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## Fuzzy Multi-criteria Selection of Alternatives by Quasi-best Case as the Basis for Choosing Robotic Machine-Assembling Technologies

The process of choosing the robotic machine-assembling technologies (RMAT) is implemented as the fuzzy multi-criteria selection of alternatives by the suggested previously method of quasi-best case. The basic concept feature of the given method is the developed specific correlations which are based on the corresponding comparisons to better alternative P<sup>o</sup>nd to the most important criterion. All the mentioned above determines the practical and scientific value of this paper. The results of strict ranking of the elements of local criteria discrete set (LCDS) are input dP<sup>o</sup>ta. It is performed by the method of expert survey and at the same time demonstrates RMAT phenomenon. The idea of selections the process of ordering constituents of initially unordered LCDS elements which finally form ordered set. The selection is performed within the set of these elements. The obtained ordering of RMAT phenomenon as a result of selection is recommended to be analyzed in the process of choosing. The base of solving the task of RMAT selection is its formalized description and on its base the generalized content formalisms of quasi-best case method are determined. The performance of the presented the theoretical issues is demonstrated step-by-step with the real example of the automated RMAT selection.

*Keywords:* alternative, automation, selection, local criterion, fuzziness, optimization, robotic machine-assembling technology, quasi-best case.

### 1 Introduction

Industrial robots (IR) are widely used in modern automated machine - assembling enterprises of machine and instrument engineering which implement robotic machine-assembling technologies (RMAT). The International Federation of Robotics (IFR) reports that annual increment in release and introduction of such universal and expensive means of industrial automation, which IR of various design and technology performance are [1], is about (14–16)% [2, 3] for recent years. The conduct of different by content and formulation researches is important and topical in order to improve the effectiveness and further development of robotic machine-assembling enterprises. It is recommended to develop either latest or using and modifying known approaches, methods and techniques by adapting them to specifics of formulation and content of the tasks solved.

One of the problems which occur here is the problem of proper RMAT selection taking into account their final set that is generated before [4]. It implies the preliminary determination and analysis of the ordered sequence of the selection local criteria (see further).

Every RMAT is presented with the set of phenomena which are local criteria discrete set (LCDS)  $S = (S_j | j = \overline{1, m})$ . Its components are the following [5]:  $Gm$  — geometric;  $Kn$  — kinematic;  $Dn$  — dynamic;  $Ct$  — control;  $En$  — energy;  $Tr$  — trajectory;  $\tau(Q)$  — time (productivity);  $Rl$  — reliability;  $Ec$  — economy;  $Ac$  — accuracy;  $Fc$  — force;  $Fopt$  — the component which is determined by other types of optimization criteria (e.g. technical and economic [6] etc.):

$$S = (Gm, Kn, Dn, Ct, En, Tr, \tau(Q), Rl, Ec, Ac, Fc, Fopt). \quad (1)$$

The complexity and content feature of RMAT selection tasks can be explained: by the necessity to take into account the desired multi-criteria of the extremity of every LCDS criterion; by the obvious ambivalence related to the ordering of LCDS elements consideration; by the necessity of decision-making within the set with the obtained alternatives. It can be possible provided that there are various criteria with different physical nature and different measurement scales (see above). The presented task is a task of multi-objective optimization in terms of prior ambivalence. The methods of finding solutions to such tasks are featured by variability and multiplicity [7].

The idea of many approaches to solving such tasks is in using information obtained from the experts as a result of sampling by survey method [8]. Here, the calculated value of Kendall concordance coefficient  $W$  [8] is the correlation of expert opinions. The ordered sequence (list) of local criteria is the desired solution. This sequence is formed at the correlation of expert opinions  $W \approx 1$  (ideally  $W = 1$ ). The using of rank correlation method is possible in other case [8]. The criterion for decision-making here is also the value of  $W$  with its indicated interpretation. If  $W$  is  $\ll 1$ , using of other approaches is possible. The examples of such approaches can be the method of pair-wise comparison of alternatives, which is based on the ideas of Bellman-Zade [9, 10], and Saaty hierarchies [11] and also fuzzy multi-criteria selection of alternatives by worst-case suggested by Rotstein [12]. The first ones among them (methods of Bellman-Zade and Saaty) are time-consuming and it is due to the performance of total alternative enumerating at the pair-wise comparison and the long processing of matrix information with the further computation of membership function as to every single expert as well as to every single alternative. The latter approach (Rotstein method) does not require time-consuming matrix formation of pair-wise comparison and further processing of this information. Relatively simple computation a ratios correlations are used instead. It is profoundly compared to the worst alternative and the least important criterion [12].

The scale of corresponding rates in the given task is 12-score one (by the number of elements within LCDS, see the expression (1). It is the scale used by every among 10 experts ( $n = 10$ ) to estimate each local criterion of LCDS without repeating estimations for various local criteria, i.e. the strict ranking [8] of LCDS is performed. Here the least important criterion obtained score 1 and the most important one got 12.

The result analysis of expert survey (see Table 1) as to consistency of experts by Kendall concordance coefficient for the primary processing ( $W = 0,204$ ) and by rank correlation ( $W = 0,271$ ) demonstrates the discordance of the experts' opinions. Such discordance does not contradict the possibility of using other methods of fuzzy multi-criteria selection of alternatives, e.g. by the method of the worst-case [12], that has been already used while selecting RMAT fuzzy multi-criteria [13].

The main idea of the method of quasi-best case [14] applied here is the answer to the regular question: why would not make another principle as the basis of the process of fuzzy multi-criteria selection of alternatives, if the optimal solution used by any approaches at fuzzy multi-criteria alternative selection is either de-jure or de-facto unknown. The principle of comparison of each from local criterion of LCDS to other local criterion, for example, the best one, can serve as a solution. The method is named as "quasi-best" due to the relativity of the term "best".

Table 1

Matrix  $M_c[n \times m]$  of expert survey results  $(i; c_j)$

	$S_1 = Gm$	$S_2 = Kn$	$S_3 = Dn$	$S_4 = Ct$	$S_5 = En$	$S_6 = Tr$	$S_7 = \tau(Q)$	$S_8 = Rl$	$S_9 = Ec$	$S_{10} = Ac$	$S_{11} = Fc$	$S_{12} = Fopt$
$E_1$	11	12	3	10	4	9	8	5	6	7	2	1
$E_2$	5	4	2	3	7	6	10	9	11	8	1	12
$E_3$	9	10	8	7	5	12	6	4	1	11	3	2
$E_4$	5	9	10	12	6	4	7	3	8	1	11	2
$E_5$	12	11	4	5	6	7	2	8	3	9	10	1
$E_6$	3	8	4	2	12	5	11	7	9	10	6	1
$E_7$	9	8	7	10	5	4	2	12	1	11	6	3
$E_8$	7	10	6	5	12	11	9	3	2	8	1	4
$E_9$	4	5	7	8	9	1	6	12	10	11	2	3
$E_{10}$	9	12	3	5	7	11	10	2	6	8	4	1
$\sum S_j$	74	89	54	67	73	70	71	65	57	84	46	30

In general, the substantive *features* of the method of quasi-best case [14] used is the decrease of subjectivity level at the process of forming the ordered set of LCDS criteria and using the corresponding comparisons to the best alternative as well as to the most important criterion. It is the most important while solving the given task.

