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On the hyperbolic type differential equation with time involution

In the present paper, the initial value problem for the hyperbolic type involutory in t second order linear partial differential equation is studied. The initial value problem for the fourth order partial differential equations equivalent to this problem is obtained. The stability estimates for the solution and its first and second order derivatives of this problem are established.

Keywords: involutory type hyperbolic equation, stability, Banach space.

Introduction

Delay differential equations are universal phenomenon applied their models in engineering systems to behave like a real process [1–6].

Involutary differential equations have been studied in several papers [7–11]. In the paper [10], the boundedness of the solution of the initial value problem

$$y''(t) = f(t, y(t), y(u(t))),\ t \in I = (-\infty, \infty),\ y(t_0) = y_0,\ y'(t_0) = y'_0$$

for the second order ordinary differential equation with involution was investigated. Theorem on stability estimates for the solution of the initial value problem for the second order ordinary linear differential equation with involution was proved. Finally, theorem on existence and uniqueness of bounded solution of initial value problem for the second order nonlinear ordinary differential equation with involution was established. Presently, spectral questions of differential equations with involution were studied in papers [12–20].

Delay hyperbolic differential equations have been investigated in several papers [21–25]. Partial differential equations with involution terms have deeply different properties of solutions then without involution terms [26, 27]. Therefore, it is important to study properties of partial differential equations with involution.

In the present paper, the stability of the solution of the initial value problem for the hyperbolic type time involution partial differential equation

$$\begin{cases} \frac{\partial^2 u(t,x)}{\partial t^2} - a u_{xx}(t,x) - b u_{xx}(-t,x) = g(t,x),\ t,\ x \in I, \\ u(0,x) = \varphi(x),\ u_t(0,x) = \psi(x),\ x \in I \end{cases} \tag{1a}$$

is investigated. Here, $g(t,x)\,(t, x \in I)$, $\varphi(x)$ and $\psi(x)$ are given smooth functions. The stability estimates for the solution and its first and second order derivatives of this problem are established.

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1 Stability of problem (1a)

Theorem 1.1. Assume that $g(t,x)$ is a continuously differentiable and bounded function and $g(0,x)=0$ and $\varphi(x)$ is a twice continuously differentiable and bounded function and $\psi(x)$ is a continuously differentiable and bounded function and $|b|<a, a\in(0,\infty)$. Then, for solutions of problem (1a) the following stability estimates hold:

$$\sup_{t,x\in I}|u(t,x)| \leq M_1(a,b)\left[\sup_{x\in I}|\varphi(x)| + \int_{-\infty}^{\infty}|\psi(y)|\,dy + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|g(y,x)|\,dydx\right], \tag{1b}$$

$$\sup_{t,x\in I}|u_t(t,x)| + \sup_{t,x\in I}|u_x(t,x)| \leq M_1(a,b)\left[\sup_{x\in I}|\varphi_x(x)| + \sup_{x\in I}|\psi(x)| + \sup_{x\in I}\int_{-\infty}^{\infty}|g(y,x)|\,dy\right], \tag{1c}$$

$$\sup_{t,x\in I}|u_{tt}(t,x)| + \sup_{t,x\in I}|u_{xx}(t,x)| + \sup_{t,x\in I}|u_{tx}(t,x)| \leq M_2(a,b)\left[\sup_{x\in I}|\varphi_{xx}(x)| + \sup_{x\in I}|\psi_x(x)| + \sup_{t,x\in I}|g(t,x)|\right]. \tag{1d}$$

Proof. Problem (1a) can be written as abstract initial value problem

$$\begin{cases} \frac{d^2u(t)}{dt^2} + aAu(t) + bAu(-t) = g(t),\ t\in I, \\ u(0)=\varphi,\ u'(0)=\psi \end{cases} \tag{2a}$$

in a Banach space $C(I)$ of all continuous and bounded functions $f(x)$ defined on I with norm

