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Properties of lattices of the existential formulas of Jonsson fragments

This article is devoted to studying of the properties of model-theoretic concepts of a fragment of the Jonsson sets and their application to the lattices of existential formulas. The concept of the Jonsson set allocates as a special subset of the semantic models for the considered the Jonsson theory. Next, we present some model-theoretic properties of the fragments of the considered Jonsson sets that were previously considered for the Jonsson theories. These properties describe the relationship between the lattices of the existential formulas and a center of the considered fragment.

Key words: Jonsson set, semantic model, lattices of existential formulas, center of fragment, complement, weak complement, the pseudo-complement, the Stone algebra.

Introduction. This article is devoted to studying of the properties of model-theoretic concepts of fragment Jonsson sets and its application. Jonsson concept set was defined in [1] and further results were obtained, which were presented in [2–4]. And we will look at some model-theoretic properties of fragments considered Jonsson sets that were previously considered to Jonsson theories [5–9].

On the other hand natural examples of the Jonsson theories are many, this is, for example, the theories of Boolean algebras, Abelian groups, fixed-field characteristics, polygons, and so on. All of these examples are important in algebra, and the various branches of mathematics. As you can see, from listed a scope of application of the technique developed for studying the Jonsson theories can be quite broad.

Thus, all of the above suggests that the study of the model-theoretic properties of the Jonsson theories is an urgent task.

It is well known that the Jonsson theory are a natural subclass of this broad class of theories, as a class of the inductive theories. If the case study of complete theories we are mainly dealing with two objects is the theory itself and its model, in the case study The Jonsson theory as models we consider the class of existentially closed models of the theory, as well as a necessary condition for a certain completeness of this theory in logical sense. At least, this theory must be existentially complete.

As mentioned above, the basic theories of algebra examples are examples of the inductive (Jonsson) theories and they tend to represent an example of incomplete theories. This modern device model theory developed mainly for the complete theories, and so the study of modern technology incomplete theories noticeably less meaningful than for the complete theories.

Studying the inductive theories [5], it follows that the Jonsson theory as a subclass of inductive theories are such a part where there are the certain methods of investigation incomplete theories, namely the method of transfer of the first-order properties of the Jonsson center on the Jonsson theory itself. This method of research and in the study the Jonsson theories and unrelated to the contents of this article, we refer the reader to the following sources [6, 7].

We give a definition of the fragment:

We say that all the $\forall\exists$-consequences of the Jonsson theory create a Jonsson fragment of this theory, if the deductive closure of $\forall\exists$-consequences is the Jonsson theory.

Due to the fact that this is not always true (that it will the Jonsson theory), it would be interesting to be able to allocate in an arbitrary theory a part which will the Jonsson theory. This problem is the place to be if only because of the fact that morlizatsiya arbitrary theory that it provides us morlizatsiya arbitrary The Jonsson theory is a theory, moreover, the resulting theory is perfect. [5]

Another way of obtaining the Jonsson theory is the use of the fact that any countable model of the theory of inductive necessarily isomorphic to invest in some existentially closed model of the theory [5]. Next, we consider all $\forall\exists$-offers true in this model. Then in the case the inductive Jonsson theory is well known that $\forall\exists$-true offers in this existentially closed model form the Jonsson theory.

Recall that the Jonsson theory is called perfect if the semantic model saturated.

At this point quite well studied are committed the Jonsson theory. To them has been proven criteria while [6], which provided many model-theoretic facts about the Jonsson theory and its center. There is a complete description of how the center of such theories and models of their classes.

Let *T*-Jonsson perfect theory for the complete existential sentences in the language *L*, and it has a semantic model *C*.

We say that the set *X* - Σ-definable if it is definable some existential formula.

a) The set *X* is called the Jonsson theory *T* if it satisfies the following properties:

X is Σ-definable subset of *C*;

dsl (*X*) is a medium-existentially closed submodel *C*.

The definition the Jonsson sets can see that they work very simply in terms of rank Morley [3, 4]. It turns out that the elements of the set-theoretic difference (wells) and a plurality of circuit have rank 0; they are algebraic. So, this is a case where we can work with the elements, even in an incomplete way.

