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A note on epidemiologic models: SIR modeling of the COVID-19 with variable coefficients

The coronavirus disease 2019 (COVID-19) has been responsible for over three million reported cases worldwide. The construction of an appropriate mathematical (epidemiological) model for this disease is a challenging task. In this paper, we first consider susceptible — infectious — recovered (SIR) model with constant parameters and obtain an approximate solution for the SIR model with varying coefficient as it is one of the simplest models and many models are derived from this framework. The numerical experiments confirm that the proposed formulation demonstrates similar characteristic behaviour with the well-known approximations.

Keywords: infectious diseases, COVID-19, mathematical modeling, SIR model, variable coefficients.

Introduction

The coronavirus disease (COVID-19) was pronounced as a major health hazard by World Health Organization (WHO) in late December 2019 [1]. At present, this disease is affecting over 200 countries and territories around the world, and the global number of COVID-19 cases is increasing rapidly. In early December of 2019, this infectious disease has first broken out in China. Although, the disease in China seems to be under control, there are still many infections around the world. The high rate of the infection spread and the number of fatalities makes the understanding of the current epidemiological models more important than ever before. A considerable amount of research works of different complexity levels from simple to complicated ones has been devoted to defeat the disease, which include a lot of problem parameters. The introduced models encountered in the literature are typically based on systems of ordinary differential equations (ODEs) or partial differential equations (PDEs) [2, 3]. Although the PDE models allow one to describe dynamics in time and space; they are not simple to formulate, analyze, and solve.

The most relevant mathematical models relating to the spread of the disease is the susceptible — infectious — recovered (SIR) model [4–8], susceptible — exposed — infectious — removed (SEIR) model [5, 9–11], the susceptible — infectious — susceptible (SIS) model [12, 13]. The SEIR, SIR and SIS models can also reflect the dynamics of different epidemics such as Human Immunodeficiency Virus (HIV), Severe Acute Respiratory Syndrome (SARS) and they have also been used to model the COVID-19 [8, 11]. There are also other strategies such as the logistic model [14, 15], the susceptible — asymptomatic — recovered — infected — isolated infected — quarantined susceptible ($SARII_qS_q$) model [16], the susceptible — unquarantined — quarantined — confirmed (SUQC) model [17], the susceptible — exposed — insusceptible — quarantined — recovered — death (SEIQRDP) model [18] to describe the trend of COVID-19 [19]. Although many studies use ordinary differential equations (ODEs) to predict the susceptible, infected, and recovered populations, it is worth mentioning the PDE models for the spread of an epidemic. The SIR model has been studied in [20] by constructing a hyperbolic Kolmogorov PDE for the discrete-stochastic model, in the large population limit. Moreover, the dynamics of SIR type reaction-diffusion epidemic model with specific nonlinear incidence rate has been investigated in [21]. The study of suitable PDE models for the COVID-19 case will be detailed in a forthcoming work. It should also be noted that complicated models need more effort as they include a lot of variables and require a detailed analysis for their validation which makes the procedure difficult in the absence of reliable data.

In this work, we consider well-known SIR model to simulate the process of COVID-19 which is proposed by Kermack and McKendrick [22]. There are different strategies to understand the predictions of this model and

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the behavior of its solutions. Kermack and McKendrick [22] reduced this problem with constant parameters to a single differential equation and derived an approximate solution in terms of a hyperbolic secant function. The classical SIR model contains two time-invariant coefficients: The transmission rate β and the removal (recovering) rate γ which neglects the time-varying property of β and γ. However, it is too simple to effectively predict the trend of the disease. Therefore, assuming the ratio of the transmission and removal rates to remain constant when both rates are functions of time variable t, we study a time-dependent SIR model and obtain approximate solution of such model which allows changing infection and removal rates for the latest COVID-19 data.

The rest of this paper is organized as follows. In Section 1, we briefly introduce the SIR model with constant coefficients and its approximate solution. We present approximate solution of SIR model with time-dependent coefficients in Section 2. In Section 3, we provide some numerical tests to illustrate the performance of proposed formulation for both constant and variable coefficient cases.