Therefore, the adapted to the content of quasi-best case method, the content of the task for RMAT multi-criteria selection becomes the fuzzy forming the ordered set of local criteria from LCDS for their further analysis. It is performed within the set of alternatives (experts'  $\mathbb{T}^{\text{TM}}$  opinions) and their content is determined by the results of strict ranking of LCDS elements. Taking into account all the mentioned above, the purpose of the given paper is to increase feasibility and to decrease time-consuming factor of decision-making at selecting RMAT. It is recommended to use scientific and methodological set ordering of discrete local criteria of RMAT phenomenon by applying fuzzy multi-criteria selection of alternatives with the method of quasi-best case.

## 2 Formalized task statement

In its general form the essence of fuzzy multi-criteria RMAT selection [14] comes to forming LCDS element set  $S = (S_j | j = \overline{1, m})$  from initially unordered one into finally ordered set  $S_{\langle \rangle} = S_j | j = \overline{1, m}$  as a result of execution of some certain computation procedures  $\varphi = (\varphi_k | k = \overline{1, l})$  with total number  $l$ . In the given case  $l = 7$  equals the number of methodically determined steps **P $\mathcal{b}$**  (see further). Every  $\varphi_k$ -th computation procedure, as well as its corresponding step, implements the obtaining of intermediate and final results. The latter ones are calculated within the information sets of input data, namely, within the set of experts  $E = (E_i | i = \overline{1, n})$  and RMAT  $S = (S_j | j = \overline{1, m})$ . Therefore, the simplified form is:

$$\varphi : (E \times S) \rightarrow \langle S_{(j)\max} \rangle \quad (2)$$

Here  $\rightarrow$  is the symbol of surjective reflection of input data with united by Cartesian product (symbol  $\times$ ) to corresponding computation data [15], which are implemented by the mentioned above set of computation procedures  $\varphi$  from [14]:  $\varphi = (\varphi_c, \varphi_w, \varphi_\alpha, \varphi_{Ew}, \varphi_{Ew^a}, \varphi_{(j)\max}, \varphi_{\langle \rangle})$ .

Expression (2) obtains the form:

$$\begin{aligned} & (\varphi = (\varphi_k | k = \overline{1, l}) : (((((((((E = (E_i | i = \overline{1, n})) \times (S = (S_j | j = \overline{1, m})) \xrightarrow{K1} \\ & ((i c_j) \subset M_c) \xrightarrow{K2} ((i w_j) \subset M_w) \xrightarrow{K3} ((i \alpha_j) \subset M_\alpha) \xrightarrow{K4} ((E w_j) \subset M_{Ew}) \xrightarrow{K5} \\ & \left( (i w_j^{(i \alpha_j) \max}) \subset M_{Ew^\alpha} \right) \xrightarrow{K6} (i w_j^{\alpha \max}) \subset (S_{(j)\max}) \xrightarrow{K7} \langle S_{(j)\max} \rangle | \forall i = \overline{1, n}; \forall j = \overline{1, m}. \end{aligned} \quad (3)$$

Here **P $\mathcal{b}$ 1**, ..., **P $\mathcal{b}$ 7** are the designations of methodically resulted steps that correspond the implementation of the corresponding procedure set  $\varphi = (\varphi_c, \varphi_w, \varphi_\alpha, \varphi_{Ew}, \varphi_{Ew^a}, \varphi_{(j)\max}, \varphi_{\langle \rangle})$  with [14], namely:

– **P $\mathcal{b}$ 1** is in correspondence with the procedure  $\varphi_c$ , that forms matrix  $M_c$  of final results of strict expert ranking, matrix  $M_c$  elements  $(i c_j)$  are integral natural numbers. The value and significance of every number are determined by ranking conditions;

– **P $\mathcal{b}$ 2** corresponds the implementation of procedure  $\varphi_w$  which determines elements  $(i w_j)$  of matrix  $M_w$  as the weight of all alternatives by relation of ranks of all  $E_i$ -th alternatives to the rank of the best alternative  $S_{j\max}$ ;

– step **P $\mathcal{b}$ 3** (procedure  $\varphi_\alpha$ ) is used to form elements  $(i \alpha_j)$  of matrix  $M_\alpha$  as fuzzy set taking into account the significance of every  $S_j$ -th criterion due to its weight  $\alpha_j$  within the set of alternatives  $E$ ;

– the content of step **P $\mathcal{b}$ 4** is the implementation of procedure  $\varphi_{Ew}$ , that determines the importance of estimation of every  $E_i$ -th expert from the set  $E$  by determining the weights of alternatives related to  $S_j$ -th criterion, i.e. finding elements  $(E w_j)$  of matrix  $M_{Ew}$ ;

– **P $\mathcal{b}$ 5** implements procedure  $\varphi_{Ew^a}$  of determination of significance of alternative (of every  $E_i$ -th expert) by the weight  $E \alpha$  of every of them within the set of criteria  $S$  forming elements  $(i w_j^{(i \alpha_j) \max})$

of matrix  $M_{Ew^\alpha}$  as fuzzy set; the very content of step **K5** determines the significant peculiarities of quasi-best case method;

– by implementing **Pл6**, elements  $({}_i w_j^{(\alpha_j) \max})$  max of set  $(S_{(j) \max} | j = \overline{1, m})$  of fuzzy estimations for every local criterion  $S_{j \max}$  are formed. It means that unordered set of membership functions is formed within the set of their highest values and this is the content of procedure  $\varphi_{(j) \max}$  execution;

– **Pл7** is used to implement procedure  $\varphi_{<>}$  that orders elements of unordered set  $(S_{(j) \max} | j = \overline{1, m})$  obtained within **K6** into the ordered one  $S_{<>} = \langle S_{(j) \max} | j = \overline{1, m} \rangle$  by solving *max-max* task. This is the final solution to the task of fuzzy multi-criteria selection of alternatives using the method of quasi-best case.

All mentioned above matrixes and name!Cf  $M_c, M_w, M_\alpha, M_{Ew^\alpha}$  and  $M_{Ew^\alpha}$  have the dimension  $[n \times m]$  and sets  $(S_{(j) \max} | j = \overline{1, m})$  and  $\langle S_{(j) \max} | j = \overline{1, m} \rangle$  have dimension  $[1 \times m]$ .

