B is a good almost-weak (Σ^{+},Σ^{+}) - atomic model.

Lemma 1.7 [13]. If a Δ-PJ-theory T has a good almost-weak (Σ^{+},Σ^{+})-atomic model, then each $h-\Delta$-algebraically simple model T is a good almost-weak (Σ^{+},Σ^{+}) - atomic model of T.

Lemma 1.8 [13]. Let a Δ-PJ-theory be complete for Σ^{+}-sentences. Then each good almost-weak (Σ^{+},Σ^{+}) - atomic model is an $h-\Delta$-algebraically prime model.

Theorem 1.9 [5]. Let T be a Δ-PJ-theory, complete for Σ^{+}-sentences, and let T have a good almost-weak (Σ^{+},Σ^{+}) - atomic model. Then the following conditions are equivalent:

1) A is an $h-\Delta$ algebraically prime model of the theory T;

2) A is a good almost-weak (Σ^{+},Σ^{+}) *- atomic model of the theory theory .*

Let us consider the similarity of Δ-PM-theories in the enriched signature.

One of the classical mathematical problems is the question of classification of objects by studying some general features. In mathematics the role of such objects is played by sets with relations and operations given on them. With the help of mathematical logic these objects were associated with some sets of formulas of language of predicate calculus. This connection between syntax and semantics of fixing language is the essence of the model theory. Therefore, it is clear that finding the syntactic and semantic properties of a similarity can be useful in the classification of objects of model theory.

In [10] T.G. Mustafin defined the precise concept of syntactic [10, Def. 1] and semantic [10, Def. 4] similarities for complete theories. Moreover, with the help of the language of these definitions and the corresponding concepts (for example, the envelope of theory [10, Def. 12], semantic properties of theories, models, elements [10, Def. 8]), he proved that an arbitrary complete theory is syntactically similar to some theory of polygons [10, Th.4, Th.5]. In addition to this he showed that the semantic similarity preserves a number of useful properties [10].

This part of our work is related to different concepts of similarity and cosemanticness of the positive Jonsson's theories. Concepts of cosemanticness of the Jonsson's theories have been defined in [11]. In the papers [12], [6] the notions of syntactic and semantic similarities of Jonsson's and Δ-PJ-theories have been defined. In the work [13] the properties of cosemanticness of the Δ-PJ-theories have been considered.

The above notions of syntactic and semantic similarities were determined for complete theories. In the class of the Jonsson's theories this approach to the classification of the object is acceptable, but demanded certain changes in the relevant definitions in the sense of T.G. Mustafin as in [10]. This is due, first, to the fact that, generally speaking, the Jonsson's theories are not complete, and secondly, that the homogeneous-universal models of the Jonsson's theory, generally speaking, are not saturated as in the case of Western model theory.

Recall the definitions of syntactic and semantic similarity from [10].

Let Tbe a complete theory, then $\mathrm{F(T)} = \bigcup_{n<\omega} F_n(T)$, where $F_n(T)$ is a Boolean algebra of formulas with n free variables.

Let T_1 and T_2 be complete theories.

We will say that T_1 and T_2 are syntactically similar if there exists a bijection $f : F(T_1) \to F(T_2)$ such that

1) the restriction of f to $F_n(T_1)$ is an isomorphism of Boolean algebras $F_n(T_1)$ and $F_n(T_1), n < \omega$;

2) $f(\exists v_{n+1}\phi) = \exists \phi_{n+1} f(\phi), \phi \in F_{n+1}(T), n < \omega$;

3) $f(v_1 = v_2) = (v_1 = v_2)$.

A pure triple is $\langle A, \Gamma, M \rangle$, where A is not an empty set, Γ is a group of permutations of $A$and M is a family of subsets of A, such that $M \in M \Rightarrow g(M) \in M$ for each $g \in \Gamma$.

Proposition 1.10 ([10], Prop. 1). If the theories T_1 and T_2 are syntactically similar, then T_1 and T_2 are semantically similar, the converse implication fails.

We give analogues of the above definitions for the Jonsson's theories.

The following definitions of [12] are generalizations of previous definitions.

Let T be an arbitrary Jonsson's theory, then $E(T) = \bigcup_{n<\omega} E_n(T)$ where $E_n(T)$ is the lattice of $\exists$-formulas with n free variables, T^* is the center of Jonsson's theory T, i.e., $T^* = Th(C)$, where C is Jonsson's semantic model of the theory T.

Let T_1 and T_2 be Jonsson's theories.