The second point the utility of such a definition Jonsson set is that we are closing this set just get some existentially closed model. This in turn enables us to determine Jonsson first fragment from the set in question, and, in principle, have any theory.

To study the behavior of the elements in the case of wells Jonsson sets, we can always consider the consequences of ∀∃-true in the above circuits Jonsson set. In view of the above, in this case, that the review will be a lot of proposals the Jonsson theory.

Resulting in this case it will be called the Jonsson theory fragment corresponding set. It is clear that we can carry out research Jonsson fragments with respect to an initial theory, which is a new formulation of the problem study the Jonsson theory.

This article discusses the fragments Jonsson sets which are subsets of a semantic model of the Jonsson theory countable first-order language. A series of results that establish a connection between the properties of the fragment and the theory Jonsson, Jonsson Central complement of the theory and the properties of the lattice of equivalence classes of existential formulas on this fragment under consideration. In terms of the lattice formulas introduced in [10] (complementarity, pseudo-complementarity, the weak complementarity, algebra Stone), necessary and sufficient conditions for the elimination of quantifiers central complement the Jonsson theory, positive model completeness central complement the Jonsson theory, perfect Jonsson theory yonsonovosti Complement central moiety.

In the study of complete theories of one of the main methods is the use of the properties of a topological space $S_n(T)$ of ultrafilters Boolean algebra $F_n(T)$ of fixed T. With this method, we study these classical concepts of model theory as stability models and theory, the saturation model, homogeneous model, a model diagram, etc. In the case of an incomplete theory, we can consider the lattice $E_n(T)$ of existential formula, which is a sublattice of the Boolean algebra $F_n(T)$. Due nonclosure existential formulas in the general case with respect to the Boolean logic operation properties of a topological space of existential types is significantly different from the complete case. It is clear that such an approach (limit $F_n(T)$ up $E_n(T)$) is a generalization of the case when we are dealing with a complete theory. Since Jonsson theories are, generally speaking, incomplete, it would be interesting to consider the properties of the lattice existential formulas in connection with the above mentioned context (for example, in [6]). The main research tool Jonsson semantic theory is the method proposed at the time by Professor T.G.Mustafin [7], the essence of which is the translation of the central properties of the prototype of the complement for Jonsson. In this paper, in addition to the semantic method [6] and the other outcomes of the Jónsson theories [11–20]. used concepts and results from [10] V. Weispfenning.

The work consists of two sections. In the first section we present a list of those definitions and results from [10], that are required to obtain the main results of this work. In the second section discusses the fragments of the Jonsson set of the Jonsson theory and prove to them, the «Jonsson» analogs of the theorems in [10] on the basis of this article, the authors of [21–23].

Paragraph 1. Lattices of existential formulas. We introduce the definitions and give the related results on the lattice properties of existential formulas, based on [10, 24, 25].

Let *L* — first-order language. Let *T* — inductive theory of L. We denote by $E_n(L)$ the set of existential formulas of *L* with n free variables, $E(L)=\bigcup\limits_{n<\omega}E_n(L)$. Let $E_n(T)$ — distributive lattice of equivalence classes $\varphi^T=\{\psi\in E_n(L)\mid\ T\vdash\varphi\leftrightarrow\psi\}$, $\varphi\in E_n(L)$, $E(T)=\bigcup\limits_{n<\omega}E_n(T)$.

Definition 1.1 [10]. Let $\varphi^T, \psi^T \in E_n(T)$ and $\varphi^T \cap \psi^T = 0$. Then ψ^T called the complement of φ^T, if $\varphi^T \cup \psi^T = 1$; ψ^T called pseudo-complement of φ^T, if for all $\mu^T \in E_n(T)$ $\varphi^T \cap \mu^T = 0 \Rightarrow \mu^T \le \psi^T$; ψ^T called a weak complement of φ^T, if for all $\mu^T \in E_n(T)$ $(\varphi^T \cup \psi^T) \cap \mu^T = 0 \Rightarrow \mu^T = 0$.