### 3 Task solution

The process of fuzzy multi-criteria of RMAT selection is presented step by step **Pл1**, ..., **Pл7** based on the results of actual strict expert sampling. It is performed by using the proposed method in accordance with its content [14] and formalized statement of the given task (see expression (3)).

**Pл1**. Forming matrix  $M_c[n \times m]$  of the results of expert sampling. Every element  ${}_i c_j$  of this matrix is estimation (rank) of 12-score scale, which was used by every  $E_i$ -th expert to estimate every  $S_j$ -th criterion. Matrix  $M_c[n \times m]$ , as well as other matrixes, has the form of a table in the given case Table 1.

**K2**. The calculation of weights  ${}_i w_j$  of alternatives by ratio of ranks  ${}_i r_j$  of all  $E_i$ -th alternatives (Table 1) to the rank of the best alternative  ${}_i w_j \max$ . The latter one for every  $S_j$ -th criterion is determined as following:

$${}_i w_j \max = \frac{{}_i r_j \max}{\sum_{i=1}^n {}_i r_j} \mid \forall j = \overline{1, m}. \tag{4}$$

Here the nominator is actually the total of ranks (estimations) given by the experts of set  $E$  for every  $S_j$ -th criterion. For matrix  $M_c[n \times m]$  (Table 1) this is the total of elements from its every column. For example, the highest rank (score) 12 was given by expert  $E_5$  to  $S_1 = Gm$ . It means that this rating is the highest  ${}_5 r_{Gm} \max = 12$  by the given criterion within the set of all experts (alternatives). Here and further the pre subscript specifies the reference number of expert (in the given case  $i=5$ ) by Table 1. The weight of the best alternative  $Gm_5 w_{Gm} \max$  within the set of estimations of all experts (alternatives) taking into account (4) is determined with following expression:

$$\begin{aligned} \sum_{i=1}^n {}_i r_j &= \frac{{}_1 r_{Gm}}{{}_5 r_{Gm} \max} + \frac{{}_2 r_{Gm}}{{}_5 r_{Gm} \max} + \frac{{}_3 r_{Gm}}{{}_5 r_{Gm} \max} + \frac{{}_4 r_{Gm}}{{}_5 r_{Gm} \max} + \frac{{}_5 r_{Gm}}{{}_5 r_{Gm} \max} + \frac{{}_6 r_{Gm}}{{}_5 r_{Gm} \max} + \\ &+ \frac{{}_7 r_{Gm}}{{}_5 r_{Gm} \max} + \frac{{}_8 r_{Gm}}{{}_5 r_{Gm} \max} + \frac{{}_9 r_{Gm}}{{}_5 r_{Gm} \max} + \frac{{}_{10} r_{Gm}}{{}_5 r_{Gm} \max} = \\ &= \frac{{}_1 r_{Gm} + {}_2 r_{Gm} + {}_3 r_{Gm} + {}_4 r_{Gm} + {}_5 r_{Gm} + {}_6 r_{Gm} + {}_7 r_{Gm} + {}_8 r_{Gm} + {}_9 r_{Gm} + {}_{10} r_{Gm}}{{}_5 r_{Gm} \max}; \\ {}_5 w_{Gm} \max &= \frac{1}{\frac{{}_1 r_{Gm} + {}_2 r_{Gm} + {}_3 r_{Gm} + {}_4 r_{Gm} + {}_5 r_{Gm} + {}_6 r_{Gm} + {}_7 r_{Gm} + {}_8 r_{Gm} + {}_9 r_{Gm} + {}_{10} r_{Gm}}{{}_5 r_{Gm} \max}} = \\ &= \frac{1}{\frac{11}{12} + \frac{5}{12} + \frac{9}{12} + \frac{5}{12} + \frac{12}{12} + \frac{3}{12} + \frac{9}{12} + \frac{7}{12} + \frac{4}{12} + \frac{9}{12}} = \frac{12}{74} = 0,1622. \end{aligned}$$

Obtained similar to (4) weights for all other criteria of set  $S$  practically form matrix  $M_w[n \times m]$  and are added to Table 2.

Table 2

Matrix  $M_w[n \times m]$  of alternative weights ( ${}_i w_j$ ) for various criteria as fuzzy set

	<i>Gm</i>	<i>Kn</i>	<i>Dn</i>	<i>C</i>	<i>E</i>	<i>T</i>	$\tau(Q)$	<i>Rl</i>	<i>Ec</i>	<i>Ac</i>	<i>Fc</i>	<i>Fopt</i>
$E_1$	.1487	<b>.1348</b>	.0556	.1493	.0548	.1286	.1127	.0769	.1053	.0833	.0435	.0333
$E_2$	.0676	.0449	.0370	.0448	.0969	.0857	.1409	.1385	<b>.1930</b>	.0952	.0217	<b>.4000</b>
$E_3$	.1216	.1124	.1482	.1045	.0685	.1714	.0845	.0615	.0175	<b>.1310</b>	.0652	.0667
$E_4$	.0676	.1011	<b>.1852</b>	<b>.1791</b>	.0822	.0571	.0986	.0462	.1404	.0119	<b>.2391</b>	.0667
$E_5$	<b>.1622</b>	.1236	.0741	.0746	.0822	.1000	.0282	.1231	.0526	.1071	.2174	.0333
$E_6$	.0406	.0899	.0741	.0299	<b>.1644</b>	.0714	<b>.1549</b>	.1077	.1579	.1191	.1304	.0333
$E_7$	.1216	.0899	.1296	.1493	.0685	.0571	.0282	<b>.1846</b>	.0175	<b>.1310</b>	.1304	.1000
$E_8$	.0946	.1124	.1111	.0746	<b>.1644</b>	.1571	.1268	.0462	.0351	.0952	.0217	.1333
$E_9$	.0541	.0562	.1296	.1194	.1233	.0143	.0845	<b>.1846</b>	.1754	<b>.1310</b>	.0435	.1000
$E_{10}$	.1216	<b>.1348</b>	.0556	.0746	.0959	.1571	.1409	.0308	.1053	.0952	.0870	.0333

Here, the following condition is met for every  $S_j$ -th criterion (column of the table 2):

$$\left( \sum_{i=1}^n {}_i w_j | \forall j = \overline{1, m} \right) = 1 \tag{5}$$

Obtained elements of matrix  $M_w[n \times m]$  (Table 2) are quantitative estimation of membership degree for every  $S_j$ -th criterion from LCDS to fuzzy sets. They can be explanted as weights which are included into fuzzy sets (4).