We will say that T_1 and T_2 are J-syntactically similar if there exists a bijection $f : E(T_1) \to E(T_2)$ such that

1) the restriction of f to $E_n(T_1)$ is an isomorphism of the lattices $E_n(T_1)$ and $E_n(T_2), n < \omega$;

2) $f(\exists v_{n+1}\phi) = \exists \phi_{n+1} f(\phi), \phi \in F_{n+1}(T), n < \omega$;

3) $f(v_1 = v_2) = (v_1 = v_2)$.

The triple $\langle C, AutC, SubC \rangle$ is a J-semantic triple, where C is a semantic model of T, $AutC$ is the group of automorphisms of C, $AutC$ is the class of all subsets of C, which are universes of the corresponding submodels of C.

Two Jonsson's theories T_1 and T_2 are called J-semantically similar if their J-semantic triples are isomorphic as pure triples.

The correctness of this definition follows from the fact that a semantic model of a perfect Jonsson's theory is unique up to isomorphism. Otherwise, all semantic models are only elementary equivalent to each other.

We have the following result.

Theorem 1.11 [6]. Let T_1 and T_2 be $\exists$-complete Jonsson's perfect theories.

Then the following conditions are equivalent:

1) T_1 and T_2 are J-syntactically similar;

2) T_1^* and T_2^* are syntactically similar in the sense of complete theories, where T_1^* and T_2^* are centers of T_1 and T_2 respectively.

From the definition of Δ-PJ-theory T it should be noted that $ModT$ is closed under homomorphisms, but the theory T is not always Jonsson's. If it is not Jonsson's, we restrict our considerations only under Δ-continuations, i.e., in the definition of the theory T will be just the Δ-immersions.

Let T be any Δ-PJ-theory, then, $E^{+}(T)=\bigcup_{n<\omega}\sum_{n}^{+}(T)$, where $\sum_{n}^{+}(T)$ is a lattice of positive existential formulas with n free variables.

Let T_1 and T_2 be Δ-PJ- theories.

We say that T_1 and T_2 are PJ-syntactically similar if there exists a bijection $f: E^{+}(T_1)\rightarrow E^{+}(T_2)$ such that

1) The restriction of f to $E_n^{+}(T_1)$ is an isomorphism of lattices $E_n^{+}(T_1)$ and $E_n^{+}(T_2), n<\omega$;

2) $f(\exists v_{n+1}\phi)=\exists\phi_{n+1}f(\phi), \phi\in E^{+}{}_{n}(T), n<\omega$;

3) $f(v_1=v_2)=(v_1=v_2)$.

Let T_1 and T_2 be Δ-PJ- theories.

We say that T_1 and T_2 are PJ-syntactically similar if there exists a bijection $f: E^{+}(T_1)\rightarrow E^{+}(T_2)$ such that

1) The restriction of f to $E_n^{+}(T_1)$ is an isomorphism of the lattices $E_n^{+}(T_1)$ and $E_n^{+}(T_2), n<\omega$;

2) $f(\exists v_{n+1}\phi)=\exists\phi_{n+1}f(\phi), \phi\in E^{+}{}_{n}(T), n<\omega$;

3) $f(v_1=v_2)=(v_1=v_2)$.

In the frame of such definition we have the same result as above (Theorem 7 [6]) for Δ-PJ- theories.

In [11], [13] a property of a cosemanticness for models of the Jonsson's theories and respectively for Δ-PJ theories were considered.

We say that Jonsson's theories T_1 and T_2 are cosemantic $(T_1 \rhd\lhd T_2)$, if they have a common semantic model, i.e., $C_{T_1}=C_{T_2}$.

We say that the models A and B are cosemantic $(A\rhd\lhd B)$ if for any Jonsson's theory B such that $A\models T$, there is a Jonsson's theory T' cosemantic with T, such that $B\models T'$.

And vice versa.

Lemma 1.12 [13]. For any two models A and B the following implications hold:

$A\equiv B\Rightarrow A\equiv_j B\Rightarrow A\rhd\lhd B.$

Two Δ-PJ-theories T_1 and T_2 are called Δ-PJ-cosemantic, $T_1 ><_{P_j}^{\Delta} T_2$, if they have a common semantic model, in the case where T_1 and T_2 are Jonsson's theories, and have a common universal domain, when they are not Jonsson's.

Models A and B are called Δ-PJ-equivalent if for any Δ-PJ-theory $T, A\models T\Rightarrow B\models T$ and this is denoted by $A\equiv_{P_j}^{\Delta} T$.