$$\|f\|_{C(I)} = \sup_{x\in I}|f(x)|.$$

Here, positive operator A defined by the formula

$$Au = -u''(x)$$

with domain $D(A)=\{u : u(x),\ u''(x)\in C(I)\}$, $g(t)=g(t,x)$ and $u(t)=u(t,x)$ are known and unknown abstract functions defined on I with values in $C(I)$ and $\varphi=\varphi(x)$, $\psi=\psi(x)$ are unknown elements of $C(I)$. Now, we will obtain the initial value problem for the fourth order differential equation to problem (2a) under smoothness conditions of solution. Differentiating equation (2a), we get

$$\frac{d^3u(t)}{dt^3} + aAu'(t) - bAu'(-t) = g_t(t),$$

$$\frac{d^4u(t)}{dt^4} + aAu''(t) + bAu''(-t) = g_{tt}(t). \tag{3}$$

Using these equations and initial condition and equation in problem (2a), we get

$$\begin{cases} u(0)=\varphi,\ u'(0)=\psi, \\ u''(0) = -(a+b)A\varphi, \\ u'''(0) = (-a+b)A\psi + g_t(0). \end{cases}$$

Putting $-t$ instead of t in equation (2a), we get

$$u_{tt}(-t) + aAu\,(-t) + bAu\,(t) = g(-t). \tag{4}$$

Applying equations (2a), (3) and (4), we get

$$\frac{d^4u\,(t)}{dt^4} + aA\frac{d^2u\,(t)}{dt^2} + bA\left[-aAu\,(-t) - bAu\,(t) + g(-t)\right] = g_{tt}(t),$$

$$bAu\,(-t) = -\frac{d^2u\,(t)}{dt^2} - aAu\,(t) + g(t).$$

From these equations it follows equation

$$\frac{d^4u\,(t)}{dt^4} + aA\frac{d^2u\,(t)}{dt^2} - aA\left[-\frac{d^2u\,(t)}{dt^2} - aAu\,(t) + g(t)\right] - b^2A^2u\,(t) = -bAg(-t) + g_{tt}(t)$$

or

$$\frac{d^4u\,(t)}{dt^4} + 2aA\frac{d^2u\,(t)}{dt^2} + \left(a^2 - b^2\right)A^2u\,(t) = aAg(t) - bAg(-t) + g_{tt}(t).$$

Then, we have the following initial value problem for the fourth order abstract differential equation

$$\begin{cases} \frac{d^4u(t)}{dt^4} + 2aA\frac{d^2u(t)}{dt^2} + \left(a^2 - b^2\right)A^2u\,(t) = F(t), \\ F(t) = aAg(t) - bAg(-t) + g_{tt}(t),\ t \in I, \\ u(0) = \varphi,\ u'(0) = \psi,\ u''\,(0) = -\,(a+b)\,A\varphi, \\ u'''\,(0) = (-a+b)\,A\psi + g_t(0). \end{cases} \tag{5}$$

Now we will obtain solution of the initial value problem (5). It is easy to see that

$$\frac{d^4u\,(t)}{dt^4} + 2aA\frac{d^2u\,(t)}{dt^2} + \left(a^2 - b^2\right)A^2u\,(t) = \left(\frac{d^2}{dt^2} + (a - |b|)\,A\right)\left(\frac{d^2}{dt^2} + (a + |b|)\,A\right)u\,(t)\,.$$

Therefore, problem (5) can be written as abstract initial value problem

$$\begin{cases} \left(\frac{d^2}{dt^2} + (a+|b|)\,A\right)u\,(t) = v(t),\ u(0) = \varphi,\ u'(0) = \psi, \\ \left(\frac{d^2}{dt^2} + (a-|b|)\,A\right)v(t) = F(t), \\ F(t) = aAg(t) - bAg(-t) + g_{tt}(t),\ t \in I, \\ v(0) = (-b+|b|)\,A\varphi, \\ v'(0) = (b+|b|)\,A\psi + g'(0) \end{cases} \tag{6}$$

for the system of second order abstract differential equations in a Banach space $C(I)$. Problem (6) can be written as initial value problem