Obtained weights of alternatives for various criteria (see Table 2) allow to present criteria as fuzzy sets that are given within the universal sets of alternatives. It enables every  $S_j$ -th criterion forming set  $D_w$  by selecting maximum element (underlined in Table 2). Here and further some data in brackets [...] is not calculated but it is of informative character. Finally we obtain:

$$D_w = \left( \frac{{}_i w_1 \max}{]S_1[}; \dots; \frac{{}_i w_j \max}{]S_j[}; \dots; \frac{{}_i w_m \max}{]S_m[} \right) = \left( \frac{{}_i w_j \max}{]S_j[} | i = \overline{1, n}; \forall j = \overline{1, m} \right).$$

For the task of RMAT selection that is being solved in the given presentation for every  $S_j$ -th criterion, we obtain:

$$D_w = \left( \frac{0, 1622}{]Gm[}; \frac{0, 1348}{]Kn[}; \frac{0, 1852}{]Dn[}; \frac{0, 1741}{]Ct[}; \frac{0, 1644}{]En[}; \frac{0, 1714}{]Tr[}; \frac{0, 1549}{]\tau(Q)[}; \frac{0, 1846}{]Rl[}; \frac{0, 1930}{]Ec[}; \frac{0, 1310}{]Ac[}; \frac{0, 2391}{]Fc[}; \frac{0, 4000}{]Fopt[} \right).$$

The elements (i.e. the numerators)  ${}_i w_j \max$  of obtained set  $D_w$  can be explanted as a set of potentially good solutions [12].

The obtained set  $D_w$  is a fuzzy one and it contains alternative membership in relation to optimal in some sense solution. By analyzing elements of set  $D_w$  the ranking of its elements from *max* to *min* was performed and ordered set  $D_{\langle w \rangle}$  of local criteria form set  $S$  was formed. *Max-max* task is applied to demonstrate the elements of matrix  $M_w[n \times m]$  (Table 2):

$$D_{\langle w \rangle} = \left\langle \frac{0, 4000}{]Fopt[}; \frac{0, 2391}{]Fc[}; \frac{0, 1930}{]Ec[}; \frac{0, 1846}{]Rl[}; \frac{0, 1852}{]Dn[}; \frac{0, 1741}{]Ct[}; \frac{0, 1714}{]Tr[}; \frac{0, 1644}{]En[}; \frac{0, 1622}{]Gm[}; \frac{0, 1549}{]\tau(Q)[}; \frac{0, 1348}{]Kn[}; \frac{0, 1310}{]Ac[} \right\rangle.$$

**РлЗ.** Determination of significance of every single criterion within set  $S$ . Let  $\alpha_j$  be the weight for criterion  $S_j \subset S$  that characterizes its significance. Taking into account the weights of criteria from Table 2 the fuzzy set of solutions is formed as following [9]:

$$D = ({}^i w_1)^{\alpha_1} \cap ({}^i w_2)^{\alpha_2} \cap \dots \cap ({}^i w_j)^{\alpha_j} \cap \dots \cap ({}^i w_m)^{\alpha_m} \mid \forall j = \overline{1, m} = \cap_i^n ({}^i w_j)^{\alpha_j} \mid \forall j = \overline{1, m}.$$

Criteria are compared only to the most significant one (the best) among them at the next stage. Here it is accepted that the more significant the weight  $\alpha_j$  for  $S_j$ -th criterion is, the higher is its range  $R_j$  [12]:

$$\frac{\alpha_1}{R_1} = \frac{\alpha_2}{R_2} = \dots = \frac{\alpha_j}{R_j} = \dots = \frac{\alpha_m}{R_m}.$$

Let  $\alpha_j \max$  and  $R_j \max$  be weight and rank correspondingly for the most significant criterion  $S_j$ . If the requirement (5) is met regarding parameter  ${}^i \alpha_j$ , i.e.  ${}^i \alpha_j (\sum_{i=1}^n {}^i \alpha_j \mid \forall j = \overline{1, m}) = 1$  by the similar way to  ${}^i w_j$  (see expression (4)), the weights of criteria are distributed in accordance with the ranks as following:

$${}^i \alpha_{j \max} = \frac{1}{\frac{R_1}{R_{j \max}} + \frac{R_2}{R_{j \max}} + \dots + \frac{R_j}{R_{j \max}} + \dots + \frac{R_m}{R_{j \max}}} = \frac{1}{\sum_{j=1}^m \frac{R_j}{R_{j \max}}} = \frac{R_{j \max}}{\sum_{j=1}^m R_j}; \quad (6)$$

$${}^i \alpha_1 = {}^i \alpha_{j \max} \frac{R_1}{R_{j \max}}; \dots; {}^i \alpha_j = {}^i \alpha_{j \max} \frac{R_j}{R_{j \max}}; \dots; {}^i \alpha_m = {}^i \alpha_{j \max} \frac{R_m}{R_{j \max}}. \quad (7)$$

As one can see in Table 1, the total of elements in column 2 equals 89 and is the biggest one in relation to the totals of elements of other columns that characterize the ranks of other local criteria as LCDS elements. It means that criterion  $K_n$  is the most significant one resulting from the expert sampling being analyzed:

$${}^i \alpha_{j \max} = {}^i \alpha_2 = {}^i \alpha_{K_n}; \quad R_{2 \max} = \sum_{i=1}^n {}^i S_2 = 89.$$

Weights of all other local criteria of set  $S$  are calculated by using (6) and (7):

$${}^i \alpha_{K_n} = {}^i \alpha_{2 \max} = \frac{1}{\frac{74}{89} + \frac{89}{89} + \frac{54}{89} + \frac{67}{89} + \frac{73}{89} + \frac{70}{89} + \frac{71}{89} + \frac{65}{89} + \frac{57}{89} + \frac{84}{89} + \frac{46}{89} + \frac{30}{89}} = \frac{89}{780} = 0,1141;$$

$${}^i \alpha_{K_n} = {}^i \alpha_{2 \max} = {}^i \alpha_{K_n} * \frac{74}{89} = 0,1141 * \frac{74}{89} = 0,0949; \dots;$$

$${}^i \alpha_{Opt} = {}^i \alpha_{K_n} * \frac{30}{89} = 0,1141 * \frac{30}{89} = 0,0385.$$

The obtained values of weights  ${}^i \alpha_j$  for every  $S_j$ -th criterion within the set of alternatives (experts)  $E$  allow finding every fuzzy criterion within the set of alternatives as following:

$$S_j = \left( \frac{{}^i w_j^{({}^i \alpha_j)}}{{}^i E_i} \mid i = \overline{1, n}; \forall j = \overline{1, m} \right),$$

where  ${}^i w_j$  are the elements of matrix  $M_w[n \times m]$  (see Table 2);  ${}^i \alpha_j$  is the power to which all corresponding elements of matrix  $M_w[n \times m]$  are raised and it is expressed with (6) and (7).