1 The SIR Model with Constant Coefficients

In 1927, a set of equations studied by Kermack and McKendrick [22] to investigate the dynamics of an infectious disease in three groups: Susceptible (S), infectious (I), and recovered (R) whose sizes are functions of time t, that is,

$$\begin{cases} \frac{dS(t)}{dt} = -\beta\, I(t)\, S(t), \qquad S(t_0) = S_0, \\ \frac{dI(t)}{dt} = \beta\, I(t)\, S(t) - \gamma I(t), \qquad I(t_0) = I_0, \\ \frac{dR(t)}{dt} = \gamma\, I(t), \qquad R(t_0) = R_0 \end{cases} \tag{1}$$

together with a fixed population size N,

$$S(t) + I(t) + R(t) = N. \tag{2}$$

Here, $S(t)$ represents the number of susceptible individuals not yet infected with disease at t, $I(t)$ stands for the number of infectious individuals who have been infected and are in danger of spreading the disease to the susceptibles, and $R(t)$ is the number of removed (and immune) or deceased individuals. The constant parameter β stands for the infection rate, and the average infectious period is $1/\gamma$ days. The initial conditions are given by: $S(t_0) = S_0, I(t_0) = I_0, R(t_0) = R_0 \geq 0$.

The mentioned simple system is appropriate for estimating the dynamics of the COVID-19 in different countries [23–25] by using the freely available statistics provided by the European Centre for Disease Prevention Control and World Health Organisation [26]. Therefore, this model has been taken as a background by many researchers for modeling COVID-19 in various countries of the world as it provides a simple procedure. The equations are generally solved numerically. Kermack and McKendrick [22] first reduced this problem to a single differential equation as in the following way: Using problem (1) and equality (2), we have

$$\frac{dR(t)}{dt} = \gamma(N - S(t) - R(t)) \tag{3}$$

and

$$\frac{dS}{dR} = -\frac{\beta}{\gamma}\, S$$

which is a separable differential equation and it can be solved for S and then substituted to the equation (3) to get:

$$\frac{dR(t)}{dt} = \gamma(N - S_0 e^{-\frac{\beta}{\gamma}(R(t)-R_0)} - R(t)).$$

Since it is not possible to find R as an explicit function, by assuming that $\frac{\beta}{\gamma}R$ is small compared with unity, the exponential term can be expanded in powers of $\frac{\beta}{\gamma}R$. Thus, we have

$$\frac{dR(t)}{dt} = \gamma\Big(N - S_0 - R_0 + \Big(\frac{\beta}{\gamma}S_0 - 1\Big)R(t) - \frac{\beta^2}{2\gamma^2}S_0 R(t)^2 + ...\Big). \tag{4}$$

Moreover, Kermack and McKendrick [22] neglegted some terms in (4) that is,

$$\frac{dR(t)}{dt} \approx \gamma\Big(N - S_0 - R_0 + \Big(\frac{\beta}{\gamma}S_0 - 1\Big)R(t) - \frac{\beta^2}{2\gamma^2}S_0R(t)^2\Big),$$

and derived the following approximate solution of the SIR model for the removal rate, $\frac{dR}{dt}$, in terms of a hyperbolic function,

$$R(t) \approx \frac{\gamma^2}{\beta^2 S_0}\Big(\frac{\beta}{\gamma}S_0 - 1 + \sqrt{-\mu}\tanh\Big(\frac{\sqrt{-\mu}}{2}\gamma t - \phi\Big)\Big),$$

where

$$\begin{cases} \phi = \tanh^{-1}\frac{\frac{\beta}{\gamma}S_0 - 1}{\sqrt{-\mu}}, \\ \sqrt{-\mu} = \Big(\frac{\beta}{\gamma}S_0 - 1\Big)^2 + 2S_0I_0\frac{\beta^2}{\gamma^2}. \end{cases}$$

We note that the rate of infection β can be changed by vaccination or isolation of infected individuals and the rate of removal γ can be changed by the use of different medicines or treatment protocols. Moreover, changing infection rate β and removal rate γ for the latest coronavirus data may allow one to track the reproductivity of the COVID-19 through time and to assess the effectiveness of the control measures implemented by the public health authorities [27]. This can be achieved by using time-dependent $\beta(t)$ and $\gamma(t)$ functions, rather than constants β and γ which is the subject of the following section.