$$\begin{cases} \frac{\partial^2 u(t,x)}{\partial t^2} - (a+|b|)\, u_{xx}\,(t,x) = v\,(t,x)\,,\ t,\ x \in I, \\ u(0,x) = \varphi(x),\ u_t(0,x) = \psi(x), \quad x \in I, \\ \frac{\partial^2 u(t,x)}{\partial t^2} - (a-|b|)\, u_{xx}\,(t,x) = F(t,x), \\ F(t,x) = -a g_{xx}(t,x) + b g_{xx}(-t,x) + g_{tt}(t,x),\ t,\ x \in I, \\ v(0,x) = (-\,|b| + b)\,\varphi_{xx}(x), \\ v_t(0,x) = (-\,|b| - b)\,\psi_{xx}(x) + g'(0,x),\ x \in I \end{cases}$$

for the system of hyperbolic equations. Applying Dalambert's formula, we get

$$u\,(t,x) = \frac{\varphi(x+\sqrt{a+|b|}t) + \varphi(x-\sqrt{a+|b|}t)}{2} + \frac{1}{2\sqrt{a+|b|}} \int\limits_{x-\sqrt{a+|b|}t}^{x+\sqrt{a+|b|}t} \psi(\xi)d\xi$$

$$+ \int\limits_0^t \frac{-\,|b|+b}{2\sqrt{a+|b|}} \int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)} \frac{\varphi_{\xi\xi}(\xi+\sqrt{a-|b|}\tau) + \varphi_{\xi\xi}(\xi-\sqrt{a-|b|}\tau)}{2} d\xi d\tau$$

$$- \int\limits_0^t \frac{1}{4\sqrt{a^2-b^2}} \int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)} \int\limits_{\xi-\sqrt{a-|b|}\tau}^{\xi+\sqrt{a-|b|}\tau} (|b|+b)\,\psi_{\lambda\lambda}(\lambda) d\lambda d\xi d\tau$$

$$+ \int\limits_0^t \frac{1}{4\sqrt{a^2-b^2}} \int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)} \int\limits_{\xi-\sqrt{a-|b|}\tau}^{\xi+\sqrt{a-|b|}\tau} g'(0,\lambda) d\lambda d\xi d\tau$$

$$\int\limits_0^t \frac{1}{4\sqrt{a^2-b^2}} \int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)} \int\limits_0^\tau \int\limits_{\xi-\sqrt{a-|b|}(\tau-p)}^{\xi+\sqrt{a-|b|}(\tau-p)} F(p,\lambda) d\lambda dp d\xi d\tau.$$

$$= J_1(t,x) + J_2(t,x) + J_3(t,x) + J_4(t,x),$$

where

$$J_1(t,x) = \frac{\varphi(x+\sqrt{a+|b|}t) + \varphi(x-\sqrt{a+|b|}t)}{2} + \frac{1}{2\sqrt{a+|b|}} \int\limits_{x-\sqrt{a+|b|}t}^{x+\sqrt{a+|b|}t} \psi(\xi)d\xi, \tag{7}$$

$$J_2(t,x) = \int\limits_0^t \frac{-\,|b|+b}{2\sqrt{a+|b|}} \int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)} \frac{\varphi_{\xi\xi}(\xi+\sqrt{a-|b|}\tau) + \varphi_{\xi\xi}(\xi-\sqrt{a-|b|}\tau)}{2} d\xi d\tau,$$

$$J_3(t,x) = -\int\limits_0^t \frac{1}{4\sqrt{a^2-b^2}} \int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)} \int\limits_{\xi-\sqrt{a-|b|}\tau}^{\xi+\sqrt{a-|b|}\tau} (|b|+b)\,\psi_{\lambda\lambda}(\lambda) d\lambda d\xi d\tau,$$

$$J_4(t,x) = \int\limits_0^t \frac{1}{4\sqrt{a^2-b^2}} \int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)} \int\limits_0^\tau \int\limits_{\xi-\sqrt{a-|b|}(\tau-p)}^{\xi+\sqrt{a-|b|}(\tau-p)} F(p,\lambda) d\lambda dp d\xi d\tau$$

$$+\int\limits_0^t \frac{1}{4\sqrt{a^2-b^2}} \int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)} \int\limits_{\xi-\sqrt{a-|b|}\tau}^{\xi+\sqrt{a-|b|}\tau} g'(0,\lambda) d\lambda d\xi d\tau.$$