Therefore:

$$Gm = \left( \frac{{}^i w_j^{({}^i \alpha_{Gm})}}{{}^i E_1} \mid i = \overline{1, n}; \forall j = \overline{1, m} \right) =$$

$$= \left( \frac{0,1487^{0.0949}}{E_1}; \frac{0,0676^{0.0949}}{E_2}; \frac{0,1216^{0.0949}}{E_3}; \frac{0,0666^{0.0949}}{E_4}; \frac{0,1622^{0.0949}}{E_5}; \frac{0,0405^{0.0949}}{E_6}; \right.$$

$$\left. \frac{0,1216^{0.0949}}{E_7}; \frac{0,0946^{0.0949}}{E_8}; \frac{0,0541^{0.0949}}{E_9}; \frac{0,1216^{0.0949}}{E_{10}} \right),$$

otherwise we obtain:

$$Gm = \left( \frac{0,8346}{]E_1[}; \frac{0,7744}{]E_2[}; \frac{0,8188}{]E_3[}; \frac{0,7744}{]E_4[}; \frac{0,8415}{]E_5[}; \frac{0,7378}{]E_6[}; \frac{0,8188}{]E_7[}; \frac{0,7995}{]E_8[}; \frac{0,7582}{]E_9[}; \frac{0,8188}{]E_{10}[} \right).$$

The calculations for all the other local criteria within LCDS, i.e. within set  $S$ , are performed similarly and they are transferred to Table 3. Matrix  $M_\alpha[n \times m]$  is formed and its elements are significance of every  $S_j$ -th criterion within LCDS due to its weight  $\alpha_j$  within the set of alternatives  $E$ .

Table 3

**Matrix  $M_\alpha[n \times m]$  of significance of every of its criteria due to its weight  $\alpha_j$  within the set of alternatives  $E$  as a fuzzy set**

	$Gm$	$Kn$	$Dn$	$C$	$E$	$T$	$\tau(Q)$	$Rl$	$Ec$	$Ac$	$Fc$	$Fopt$
$E_1$	.8346	.7956	.8187	.8493	.7620	.8319	.8198	.8076	.8483	.7652	.8312	<u>.8374</u>
$E_2$	.7744	.7019	.7960	.7658	.2030	.8021	.8366	.8481	.8867	.7763	.7979	<u>.9654</u>
$E_3$	.8188	.7792	.8762	.8236	.7781	.8536	.7986	.7927	.7442	.8034	.8513	<u>.0667</u>
$E_4$	.7744	.7699	.8898	.8627	.7915	.7735	.8099	.7739	.8663	.6205	<u>.9191</u>	.9011
$E_5$	.8415	.7878	.8351	.8002	.7915	.8133	.7226	.8398	.8064	.7862	<u>.9139</u>	.8774
$E_6$	.7379	.7597	.8351	.7396	.8445	.7891	.8439	.8305	.8738	.7952	<u>.8868</u>	.8774
$E_7$	.8188	.7597	.8681	.8493	.7781	.7735	.7226	.8687	.7442	.8034	.8868	<u>.9153</u>
$E_8$	.7995	.7792	.8589	.8002	.8445	.8470	.8286	.7739	.7829	.7763	.7979	<u>.9254</u>
$E_9$	.7582	.7200	.8681	.8331	.8221	.6830	.7986	.8687	.8806	.8034	.8312	<u>.9153</u>
$E_{10}$	.8188	.7956	.8187	.8002	.8030	.8470	.8357	.7482	.8483	.7763	.8659	<u>.8774</u>

Values of matrix  $M_\alpha[n \times m]$  elements (see Table 3) give the possibility to form set  $D_I$ . Its elements indicate the degree of membership as to optimal solution. It means that they contain maximum elements of matrix  $M_\alpha[n \times m]$  and they are underlined for every  $E_i$ -th expert (see denominator of every element in brackets  $]E_i[$  of set  $D_I$ ):

$$D_I = \left( \frac{0,8774}{]E_1[}; \frac{0,9654}{]E_2[}; \frac{0,9011}{]E_3[}; \frac{0,9191}{]E_4[}; \frac{0,9139}{]E_5[}; \frac{0,8868}{]E_6[}; \frac{0,9153}{]E_7[}; \frac{0,9254}{]E_8[}; \frac{0,9153}{]E_9[}; \frac{0,8774}{]E_{10}[} \right).$$

Ordering of elements of set  $D_I$  from  $max$  to  $min$  gives ordered set  $D_{(I)}$  that characterizes ordered significance of experts by the degree of membership of alternatives to optimal solution:

$$D_{(I)} = \left\langle \frac{0,9654}{]E_2[}; \frac{0,9254}{]E_8[}; \frac{0,9191}{]E_4[}; \left( \frac{0,9153}{]E_7[}; \frac{0,9153}{]E_9[} \right); \frac{0,9139}{]E_5[}; \frac{0,9011}{]E_3[}; \frac{0,8868}{]E_6[}; \left( \frac{0,8774}{]E_1[}; \frac{0,8774}{]E_{10}[} \right) \right\rangle.$$

**K4.** Determination of assessment significance ( ${}_i w_j$ ) of every  $E_i$ -th expert within their set  $E = (E_i | i = \overline{1, n})$ , i.e. calculation of alternative weights regarding every  $S_j$ -th criterion within LCDS. The content of this step is similar to **K3** execution taking into account the essence of step **K2** and data of Table 1, but it relates every  $E_i$ -th expert.

Table 1 is used to determine the biggest total of elements of columns for all local criteria within LCDS. It means that elements of set  $S$  are taken to determine rank  ${}_i r_j$  of corresponding  $S_j$ -th criterion. Obtained data is used for further calculation while performing step **K4**.