Definition 1.2 [10].

1) φ^T is called complemented if φ^T has a complement.
2) φ^T is called weakly complemented if φ^T has a weak complement.
3) φ^T is called pseudo-complemented if φ^T has a pseudo-complement.
4) $E_n(T)$ is complemented, if each $\varphi^T \in E_n(T)$ is complement.
5) $E_n(T)$ is called weakly complemented, if every $\varphi^T \in E_n(T)$ is weakly complement.
6) $E_n(T)$ is called pseudo-complemented if each $\varphi^T \in E_n(T)$ is pseudo-complement.

Next, consider the formula, stable with respect to extensions of models and sub-models.

Definition 1.3 [10]. The formula $\varphi(x_1,...,x_n)$ is called resistant with respect to extensions models in $ModT$, if for any models A and B of T such that $A \subset B$, and for any $a_1,...,a_n \in A$ of that $A \models \varphi[a_1,...,a_n] \Rightarrow B \models \varphi[a_1,...,a_n]$.

Theorem 1.1 [10]. The formula φ is stable under extensions models in $ModT$ if and only if there is an existential formula ψ such that $T \vdash \varphi \leftrightarrow \psi$.

Definition 1.4 [10]. The formula $\varphi(x_1,...,x_n)$ is called resistant with respect to the sub-models in $ModT$, if for any models A and B of T such that $A \subset B$, and for any $a_1,...,a_n \in A$ from that $B \models \varphi[a_1,...,a_n] \Rightarrow A \models \varphi[a_1,...,a_n]$.

Theorem 1.2 [10]. The formula φ is resistant with respect to the sub-models in $ModT$ if and only if there exists a universal formula ψ such that $T \vdash \varphi \leftrightarrow \psi$.

Consider the notion of invariant formula and the relationship between invariant of existential formula and complementarity of its class in $E(T)$.

Definition 1.5 [10]. The formula φ is called invariant in $ModT$, if it is resistant at the same time with respect to extensions models in $ModT$ and relatively sub-models in $ModT$.

Theorem 1.3 [10]. The existential formula φ is invariant in the $ModT$ if and only if φ^T is a complement in $E(T)$.

Theorem 1.4 [10]. The existential formula φ is invariant in the $Mod(Th_{\forall\exists}(E_T))$, where E_T — class of existentially closed models of *T* if and only if φ^T is weakly complemented in $E(T)$.

We introduce the necessary definitions and formulate known results that establish a link between the model completeness, quantifier elimination, positive model completeness of T and the properties of the lattice existential formulas $E_n(T)$.

Definition 1.6 [25]. The theory *T* is model complete, if $T \cup \Delta_A$ is complete in the language L_A for any model *A* of *T*.

Theorem 1.5 [25].

1) The theory *T* is model-complete if and only if every formula is stable relatively the sub-models in the $ModT$.

2) The theory *T* is model-complete if and only if every formula is stable relatively to extensions of models in the $ModT$.

Definition 1.7 [25]. It is said that the theory T admits elimination of quantifiers in *L*, if for every formula $\varphi(x_1,...,x_n)$ of *L* there is a quantifier-free formula is such that $T \vdash \forall x_1 ... \forall x_n (\varphi(x_1,...,x_n) \leftrightarrow \psi(x_1,...,x_n))$.

Theorem 1.6 [5].

1) Let *T*' '— a model companion of *T*, where *T* — a universal theory. In this case, *T*' '— model completion of *T* if and only if the theory *T* admits elimination of quantifiers.

2) Let *T*' — a model companion of the theory *T*. In this case *T*' — model completion of *T* if and only if the theory *T* has amalgamation property.

Definition 1.8 [25]. The theory T is called a submodel complete if $T \cup \Delta_A$ is complete in L_A for any submodel A model of T.

Theorem 1.7 [25]. The theory T is a submodel complete if and only if T admits elimination of quantifiers.

Theorem 1.8 [10]. The theory T is a submodel complete if and only if each $\varphi^T \in E_n(T)$ has a quantifier-free addition.