2 The SIR Model with Time-Dependent Coefficients

In this section, we consider a generalized version of the SIR model in which the infectious rate β and the removal rate γ may vary with respect to time when the ratio $\frac{\beta(t)}{\gamma(t)}$ remains constant. Replacing β and γ by $\beta(t)$ and $\gamma(t)$ in problem (1) yields,

$$\begin{cases} \frac{dS(t)}{dt} = -\beta(t)\ I(t)\ S(t), \qquad S(t_0) = S_0, \\ \frac{dI(t)}{dt} = \beta(t)\ I(t)\ S(t) - \gamma(t)I(t), \qquad I(t_0) = I_0, \\ \frac{dR(t)}{dt} = \gamma(t)\ I(t), \qquad R(t_0) = R_0. \end{cases} \tag{5}$$

By following the steps in Section 1, we wish to solve the following problem which is a Riccati differential equation that is quadratic in the unknown function R:

$$\frac{dR(t)}{dt} - \gamma(t)\Big(N - S_0 - R_0 + \Big(\frac{\beta(t)}{\gamma(t)}S_0 - 1\Big)R(t) - \frac{\beta(t)^2}{2\gamma(t)^2}S_0R(t)^2\Big) \approx 0. \tag{6}$$

Then, it is easy to verify that the solution of problem (6) is given by,

$$R(t) = \frac{-1 + S_0q - \sqrt{\psi}\tan\Big(\sqrt{\psi}\,\Big(C_1 + \int_0^t \frac{1}{2}\gamma(x)\,dx\Big)\Big)}{S_0q^2}, \tag{7}$$

where

$$\begin{cases} q = \frac{\beta(t)}{\gamma(t)}, \\ \psi = -1 + S_0q(q(-2N + 2R_0 + S_0) + 2), \\ C_1 = \frac{1}{\sqrt{\psi}}\arctan(\frac{S_0q - 1}{\sqrt{\psi}}). \end{cases}$$

Once the unknown function $R(t)$ is calculated, $I(t)$ and $S(t)$ can be obtained by using problem (5):

$$
\begin{aligned}
I(t) &= \frac{1}{\gamma(t)} \frac{dR(t)}{dt}, \\
S(t) &= S_0 e^{-qR(t)}.
\end{aligned}
$$

We note that, due to the nature of the problem such a SIR model with time-dependent coefficients is much better to track the disease spread, control, and predict the future trend of the disease.

Remark 1. It is worth mentioning that the mean square analytical solution of a Riccati equation of random coefficients under some assumptions is studied in [28]. Moreover, the solution (7) can be seen as a solution of SIR model in a different form when the infectious rate β and the removal rate γ are constant. For more details, we refer the readers to [22].

3 Numerical Results

3.1 Experiments with Constant Coefficients

In this case, we report some numerical experiments and compare the performance of two numerical solutions: The solution by Kermack and McKendrick and the present formulation for the SIR model with constant coefficients. We remark that the numerical results are encouraging and the proposed formulation has similar features with the solution obtained by Kermack and McKendrick.