Now, we will estimate $J_k(t,x)$, $k=1,2,3,4$, separately. First, we start with estimates for $J_1(t,x)$. Applying the triangle inequality and formula (7), we get

$$|J_1(t,x)| \leq M_1(a,b)\left[\sup_{x\in I}|\varphi(x)| + \int\limits_{-\infty}^{\infty}|\psi(y)|\,dy\right],$$

$$|J_{1,t}(t,x)|, |J_{1,x}(t,x)| \leq M_{11}(a,b)\left[\sup_{x\in I}|\varphi_x(x)| + \sup_{x\in I}|\psi(x)|\right],$$

$$|J_{1,tt}(t,x)|, |J_{1,tx}(t,x)|, |J_{1,xx}(t,x)| \leq M_{111}(a,b)\left[\sup_{x\in I}|\varphi_{xx}(x)| + \sup_{x\in I}|\psi_x(x)|\right]$$

for any t, $x \in I$. Second, we will estimate $J_2(t,x)$. We have that

$$J_2(t,x) = \frac{|b|-b}{2\sqrt{a+|b|}}\left[\varphi\left(x+\sqrt{a+|b|}t\right)\right.$$

$$\left.+\varphi\left(x-\sqrt{a+|b|}t\right) - \varphi\left(x+\sqrt{a-|b|}t\right) - \varphi\left(x-\sqrt{a-|b|}t\right)\right].$$

Applying the triangle inequality and formula (7), we get

$$|J_2(t,x)| \leq M_2(a,b)\sup_{x\in I}|\varphi(x)|,$$

$$|J_{2,t}(t,x)|, |J_{2,x}(t,x)| \leq M_2(a,b)\sup_{x\in I}|\varphi_x(x)|,$$

$$|J_{2,tt}(t,x)|, |J_{2,tx}(t,x)|, |J_{2,xx}(t,x)| \leq M_2(a,b)\sup_{x\in I}|\varphi_{xx}(x)|$$

for any t, $x \in I$. Third, we will estimate $J_3(t,x)$. We have that

$$J_3(t,x) = \int\limits_0^t \frac{|b|+b}{4\sqrt{a^2-b^2}}[\psi\left(x+\sqrt{a+|b|}(t-\tau)+\sqrt{a+|b|}\tau\right) + \psi\left(x-\sqrt{a+|b|}(t-\tau)-\sqrt{a+|b|}\tau\right)$$

$$-\psi\left(x-\sqrt{a+|b|}(t-\tau)+\sqrt{a-|b|}\tau\right) + \psi\left(x+\sqrt{a+|b|}(t-\tau)-\sqrt{a-|b|}\tau\right)]d\tau. \tag{8}$$

Applying the triangle inequality and formula (8), we get

$$|J_3(t,x)| \leq M_3(a,b)\int\limits_{-\infty}^{\infty}|\psi(y)|\,dy,$$

$$|J_{3,t}(t,x)|\,,|J_{3,x}(t,x)|\leq M_3(a,b)\sup_{x\in I}|\psi(x)|\,,$$
$$|J_{3,tt}(t,x)|\,,|J_{3,tx}(t,x)|\,,|J_{3,xx}(t,x)|\leq M_3(a,b)\sup_{x\in I}|\psi_x(x)|$$

for any t, $x\in I$. Fourth, we will estimate $J_4(t,x)$. We have that

$$J_4(t,x)=\frac{1}{4\sqrt{a^2-b^2}}\int\limits_0^t\int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)}\int\limits_0^\tau\int\limits_{\xi-\sqrt{a-|b|}(\tau-r)}^{\xi+\sqrt{a-|b|}(\tau-r)}\left[-ag_{\lambda\lambda}(r,\lambda)+bg_{\lambda\lambda}(-r,\lambda)\right]d\lambda dr d\xi d\tau$$
$$+\frac{1}{4\sqrt{a^2-b^2}}\int\limits_0^t\int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)}\int\limits_0^\tau\int\limits_{\xi-\sqrt{a-|b|}(\tau-r)}^{\xi+\sqrt{a-|b|}(\tau-r)}g_{rr}(r,\lambda)d\lambda dr d\xi d\tau$$
$$+\frac{1}{4\sqrt{a^2-b^2}}\int\limits_0^t\int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)}\int\limits_{\xi-\sqrt{a-|b|}\tau}^{\xi+\sqrt{a-|b|}\tau}g'(0,\lambda)d\lambda d\xi d\tau.$$