Therefore, local criterion  $Kn \quad {}_1 r_{j \max} = {}_1 r_{Kn} = 12$  obtains the highest estimation for  $E_i$ -th expert (lower left index in  ${}_i r_j$  and in  ${}_i w_j$ ). It makes the following calculations determine weights of alternatives for  $E_1$  possible:

$$\begin{aligned} {}_1 w_{Kn} &= \frac{1}{\frac{{}_1 r_{Gm}}{r_{Kn}} + \frac{{}_1 r_{Kn}}{r_{Kn}} + \frac{{}_1 r_{Dn}}{r_{Kn}} + \frac{{}_1 r_{Ct}}{r_{Kn}} + \frac{{}_1 r_{En}}{r_{Kn}} + \frac{{}_1 r_{Tr}}{r_{Kn}} + \frac{{}_1 r_{\tau(Q)}}{r_{Kn}} + \frac{{}_1 r_{Rl}}{r_{Kn}} + \frac{{}_1 r_{Ec}}{r_{Kn}} + \frac{{}_1 r_{Ac}}{r_{Kn}} + \frac{{}_1 r_{Fc}}{r_{Kn}} + \frac{{}_1 r_{Fopt}}{r_{Kn}}} \\ &= \frac{{}_1 r_{Kn}}{\sum_{j=1}^m {}_i r_j} | \forall i = \overline{1, n}. \end{aligned}$$

In general case we get the following for every  $E_i$ -th expert:

$${}_i w_j = \frac{{}_i r_j \max}{\sum_{j=1}^m {}_i r_j} | \forall i = \overline{1, n}. \tag{8}$$

The calculations for determining alternative weights for expert  $E_1$  for all local criteria, in other words criteria being analyzed within set  $S$ , are the following:

$${}_1 w_{Kn} = \frac{12}{78} = 0,1539; \quad {}_1 w_{Gm} = {}_1 w_{Kn} * \frac{r_{Gm}}{r_{Kn}} = 0,1539 * \frac{11}{12} = 0,1410; \quad \text{and so on}$$

$${}_1 w_{Fopt} = {}_1 w_{Kn}, \dots, \quad {}_1 w_{Fopt} = {}_1 w_{Kn} * \frac{r_{Fopt}}{r_{Kn}} = 0,1539 * \frac{1}{12} = 0,0128 .$$

Other elements  ${}_i w_j$  are calculated similarly following expression (8) for all criteria of set  $S$  for every expert  $E_i \in E$  and are transferred to Table 4. Maximum values of elements in every line are underlined.

**K5.** Determination of alternative significance (expert opinion) by determination of alternative weights in relation to every criterion. The content of the given step is the actions similar to **K3** actions and they relate not every  $S_j$ -th criterion, but every  $E_i$ -th alternative. Here every element of each  $i$ -th line (fuzzy information from every  $E_i$ -th expert) of matrix  $M_{E^w}[n \times m]$  (Table 4) is raised to power which is maximum element of corresponding line ( $E_i$ -th expert) of matrix  $M_\alpha[n \times m]$ , i.e.  $({}_i a_j) \max$  is underlined (see Table 3). The following set is formed in such way

$${}_i w_j^{({}_i a_j) \max} = \left( \frac{{}_i w_j^{({}_i a_j) \max}}{]S_j[} | \forall j = \overline{1, m}; i = \overline{1, n} \right). \tag{9}$$

Table 4

**Matrix  $M_{E^w}[n \times m]$  of weights of expert opinion  ${}_i w_j$  in relation to every criterion as fuzzy set**

	<i>Gm</i>	<i>Kn</i>	<i>Dn</i>	<i>C</i>	<i>E</i>	<i>T</i>	$\tau(Q)$	<i>Rl</i>	<i>Ec</i>	<i>Ac</i>	<i>Fc</i>	<i>Fopt</i>
$E_1$	.1410	<u>.1539</u>	.0385	.1282	.0513	.1154	.1026	.0641	.0769	.0897	.0256	.0128
$E_2$	.0641	.0513	.0256	.0385	.0897	.0769	.1282	.1154	<u>.1410</u>	.1026	.0128	.1539
$E_3$	.1154	.1282	.1026	.0897	.0641	<u>.1539</u>	.0769	.0513	.0128	<u>.1410</u>	.0385	.0256
$E_4$	.0641	.1154	<u>.1282</u>	<u>.1539</u>	.0769	.0513	.0897	.0385	.1026	.0128	<u>.1410</u>	.0256
$E_5$	<u>.1539</u>	.1410	.0513	.0641	.0769	.0897	.0256	.1026	.0385	.1154	.1282	.0128
$E_6$	.0385	.1026	.0513	.0256	<u>.1539</u>	.0641	<u>.1410</u>	.0897	.1154	.1282	.0769	.0128
$E_7$	.1154	.1026	.0897	.1282	.0641	.0513	.0256	<u>.1539</u>	.0128	<u>.1410</u>	.0769	.0385
$E_8$	.0897	.1282	.0769	.0641	<u>.1539</u>	.1410	.1154	.0385	.0256	.1026	.0128	.0513
$E_9$	.0513	.0641	.0897	.1026	.1154	.0128	.0769	<u>.1539</u>	.1282	<u>.1410</u>	.0256	.0385
$E_{10}$	.1154	<u>.1539</u>	.0385	.0641	.0897	.1410	.1282	.0257	.0769	.1026	.9513	.0128

For example, each element in Table 4 of matrix  $M_{E^w}[n \times m]$ , i.e. 0,1410 for *Gm* (0,1539 for *Kn* and so on) is raised to power  $({}_i a_j) \max = ({}_1 a_{Gm}) \max = 0,8774$  for every expert  $E_1$ . Therefore, we have the following for  $E_1$ :

$${}_1 w_j^{({}_1 a_j) \max} = \left( \frac{0,1410^{0,8774}}{]Gm[}; \frac{0,1539^{0,8774}}{]Kn[}; \frac{0,0385^{0,8774}}{]Dn[}; \frac{0,1282^{0,8774}}{]Ct[}; \frac{0,0513^{0,8774}}{]En[}; \right.$$

$$\frac{0,1154^{0,8774}}{]Tr[}; \frac{0,1026^{0,8774}}{] \tau(Q)[}; \frac{0,0641^{0,8774}}{]Rl[}; \frac{0,0769^{0,8774}}{]Ec[}; \frac{0,0897^{0,8774}}{]Ac[};$$

$$\left. \frac{0,0256^{0,8774}}{]Fc[}; \frac{0,0128^{0,8774}}{]Fopt[} \right),$$

otherwise, after calculations have been performed:

$${}^1w_j^{(i\alpha_j)\max} = \left( \frac{0,1793}{]Gm[}; \frac{0,1935}{]Kn[}; \frac{0,0574}{]Dn[}; \frac{0,1649}{]Ct[}; \frac{0,0738}{]En[}; \frac{0,1504}{]Tr[}; \frac{0,1356}{] \tau(Q)[}; \frac{0,0878}{]Rl[}; \frac{0,1054}{]Ec[}; \frac{0,1206}{]Ac[}; \frac{0,0402}{]Fc[}; \frac{0,0219}{]Fopt[} \right).$$

Similar calculations are executed for other experts of set  $E$  by expression (9) and all data is transferred to Table 5. The elements of this table are  $({}^i w_j^{(i\alpha_j)\max})$  and they form matrix  $M_{Ew^\alpha} [n \times m]$  by implementing computation procedure of  $\varphi_{Ew^\alpha}$  [14].