Definition 1.9 [22]. The theory T is a positive model complete if it is model complete and each existential formula of L is equivalent in T to existential positive formula.

In the following theorems obtained in [1], communication is established between the above-defined concepts and properties of the lattice existential formulas $E_n(T)$.

Theorem 1.9 [10]. The theory T is positive complete if and only if everyone $\varphi^T \in E_n(T)$ has a positive existential addition.

Theorem 1.10 [10]. The theory T has a model companion if and only if $E_n(T)$ is weakly complemented.

Definition 1.10 [24]. A lattice is called the Stone algebra if for any of its elements the following is true: the pseudo-addition of a pseudo-element supplement equal to the element itself.

Theorem 1.11 [10]. The theory T has a model complement if and only if $E_n(T)$ — the Stone algebra.

Theorem 1.12 [10]. The theory $T_\forall$ has a model complement if and only if each $\varphi^T \in E_n(T)$ has a weak quantifier-free addition.

Paragraph 2. The Jonsson sets, their fragments and connection with the considered Jonsson theory in terms of properties of lattices of existential formulas of these theories.

Consider the Jonsson theories and establish a connection between the properties of the Jonsson theory, Central replenish of the Jonsson theory and properties of the lattice of equivalence classes of existential formulas on this theory. To do this, we will use the results of [15–17].

Let us give the following definitions.

Definition 2.1. The theory T is said Jonsson, if

1) T has an infinite model;
2) T $\forall\exists$-axiomatizable;
3) T has the joint embedding property (JEP), that is, any two models $A \models T$ and $B \models T$ isomorphicaly embedded in some model $C \models T$;
4) T has the property of amalgamation (AR), ie if for any $A, B, C \models T$ such that $f_1 : A \to B$, $f_2 : A \to C$ are isomorphic embedding $g_1 : B \to D$, $g_2 : C \to D$ such that $g_1 f_1 = g_2 f_2$.

Definition 2.2 [9]. The semantic model C_T of the Jonsson Theory T called ω^+ - homogeneous — universal model of the theory T (in the sense of [19]).

The following definitions are given in [10].

Definition 2.3 [24]. Let $\kappa \geq \omega$. The model M of the theory T is called

- κ-universal for T if every model of T of cardinality strictly less κ is isomorphically embedded in M;
- κ-homogeneous for T if for any two models A and $A1$ of T, M submodels are strictly less power and isomorphism $f{:}A \to A1$, for each extension in the Model A, which is a model and a submodel M T is strictly less power, there is an extension $B1$ of the model $A1$, which is a submodel of M and an isomorphism $g{:}B \to B1$, extending f.

Definition 2.4 [24]. The homogeneous-universal model for the T called κ-homogeneous- universal model for T with cardinality κ where $\kappa \geq \omega$.

Definition 2.5 [9]. The centre (central complement) of the Jonsson theory T is denoted as $T^* = Th(C_T)$.

Definition 2.6 [9]. The Jonsson theory T is called perfect if every semantic model C_T is a saturated model of T^*.

In [5] established a connection between the perfect of the Jonsson theory and existence of its model companion. In the future, we will need the following statements.

Theorem 2.1 [9]. Let T — the Jonsson theory. Then the following conditions are equivalent:

1) T is perfect;
2) T has a model companion.

In [8, 9] established a connection between the completeness and completeness of the model Jonsson theory.

Theorem 2.2. Let T is a perfect the Jonsson theory. Then the following conditions are equivalent:

1) *T* is complete;

2) *T* is model complete.

In [5] established a connection between the perfect of the Jonsson theory and properties of lattice $E_n(T)$. The following assertion holds.

Theorem 2.3. Let *T* — complete for $\exists$-sentences of the Jonsson theory. Then the following conditions are equivalent:

T is perfect;

*T** is model-complete;

$E_n(T)$ -Boolean algebra,

where the completeness of the theory for $\exists$-sentences means that any two models of this theory relatively the existential sentences do not differ from each other.

In connection with the above results on the introduced concepts, we obtained results relating the concepts of [10] with the theories and the Jonsson fragments of Jonsson subsets according to their semantic models.