Experiment 1a:

We first investigate the following test problem in [6],

$$
\begin{cases}
\frac{dS(t)}{dt} = -\beta \, I(t) \, S(t), \qquad S(0) = S_0, \\
\frac{dI(t)}{dt} = \beta \, I(t) \, S(t) - \gamma \, I(t), \qquad I(0) = I_0, \\
\frac{dR(t)}{dt} = \gamma \, I(t), \qquad R(0) = R_0.
\end{cases}
$$

In this study, the authors study the spread of ongoing COVID-19 when $t \in [0, 200]$; $\beta = 2/14$; $\gamma = 1/14$; $S_0 = 0.999$; $I_0 = 0.001$; $R_0 = 0$; $N = 1$ (normalized version). In Figure 1, we represent elevation plots of the solutions obtained with the present formulation and the Kermack and McKendrick formulation. The results of the numerical experiments have similar features with the strategies which shows that it can be used to estimate COVID-19 epidemic trend.

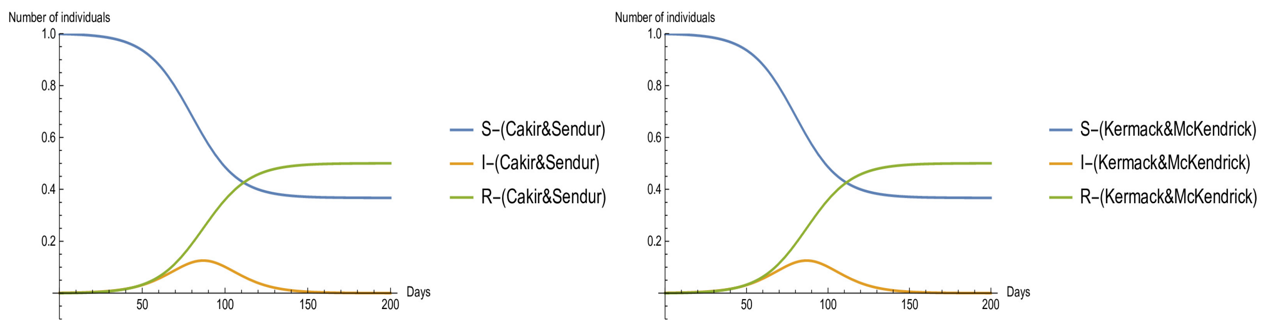

Figure 1. SIR model solutions. Left: Solution by Cakir and Sendur, Right: Solution by Kermack and McKendrick

Experiment 1b:

We consider the test problem in [25]. In this study, the data for the COVID-19 disease outbreak is adjusted the Kermack and McKendrick approximation of the SIR model. We set the problem parameters to find the solution of the SIR model for different countries in Table 1. We present only the elevation plots for China in Figure 2 as the data for other countries produces similar features in capturing the behavior of the solution. The results show that the SIR model is a good choice to get a better understanding of COVID-19.

Table 1

The parameters to solve SIR model for different countries

Country	γ	β	S_0	I_0
China	0.08	4.564 e-06	85631	5
Spain	0.08	8.416 e-07	265551	1
Italy	0.08	8.597 e-07	258511	1
France	0.08	1.096 e-06	179659	3
Germany	0.08	1.053 e-06	206003	1
Argentina	0.019	5.784 e-07	155575	200
Mexico	0.0908	3.174 e-07	479575	13

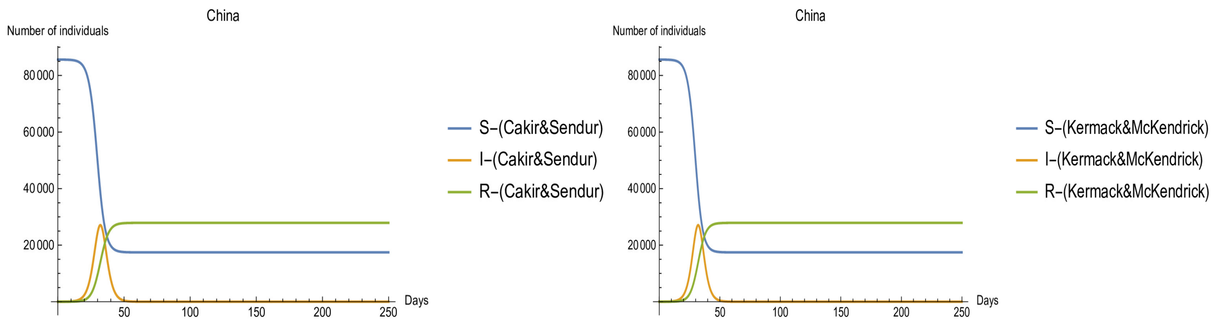

Figure 2. SIR model solutions for China. Left: Solution by Cakir and Sendur, Right: Solution by Kermack and McKendrick