**K6.** Obtaining the fuzzy sets (solutions) of matrix  $M_{Ew^\alpha} [n \times m]$  for every  $S_j$ -th criterion. Maximum value  $({}^i w_j^{(i\alpha_j)\max}) \max$  is selected and underlined (see Table 5) for every  $S_j$ -th criterion of matrix  $M_{Ew^\alpha} [n \times m]$ . Procedure  $\varphi_{(j)\max}$  is used to form ordered (in context of selection the maximum value  $({}^i w_j^{(i\alpha_j)\max}) \max$  for every  $S_j$ -th criterion) set  $(S_{(j)\max} | j = \overline{1, m})$  of solutions:

$$(S_{(j)\max} | j = \overline{1, m}) = \left( \frac{({}^i w_j^{(i\alpha_j)\max}) \max}{]S_j[} | \forall j = \overline{1, m}; i = \overline{1, n} \right).$$

We obtain the following for the example considered in Table 5:

$$(S_{(j)\max} | j = \overline{1, m}) = \left( \frac{0,1807}{]Gm[}; \frac{0,1935}{]Kn[}; \frac{0,1514}{]Dn[}; \frac{0,1790}{]Ct[}; \frac{0,1902}{]En[}; \frac{0,1851}{]Tr[}; \frac{0,1760}{] \tau(Q)[}; \frac{0,1803}{]Rl[}; \frac{0,1526}{]Ec[}; \frac{0,1712}{]Ac[}; \frac{0,1653}{]Fc[}; \frac{0,1642}{]Fopt[} \right).$$

Table 5

**Matrix  $M_{Ew^\alpha} [n \times m]$  of weights of expert opinion  ${}^i w_j$  in relation to every criterion as fuzzy set Matrix  $M_{Ew^\alpha} [n \times m]$  of significance of alternatives (expert opinions) by determining weighs of alternatives regarding every  $S_j$ -th criterion  $({}^i w_j^{(i\alpha_j)\max})$  as fuzzy set**

	<i>Gm</i>	<i>Kn</i>	<i>Dn</i>	<i>C</i>	<i>E</i>	<i>T</i>	$\tau(Q)$	<i>Rl</i>	<i>Ec</i>	<i>Ac</i>	<i>Fc</i>	<i>Fopt</i>
$E_1$	.1793	<u>.1935</u>	.0574	.1649	.0738	.1504	.1356	.0898	.1054	.1206	.0402	.0219
$E_2$	.0705	.0568	.0291	.0431	.0976	.0840	.1377	.1243	.1509	.1110	.0149	<u>.1642</u>
$E_3$	.1429	.1571	.1285	.1139	.0841	<u>.1851</u>	.0991	.0688	.0197	<u>.1712</u>	.0531	.0368
$E_4$	.0801	.1374	<u>.1514</u>	<u>.1790</u>	.0947	.0652	.1091	.0501	.1233	.0182	<u>.1653</u>	.0345
$E_5$	<u>.1807</u>	.1669	.0662	.0812	.0959	.1104	.0352	.1248	.0509	.1390	.1530	.0187
$E_6$	.0556	.1327	.0718	.0388	<u>.1902</u>	.0875	<u>.1760</u>	.1179	.1473	.1618	.1028	.0210
$E_7$	.1386	.1244	.1101	.1526	.0809	.0660	.0350	<u>.1803</u>	.0186	<u>.1665</u>	.0956	.05078
$E_8$	.1074	.1494	.0931	.0787	.1769	.1632	.1355	.0491	.0337	.1216	.0177	.0640
$E_9$	.0660	.0809	.1101	.1244	.1386	.0186	.0956	<u>.1830</u>	<u>.1526</u>	.1665	.0350	.0507
$E_{10}$	.1504	<u>.1935</u>	.0574	.0898	.1206	.1793	.1649	.0402	.1054	.1356	.0738	.0029

**K7.** Ordering of elements of set  $(S_{(j)\max} | j = \overline{1, m})$  by implementing procedure  $\varphi_{<>}$  applying the rule from *max* to *min* in relation to fuzzy estimations (nominators of each from elements within set  $(S_{(j)\max} | j = \overline{1, m})$  of every  $S_j$ -th criterion of LCDS. It means that *max-min* task regarding elements of matrix  $M_{Ew^\alpha} [n \times m]$  is solved by ordering elements of set  $(S_{(j)\max} | j = \overline{1, m})$  forming ordered set  $\langle S_{(j)\max} | j = \overline{1, m} \rangle$  :

$$\langle S_{(j)\max} | j = \overline{1, m} \rangle = \left\langle \frac{({}^i w_j^{(i\alpha_j)\max}) \max}{]S_j[} | \forall j = \overline{1, m}; i = \overline{1, n} \right\rangle.$$

We have the following in the given case:

$$\langle S_{(j) \max} | j = \overline{1, m} \rangle = \left\langle \frac{0, 1935}{]Kn[}; \frac{0, 1902}{]En[}; \frac{0, 1851}{]Tr[}; \frac{0, 1807}{]Gm[}; \frac{0, 1803}{]Rl[}; \frac{0, 1790}{]Ct[}; \right. \\ \left. \frac{0, 1760}{]\tau(Q)[}; \frac{0, 1712}{]Ac[}; \frac{0, 1653}{]Fc[}; \frac{0, 1642}{]Fopt[}; \frac{0, 1526}{]Ec[}; \frac{0, 1514}{]Dn[} \right\rangle. \quad (10)$$

Ordered list of local criteria in brackets ]...[ is the solution to the task being solved, which is the task of multi-criteria of RMAT selection by the method of quasi-best case. It means that the final result of solution to the task being solved taking into account (10) is the following (index *QBMS* demonstrates the result obtained with the method of quasi-best case):

$$\langle S_{(j) \max} | j = \overline{1, m} \rangle_{QBMS} = \langle Kn, En, Tr, Gm, Rl, Ct, \tau(Q), Ac, Fc, Fopt, Ec, Dn \rangle. \quad (11)$$

#### 4 The results obtained and their discussion

The ordered set of LCDS elements is obtained by expression (11) after having solved the given task using the method of quasi-best case (see item 3).

As it can be seen, the mapping of elements of sets (*S*) and  $\langle S_{(j) \max} | j = \overline{1, m} \rangle_{QBMS}$  by expressions (1) and (11) vary without coincidence in relation to places of local criteria within these mappings excluding criterion  $\tau(Q)$ . Obviously, it does not demonstrate infeasibility of the method used [14].