Given a certain the Jonsson theory *T* and *X* Jonsson subset of its semantic model. *M* is existentially closed model where dcl(*X*) = *M*.

Consider $Th_{\forall E}(M) = T_M$.

In the following theorem in terms of the lattice existential formulas $E_n(T_M)$ found necessary and sufficient conditions for the elimination of quantifiers Jonsson central complement of *T* and positive model completeness central complement of Jonsson theory *T*.

Theorem 2.4. Let T_M — complete for $\exists$-sentences the Jonsson theory *T** — the center of the theory T_M. Then

*T** admits elimination of quantifiers if and only if each has $\varphi^T \in E_n(T_M)$ a quantifier-free complement;

*T** is positive model-complete if and only if each $\varphi^T \in E_n(T_M)$ has a positive existential complement.

Proof.

Let *T* admits elimination of quantifiers. Then by Theorem 1.7. *T** is a submodel complete. Then the theory *T** in the definition is model complete, and by Theorem 2.3 $E_n(T_M)$ is a Boolean algebra, ie, everyone $\varphi^T \in E_n(T_M)$ has some addition. Due to the elimination of quantifiers *T**, as *T** — completion of the theory T_M, then and relatevly on the theory T_M each $E_n(T_M)$ has some quantifier-free addition.

Conversely, suppose that everyone $\varphi^T \in E_n(T_M)$ has the quantifier-free addition. Then $E_n(T_M)$ a Boolean algebra, then by Theorem 2.3, T * is model-complete, and then, in turn, by virtue of paragraph 2 of Theorem 1.5. we have that any formula on the theory T * is equivalent to some existential formula, ie, It belongs to the class of this formula. By $\exists$-completeness theory T_M $E_n(T_M) = E_n(T^*)$ .Consequently, due to the fact that everyone $\varphi^T \in E_n(T_M)$ has the quantifier-free addition and $E_n(T_M)$ is a Boolean algebra, any formula in $E_n(T^*)$ an unquantified. Thus, the theory *T** admits elimination of quantifiers.

Let the theory *T** positive model complete. Then, by definition 1.9. theory *T** is model complete and for each existential formula φ there is a positive existential formula ψ such that $T^* \models \varphi \leftrightarrow \psi$. By Theorem 2.3 $E_n(T_M)$ is a Boolean algebra, ie, everyone $\varphi^T \in E_n(T_M)$ has an existential addition, and because for each existential formula φ there is a positive existential formula ψ such that $T^* \models \varphi \leftrightarrow \psi$, we get that everyone $\varphi^T \in E_n(T_M)$ has a positive existential addition. Thus, a necessary condition of paragraph 2 is proved.

Let us prove the sufficiency of paragraph 2. Let everyone $\varphi^T \in E_n(T_M)$ has a positive existential addition. Then by Theorem 1.9. positive theory T is model-complete, and therefore, by definition, a model is complete. Then by Theorem 2.2. we have, that the theory T_M) is complete, and so the theory T^* is the central theory of the completion T_M, we find that $T_M = T^*$. Thus, a positive T^* is model complete.

The proof of Theorem 2.4. ending.

In the following theorem in terms of the lattice existential formulas $E_n(T_M)$ found necessary and sufficient conditions of perfect the Jonsson theory T.

Theorem 2.5. Let T_M — the Jonsson theory. Then the following conditions are equivalent:

T_M — perfect;

$E_n(T_M)$ — weakly complemented;

$E_n(T_M)$ — the Stone algebra.

Proof. We prove in 1) 2). Let the Jonsson theory T_M is perfect, then by Theorem 2.1, it has a model companion ${T_M}^M$. From [4] that ${T_M}^M = {T_M}^0$, where ${T_M}^0 = Th_{\forall\exists}(E_{T_M})$ — Kaiser shell of the Jonsson theory T_M. Since the definition of the model companion ${T_M}^M$ is model complete, we have by under paragraph 1 of Theorem 1.5, that every formula of the language is stable relatively to sub-models in $Mod{T_M}^M$. Consequently, each existential formula of the language is stable with relatively to sub-models in $Mod{T_M}^M$, at the same time, each existential formula of the language is stable relatively extensions models in $Mod{T_M}^M$, and therefore, by definition 1.5 this formula is invariant in $Mod{T_M}^M$. Hence, by Theorem 1.4. it follows that each existential formula weakly complemented. Thus, $E_n(T_M)$ weakly complemented.