3.2 Experiments with Time-Dependent Coefficients

In this case, we report some numerical experiments to display the performance of the present formulation for the SIR model with time dependent coefficients when $\beta(t) = q\,p\,t^r$; $\gamma(t) = p\,t^r$ for several values of p, q, r. With the above choice, the solution can be rewritten in the following form:

$$R(t) = \frac{-1 + qS_0 - \sqrt{\varphi}\tan\left(\frac{p\sqrt{\varphi}}{2(r+1)}t^{r+1} + \tan^{-1}\left(\frac{qS_0-1}{\sqrt{\varphi}}\right)\right)}{q^2 S_0},$$

where

$$\varphi = -1 + qS_0(q(-2N + 2R_0 + S_0) + 2).$$

Once the unknown function $R(t)$ is calculated, $I(t)$ and $S(t)$ can be obtained by following the steps in Section 2.

Experiment 2a:

We investigate the following test problem with time-dependent coefficients:

$$\begin{cases} \frac{dS(t)}{dt} = -\beta(t)\, I(t)\, S(t), \qquad S(t_0) = S_0, \\ \frac{dI(t)}{dt} = \beta(t)\, I(t)\, S(t) - \gamma(t)I(t), \qquad I(t_0) = I_0, \\ \frac{dR(t)}{dt} = \gamma(t)\, I(t), \qquad R(t_0) = R_0, \end{cases}$$

when $t \in [0, 200]$; $t_0 = 0$; $S_0 = 0.999$; $I_0 = 0.001$; $R_0 = 0$.

In Figure 3, we first set $q = 2$, $p = 0.02$ and illustrate the behavior of the solution obtained with the present formulation for increasing values of $r : 0 < r \leq 1$. We observe high number of infectious individuals at later stages when r is smaller and high number of infected individuals at early stage when r is increasing.