It makes sense to compare the obtained result  $\langle S_{(j) \max} | j = \overline{1, m} \rangle_{QBMS}$  to the result of ordered set  $\langle S_{(j) \max} | j = \overline{1, m} \rangle_{WMS}$ , which was obtained while selecting RMAT using the method of the worst case [13] (to index *WMS*):

$$\langle S_{(j) \max} | j = \overline{1, m} \rangle_{WMS} = \langle Kn, En, Gm, Dn, \tau(Q), Ct, Ac, Rl, Tr, Ec, Fopt, Fc \rangle. \quad (12)$$

As it is seen, the places (order number) of only three criteria coincide within the sets  $\langle S_{(j) \max} | j = \overline{1, m} \rangle_{QBMS}$  and  $\langle S_{(j) \max} | j = \overline{1, m} \rangle_{WMS}$ . These criteria are: *Kn, En and Ct*. This does not also demonstrate the infeasibility of quasi-best method used. The comparison of these sets in general demonstrates different final results. It proves feasibility of both methods of fuzzy multi-criteria of RMAT selection as well as the feasibility of quasi-best case method adapted to the particularity of topical area.

Generally, the comparison of the elements of sets  $S = (S_j | j = \overline{1, m})$  by (1),  $\langle S_{(j) \max} | j = \overline{1, m} \rangle_{QBMS}$  by (11) and  $\langle S_{(j) \max} | j = \overline{1, m} \rangle_{WMS}$  by (12) is not contradictory. The result of analysis of these sets first of all reproduces the features of solving the tasks of such content and formulation (see item 3).

Therefore, it is recommended to make decision for stated input data taking into account the obtained ordered set of LCDS elements by expression (11) at fuzzy multi-criteria selection of RMAT using the developed and applied method of quasi-best case. It means that local criteria (RMAT phenomena) have to be analyzed by the following ordered sequence: kinematics (*KC*), energy components (*En*), geometrical parameters (*Gm*) and so on, finishing with dynamics components (*Dn*). It is obvious that the number of RMAT being analyzed by every local criterion may be significantly decreased due to the selection of infeasible RMAT by every criterion. This occurs while analyzing previously synthesized final set of RMAT [4] as a result of RMAT analysis by every local criterion in sequence of set [11] elements. All the mentioned above causes complexity decrease and makes decision-making rational even at non-automated solution of the task of fuzzy multi-criteria selection of RMAT using the method of quasi-best case.

The results obtained demonstrate the achieved purpose of the given paper (see item 2).

The following *directions of further researches* have been determined as results of conducted researches:

- the development of set of related methods of fuzzy multi-criteria alternative selection based on the results of strict and non-strict expert sampling and does not fundamentally contradict the possibility to automate fuzzy multi-criteria selection of alternatives;

- the known and the latest approaches related to fuzzy multi-criteria selection of alternatives are automatically implemented in the form of computer software.

### 5 Conclusions

1. The approach of fuzzy multi-criteria selection of alternatives by quasi-best case was chosen as theoretical and methodological basis to solve the task of RMAT selection taking into account its essence and formulation. This approach was used because it is invariant one in relation to the content of the task, to the origin and the number of discrete local criteria. The selection is performed within the finite sets of the mentioned above criteria. One more reason to choose the described method is its ability to provide the lower level of subjectivism and to increase the reasonableness of decisions made while ordering criteria from their LCDS.

2. The key points of the method used are adapted to solving tasks of fuzzy multi-criteria selection of RMAT. It was performed for the first time by applying the executed formalization of generalizations of content features of the task components. The mentioned above task, in its turn, can be further implemented in an automated way.

3. The performed formalization of RMAT selection task, which is being solved, is implemented with meaningfully grounded steps. The content of these steps is the methodological basis of fuzzy multi-criteria RMAT selection by the method of quasi-best case.

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### **Роботтандырылған механикалық құрастыру технологиясын таңдау негізінде баламаны квази-жақсы жағдай әдісімен анық емес көпкритерилі таңдау**

Роботтандырылған механикалық құрастыру технологиясын (РМҚТ) таңдау процесі бұрын ұсынылған баламаны квази-жақсы жағдай әдісімен анық емес көпкритерилі таңдаумен жүзеге асты. Жұмыстың практикалық және ғылыми құндылығын анықтайтын әдістің негізгі мазмұнды ерекшелігі ол неғұрлым ерекше критериймен ең жақсы баламамен салыстыру арқылы негізделген арнайы әзірленген қатынастар болып табылады. Бастапқы деректер ретінде РМҚТ көріністері болатын, эксперттік сауалнама әдісімен орындалған локальды критерийлердің дискреттік жиынының (ЛКДЖ) элементтерін қатаң ранжирлеу нәтижелері болды. Таңдау мазмұны болып таңдау орындалатын жиында реттелген ақырлы жиынға ЛКДЖ ретсіз элементтерінің бастапқы құраушыларын реттеу процесі табылған. Алынған РМҚТ көрінісінің тізбегі таңдау процесінде таңдау нәтижесі ретінде саралауға ұсынылған. РМҚТ таңдау есебінің шешуінің негізі болып оның қалыптастырылған қойылымы және оның негізінде алғаш рет анықталған квази-жақсы жағдай әдісінің мазмұнының жалпылыңған формализмдері болды. Келтірілген теориялық жағдайлардың қадам сайын жұмыс қабілеттілігі РМҚТ автоматтандырылған таңдаудың нақты мысалымен көрсетілген.

*Кілт сөздер:* балама, автоматтандыру, таңдау, локальды критерий, анық емес, тиімділеу, роботтандырылған механикалық құрастыру технологиясы, квази-жақсы таңдау.

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### **Нечёткий многокритериальный выбор альтернатив методом квазилучшего случая как основа выбора роботизированных механосборочных технологий**

Процесс выбора роботизированных механосборочных технологий (РМСТ) реализован как нечеткий многокритериальный выбор альтернатив предложенным ранее методом квазилучшего случая. Основной содержательной особенностью данного метода, определяющей практическую и научную ценность

данной работы, являются разработанные специальные соотношения, основанные на соответствующих сравнениях с лучшей альтернативой и с наиболее важным критерием. Входными данными являются результаты строгого ранжирования элементов дискретного множества локальных критериев (ДМЛК), являющиеся проявлениями РМСТ, выполненных методом экспертного опроса. Содержанием выбора является процесс упорядочения составляющих изначально неупорядоченных элементов ДМЛК, на множестве которых выполняется выбор, в конечное упорядоченное множество. Полученная последовательность проявлений РМСТ как результат выбора рекомендована к анализу в процессе выбора. Основой решения задачи выбора РМСТ является ее формализованная постановка и впервые определенные на ее основе обобщенные формализмы содержания метода квазилучшего случая. Работоспособность изложенных теоретических положений пошагово продемонстрирована реальным примером автоматизированного выбора РМСТ.

*Ключевые слова:* альтернатива, автоматизация, выбор, локальный критерий, нечеткость, оптимизация, роботизированная механосборочная технология, квазилучший выбор.

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