We prove from 2) to 1). If $E_n(T_M)$ weakly complemented, by Theorem 1.10. theory T_M has a model companion. Then by Theorem 2.1 T_M perfect. Thus, 1) is equivalent to 2).

We prove from 1) to 3). Note that by under paragraph 2 of Theorem 1.9.the model companion of the Jonsson theory is its a model completion. Then from complete of theory T by Theorem 1.11 implies that $E_n(T_M)$ — the Stone algebra.0

We prove from 3) to 1). If $E_n(T_M)$ — the Stone algebra, Theorem 1.11. T_M theory has a model companion, and therefore, by Theorem 2.1. Theory T_M perfect.

The proof of Theorem 2.5. ending.

In the following theorem in terms of the lattice of formulas found necessary and sufficient conditions of yonsonovosti center of the Jonsson theory.

Theorem 2.6. Let T_M — the Jonsson theory. Then the following conditions are equivalent:

T^* — the Jonsson theory;

everyone $\varphi^T \in E_n(T_M)$ has a weak quantifier-free complement.

To prove the necessity we need the following statement:

The fact (*) [6]. If a model companion ${T_M}^M$ defined, then defined the model companion $(T_{M\forall})^M$ and ${T_M}^M = (T_{M\forall})^M$.

Proof. We prove from1) to 2). Let T^* — the Jonsson theory, while from [9] that the theory T_M perfect. Then by Theorem 2.1 the theory T_M has a model companion of T^* is equal to by paragraph 2 of Theorem 1.6. is the model completion of the theory T_M. By virtue of the mutual consistency of the model theory T_M and the theory $T_{M\forall}$ — all consequences of the theory of universal T_M and fact (*) model completion of the theory T_M is a model completion theory T_M. Then, by Theorem 1.12. each $\varphi^T \in E_n(T_M)$ has a weak quantifier-free complement.

We prove from 2) to 1). Let everyone $\varphi^T \in E_n(T_M)$ has a weak quantifier-free addition. Then everyone $\varphi^T \in E_n(T_M)$ has a weak addition, ie $E_n(T_M)$ weakly complemented. Then by Theorem 2.5. theory T_M perfect. Then, from [4] follows that the theory T^* is the Jonsson theory.

The proof of Theorem 2.6. ending.

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Йонсондық фрагментінің экзистенциалдық формулалар торларының қасиеттері

Мақала йонсондық жиынның фрагмент ұғымының теориялық-модельдік қасиеттерін зерттеуге және оларды экзистенциалды формулалардың торына қолдануна арналған. Йонсондық жиынның ұғымы қарастырылып отырған йонсондық теорияның семантикалық моделінің арнайы ішкі жиындарын бөліп алады. Әрі қарай бұрын қарастырылған йонсондық теориялар үшін қарастырылып отырған йонсондық жиындар фрагменттерінің кейбір теориялық-модельдік қасиеттерін көрсетеміз. Бұл қасиеттер экзистенциалды формулалар торлары мен қарастырылып отырған фрагментттің арасындағы байланыстарды жан-жақты сипаттайды.

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Свойства решёток экзистенциальных формул йонсоновского фрагмента

Статья посвящена изучению теоретико-модельных свойств понятия фрагмента йонсоновского множества и их применения к решетке экзистенциальных формул. Понятие йонсоновского множества выделяет специальные подмножества семантической модели рассматриваемой йонсоновской теории. Далее мы приводим некоторые теоретико-модельные свойства фрагментов рассматриваемых йонсоновских множеств, которые были ранее рассмотрены для йонсоновских теорий. Эти свойства описывают связи между решётками экзистенциальных формул и центра рассматриваемого фрагмента.

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