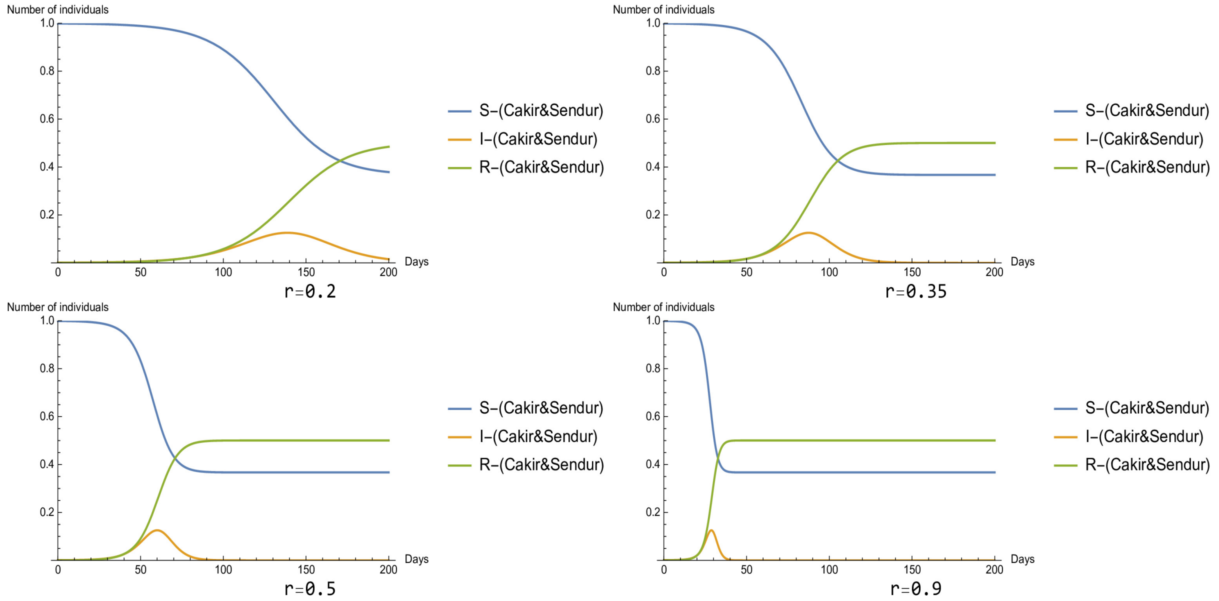

Figure 3. SIR model solutions when $\beta(t) = 2\gamma(t)$; $\gamma(t) = t^r/50$

Experiment 2b:

Next, we investigate the behavior of the solution with respect to the ratio $q = \frac{\beta}{\gamma}$. In Figure 4, we set $p = 1/100$, $r = 0.5$ and demonstrate the behavior of the solution obtained with the present formulation for increasing values of q. We note that relatively more susceptible individuals can complete the disease process without being infected when $1.4 < q < 1.6$. This situation can be explained with the existence of high-quality health care services, individuals' protection awareness and high rates of COVID-19 vaccinations. Moreover, we observe that more individuals have been infected and are in danger of spreading the disease to the susceptible for increasing values of q. The numerical results are encouraging and the approximate solution captures the characteristic behavior of the problem.

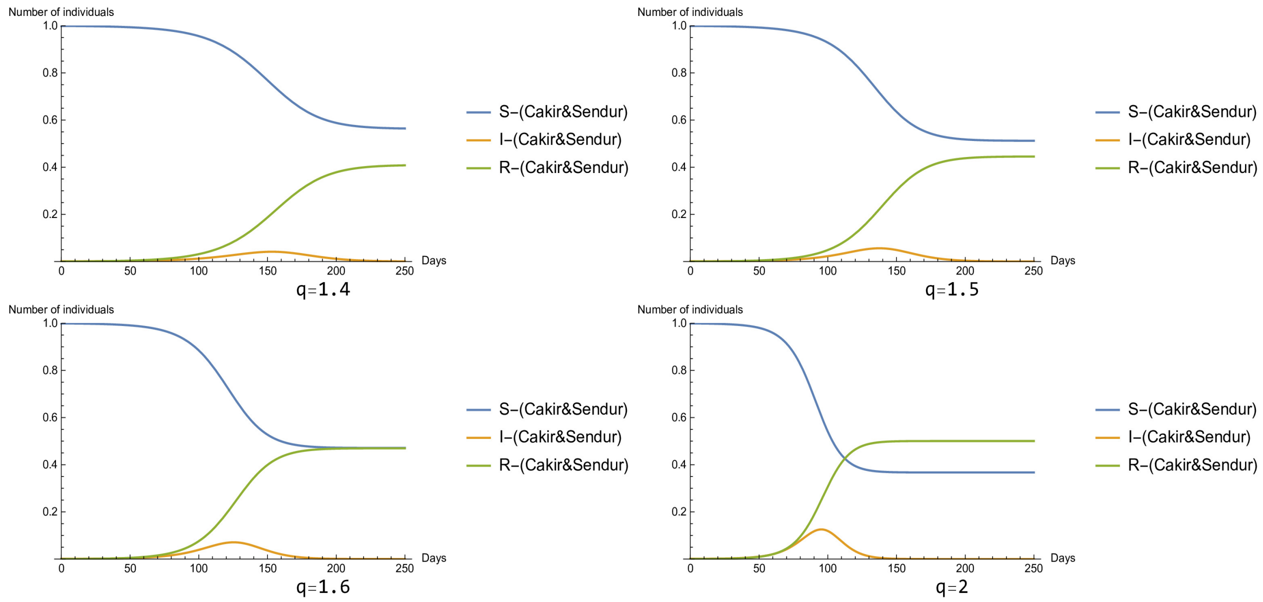

Figure 4. SIR model solutions when $\beta(t) = q\,\gamma(t)$; $\gamma(t) = t^{0.5}/100$