Applying formulas

$$\int\limits_0^\tau\int\limits_{\xi-\sqrt{a-|b|}(\tau-r)}^{\xi+\sqrt{a-|b|}(\tau-r)}\left[-ag_{\lambda\lambda}(r,\lambda)+bg_{\lambda\lambda}(-r,\lambda)\right]d\lambda dr=\frac{2a}{\sqrt{a-|b|}}g(\tau,\xi)-\frac{2b}{\sqrt{a-|b|}}g(-\tau,\xi),$$
$$\int\limits_0^\tau\int\limits_{\xi-\sqrt{a-|b|}(\tau-r)}^{\xi+\sqrt{a-|b|}(\tau-r)}g_{rr}(r,\lambda)d\lambda dr=2\sqrt{a-|b|}g(\tau,\xi)-\int\limits_{\xi-\sqrt{a-|b|}\tau}^{\xi+\sqrt{a-|b|}\tau}g'\left(0,\lambda\right)d\lambda,$$

we get

$$J_4(t,x)=\frac{1}{2\sqrt{a^2-b^2}}\int\limits_0^t\int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)}\left[\frac{a}{\sqrt{a-|b|}}g(\tau,\xi)-\frac{b}{\sqrt{a-|b|}}g(-\tau,\xi)\right]d\xi d\tau$$
$$+\frac{1}{2\sqrt{a^2-b^2}}\int\limits_0^t\int\limits_{x-\sqrt{a+|b|}(t-\tau)}^{x+\sqrt{a+|b|}(t-\tau)}\sqrt{a-|b|}g(\tau,\xi)d\xi d\tau. \qquad (9)$$

Applying the triangle inequality and formula (9), we get

$$|J_4(t,x)|\leq M_4(a,b)\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}|g(y,x)|\,dydx,$$
$$|J_{4,t}(t,x)|\,,|J_{4,x}(t,x)|\leq M_4(a,b)\sup_{x\in I}\int\limits_{-\infty}^{\infty}|g(y,x)|\,dy,$$
$$|J_{4,tt}(t,x)|\,,|J_{4,tx}(t,x)|\,,|J_{4,xx}(t,x)|\leq M_4(a,b)\sup_{t,x\in I}|g(t,x)|$$

for any t, $x\in I$. Combining the estimates for $J_k(t,x), k=1,2,3,4$, we obtain estimates (1b)-(1d).

2 Conclusion

In the present paper, the initial value problem for the hyperbolic type time involution linear partial differential equation is investigated. The equivalent initial value problem for the fourth order linear partial differential equations to the initial value problem for this second order linear partial differential equations with involution is presented. The stability estimates for the solution and its first and second order derivatives of this problem are proved.

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Уақытты инволюциялы гиперболалық типті дифференциалдық теңдеу жайында

Мақалада екінші ретті дербес туындылардағы t сызықтық теңдеудегі гиперболалық типтегі инволюциялық теңдеудің бастапқы есебі зерттеледі. Төртінші ретті дербес туындылы теңдеулер үшін осы есептің эквивалентті бастапқы есебі алынды. Жоғарыда аталған есептің шешімінің, бірінші және екінші ретті туындыларының тұрақтылық бағалаулары алынды.

Кілт сөздер: инволюциялық типті гиперболалық теңдеу, тұрақтылық, Банах кеңістігі.

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О дифференциальном уравнении гиперболического типа с инволюцией по времени

В статье изучена начальная задача для инволютивного уравнения гиперболического типа в t линейном уравнении в частных производных второго порядка. Получена эквивалентная этой задаче начальная задача для уравнений в частных производных четвертого порядка. Установлены оценки устойчивости решения и его производных первого и второго порядка указанной выше задачи.

Ключевые слова: гиперболическое уравнение инволютивного типа, устойчивость, банахово пространство.

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