Models A and B of signature Σ are called Δ-PJ-cosemantic, , $A><_{P_j}^{\Delta} B$, if for any Δ-PJ-theory T_1 such that $A\models T_1$, there exists a Δ-PJ-theory T_2 , Δ-PJ- cosemantic with T_1 and such that $B\models T_2$.

And vice versa.

And in the frame of such definition we have the same result as above (Lemma 4, [13]) for Δ-PJ-theories.

Further we present the results associated with the similarities of Δ-PJ-theories in the enriched signature which are Jonsson's. Now we consider the notions of the enrichment of signature of Δ-PJ theories and the central type as well.

Let T be an arbitrary Δ-PJ theory in first order signature σ. Let C be a semantic model of $T, A\subseteq C$. Let $\sigma_{\Gamma}(A)=\sigma\cup\{c_a \mid a\in A\}\cup\Gamma$, where $\Gamma=\{P\}\cup\{c\}$, where P is a unary predicate and c is a constant. Let us consider the following theory $T_{\Gamma}^{PJ}(A)=Th_{\forall\exists^{+}}(C,a)_{a\in A}\cup\{P(c_a \mid a\in A)\}\cup\{P(c)\}\cup\{"P\subseteq"\}$, where $\{"P\subseteq"\}$ is an infinite set of sentences expressing the fact that the interpretation of the symbol P is an existentially closed submodel in the signature σ. The requirement of existential closedness for a submodel is essential in the sense that it should not be finite. This theory is not necessarily complete.

Let us consider all completions of T^* for T in σ_Γ, where . Due to the fact that T^* is a Δ-PJ-theory, it has its centre and we call it T^C . By the restriction of T^C up to the signature σ, the theory T^C becomes of complete type. This type we call central type of the theory T. It will be noted that all semantic models are elementarily equivalent to each other.

In the language of the notion of central type we have the following theorem.

Theorem 1.13. Let T_1 and T_2 be Σ-complete, perfect, Δ-PJ-theories. Then the following conditions are equivalent:

1) T^*_1 and T^*_2 are Δ-PJ- syntactically similar;

2) T^c_1 and T^c_1 are syntactically similar (in the sense of [10]).

Proof. We also need the following facts.

Fact 1 [14]. For any complete for the existential sentences Jonsson's theory T the following conditions are equivalent:

1) T is perfect;

2) T^* model-complete.

Fact 2 [14]. For any complete for the existential sentences Jonsson's theory T the following conditions are equivalent:

1) T^* is model-complete;

2) For each $n < \omega$ $E_n(T)$ is a Boolean algebra, where $E_n(T)$ is a lattice of existential formulas with n free variables.

We note that the perfectness of T_1 and T_2 implies that T_1^* and T_2^* are Δ-PJ Jonsson's theories.

We will show 1)$\Rightarrow$ 2). We have that for every $n < \omega E_n^+(T_1^*)$ is isomorphic to $E_n^+(T_2^*)$. Let this be an isomorphism f_{1n}. By hypothesis and facts 1 and 2 for every $n < \omega E_n^+(T_1^*)$ and $E_n^+(T_2^*)$ are Boolean algebras. But because of the completeness of T_1^* and T_2^* it follows that $(T_1^*)^*$ and $(T_2^*)^*$ are model-complete by virtue of 1, and so for each $n < \omega$, for any formula $\phi(\bar{x})$ of $F_n(T_1^*)$ by Corollary 1 there is a formula $\psi(\bar{x})$ of $E_n^+(T_1^*)$ so that $(T^*)^* \models \phi \leftrightarrow \psi$.

Since the theory T_1^* is complete for positive existential sentences and $E_n^+(T_1^*) \subseteq E_n^+(T_1^*)^*$ (as $T_1^* \subseteq (T_1^*)^*$), it follows that $E_n^+(T_1^*) \subseteq E_n^+(T_1^*)^*$. Due to the fact that the theory T_2^* is complete for positive existential proposals and $E_n^+(T_2^*) \subseteq E_n^+(T_2^*)^*$ (as $T_2^* \subseteq (T_2^*)^*$), it follows that $E_n^+(T_2^*) \subseteq E_n^+(T_2^*)^*$. For each $N < \omega$, for each $\phi_1(\bar{x})$ of $F_n(T_1^*)^*$, we define the following mapping between the $F_n(T_1^*)^*$ and $F_n(T_1^*)^*$:$F_{2n}(\phi_1(\bar{x})) = f_{1n}(\psi_1(\bar{x}))$, where $F_n(T_1^*)^* \models \psi_1 \leftrightarrow \phi_1, \psi_1 \in E_n^+(T_1^*)$. It is easy to understand that by virtue of the properties of f_{1n} and what has been said above, f_{2n} is a bijection, an isomorphism between $F_n(T_1^*)^*$ and $F_n(T_2^*)^*$. Consequently, $(T_1^*)^*$ and $(T_2^*)^*$ are syntactically (in the sense of [10]). But these $F_n(T_1^*)^* = T_1^c$ and $F_n(T_2^*)^* = T_2^c$ respectively, are by definition, central types of theories.