Conclusions

The main advantage of SIR models comes from its ability to establish a balance between simplicity and usefulness. Therefore, we investigate the approximate solutions of the SIR epidemiological model which has been widely used for over 100 years. Numerical experiments confirm the good performance of the proposed formulation for a wide range of problem configurations. Consequently, the SIR model captures some features of the COVID-19 behavior and thus, it could provide guidance for the evolution of the pandemic with only two parameters. Moreover the coefficients for the transition to the infectious or recovered (removed) compartment, namely β and γ, do not remain fixed during the spread of the disease. Indeed, the transmission coefficients could be different for various cases, such as active tourism season, Christmas, the start and end periods of education, festivals, periods in which measures are applied tightly or loosened. For this reason, the SIR model with time-dependent coefficients seems much better to analyze the trend of the disease.

We note that, recently, many remarkable complicated models including a lot of parameters have been used to understand the COVID-19 cases. However, it is not easy to determine which mathematical model describes the COVID-19 outbreak best. Furthermore, a simpler model is not better or worse than a more complicated model and using complicated models may not be more reliable compared to using a simpler model. The investigation of various suitable models for the COVID-19 case, a comparison of such models ranging from simple to more complicated ones for specific countries and the highlight of their strengths and weaknesses in different situations can be considered as a future work. We also note that many studies use ordinary differential equations (ODEs) to predict the susceptible, infected, and recovered populations for the COVID-19 case. It is also remarkable to consider the spatial effects in the spread of epidemics for the mobility of people within a country and the regional levels of risk (effects of transboundary spread, face mask requirement, quarantine, lockdown, etc., among county clusters). This situation can be modelled by partial differential equations (PDEs) and it is a subject of a new research.

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Эпидемиологиялық модельдер туралы мақала: өзгермелі коэффициенттері бар SIR COVID–19 модельдеу

2019 жылғы коронавирус ауруы (COVID-19) бүкіл әлемде тіркелген үш миллионнан астам жағдайға себеп болды. Бұл аурудың жеткілікті математикалық (эпидемиологиялық) моделін құру қиын міндет. Мақалада алдымен тұрақты параметрлері бар "сезімтал — жұқпалы — қалпына келтірілген" (SIR) моделі қарастырылған және өзгермелі коэффициенті бар SIR моделіне жуық шешім алынған, өйткені бұл қарапайым модельдердің бірі және көптеген модельдер осы құрылымның туындысы болып табылады. Сандық эксперименттер ұсынылған тұжырымның белгілі жуықтауларымен ұқсас сипаттағы тәртібін көрсететінін растайды.

Кілт сөздер: жұқпалы аурулар, COVID-19, математикалық модельдеу, SIR моделі, айнымалы коэффициенттер.

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Заметка об эпидемиологических моделях: моделирование SIR COVID–19 с переменными коэффициентами

Коронавирусная болезнь 2019 года (COVID–19) стала причиной более трех миллионов зарегистрированных случаев заболевания во всем мире. Построение адекватной математической (эпидемиологической) модели этого заболевания является сложной задачей. В статье рассмотрена модель «восприимчивый — заразный — выздоровевший» (SIR) с постоянными параметрами и получено приближенное решение для модели SIR с переменным коэффициентом, поскольку это одна из самых простых моделей, и многие модели являются производными от этой структуры. Численные эксперименты подтверждают, что предложенная формулировка показывает сходное характерное поведение с известными приближениями.

Ключевые слова: инфекционные болезни, COVID–19, математическое моделирование, SIR–модель, переменные коэффициенты.