We show 2) $\Rightarrow$ 1). It is trivial since $F_n\left(T_1^*\right)^*$ is isomorphic to $F_n\left(T_2^*\right)^*$ for each $n < \omega$, and by virtue of the hypothesis and the facts 1,2. This isomorphism extends to all subalgebras.

Lemma 1.14. Any two cosemantic Jonsson's theories are J-semantically similar.

Proof. Follows from the definitions.

Lemma 1.15. If two perfect $\exists$-complete Jonsson's theories are J-syntactically similar, then they are J-semantically similar.

Proof. It follows from [10, Prop 1] and what was said above.

Definition 1.1. A property (or a notion) of theories (or models, or elements of models) is called semantic if and only if and only if it is invariant relative to semantic similarity.

Let us recall the definition of a polygon.

Definition 1.2. By a polygon over a monoid S we mean a structure with only unary functions $\langle A, f_{a:a\in S}\rangle$ such that

(i) $f_e(a) = a, \forall a \in A,$ where e is the unit of S;

(ii) $f_{\alpha\beta}(a) = f_\alpha(f_\beta(a)), \forall \alpha, \beta \in S, \forall a \in A.$

And now we can formulate the main result of this work.

Theorem 1.16. For each Σ-complete perfect Jonsson Δ-PJ theory there exists a Δ-PJ syntactically similar Σ-complete perfect Jonsson's Δ-PJ theory of polygons, such that its center is model complete.

Proof. It follows from Theorems 7, 8 and [10, Th.4, Th.5].

In connection with the following statement it will be interesting to research the notion of semantic similarity of positive Jonsson's theories.

Proposition 1.17 [10]. The following properties and notions are sematic:

(1) type;

(2) forking;

(3) λ-stability;

(4) Lascar rank;

(5) strong type;

(6) Morley sequence;

(7) orthogonality, regularity of types;

(8) $I(\aleph_\alpha, T)$ - the spectrum function.

By virtue of this notion we can say that all above mentioned properties and notions from Proposition 2 in the class of centers of $\exists^+$ complete perfect Δ-PJ theories are semantic.

Moreover, if we consider the above mentioned enrichments of signatures of such theories and we consider central types of those, we get that the situation will not change. And finally, it is appropriate to consider the Δ-PJ-analogues of the list of semantic properties and notions from classical model theory. By the way, in particular Δ-PM-positive generalizations of stability were considered. And at the end of this paper we recall that all the undefined here definitions and most unproven here statements regarding Jonsson's theories and their positive generalizations can be found in the textbook [14].

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Йонсондық теориялар және олардың модельдері үшін ұқсастықтың қасиеттері

Мақалада Δ-PJ-теориясымен олардың центрларының байланысы зерттелді. Кейбір Δ-PJ-теориясының әр түрлі компаньондарының қасиеттері және олардың осы теориямен байланысы қарастырылды. Сонымен қатар байытылған тілде Δ-PJ-теориясының централдық типтерінің ұқсастығы байқалды. Кемел Δ-PJ-теориялар класында алгебралық жайлығы кейбір атомдық модельдер сұрыпымен сәйкестігі табылды. Δ-PJ-теориялар класында синтаксистік және семантикалық ұқсастық ұғымы енгізілді және осы кластағы осы теориялардың олардың центрларымен байланысы байқалды.

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Свойства подобия для йонсоновских теорий и их моделей

В статье изучены связи Δ-PJ-теорий с их центрами, а также свойства различных компаньонов некоторых Δ-PJ-теорий с их центрами. Отмечено подобие центральных типов Δ-PJ-теорий в обогащенном языке. В классе совершенных Δ-PJ-теорий найдены свойства совпадения алгебраической простоты с некоторым сортом атомных моделей. В классе Δ-PJ-теорий было введено понятие синтактических и семантических подобий. Кроме того, были получены результаты об отношениях этих теорий в этом классе с их центрами.

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