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Wave field in a strip with symmetric located holes

In the linear formulation, the problem of the propagation of unsteady stress waves in an elastic body with symmetrically located rectangular holes is considered. Formulated in terms of stresses and velocities, the mixed problem is modeled numerically using an explicit difference scheme of the end-to-end calculation based on the method of spatial characteristics. The wave process is caused by applying an external dynamic load on the front border of the rectangular region, and the side boundaries of the region are free of stresses. The lower boundary of the rectangular region is rigidly fixed. The contour of symmetrically arranged rectangular openings is free of stress. Based on the numerical technique developed in this work, the calculated finite - difference relations of dynamic problems are obtained at the corner points of a rectangular hole, where the "smoothness"of functions "familiar"to dynamic problems is violated. At these corner points, the first and second derivatives of the desired functions suffer a discontinuity of the first kind. The isoline presents the results of changes in wave fields in an elastic body with symmetrically located rectangular holes. The concentration of dynamic stresses in the vicinity of the corner points of a rectangular hole is investigated. By numerical implementation, the stability of computational algorithms for a sufficiently large time is established.

Keywords: elastic, wave process, stress, speed, plane deformation, numerical solution.

Introduction

In recent years, the problem of developing scientifically sound and effective numerical methods for analyzing the health of structures with cuts, holes, foreign inclusions and other characteristic features has become increasingly relevant. These features make it necessary to develop new and improve traditional numerical methods for calculating and designing structures. This will make it possible to take advantage of the enormous advantages of mathematical modeling — to combine a physical experiment with a more economically viable numerical experiment and to provide answers to questions of interest to engineers with the least expenditure of funds and effort. To realize this possibility, it is necessary to solve a range of issues related to the construction of mathematical models of environments that take into account the complex features of the environment. On the other hand, the second necessary component of this approach is the creation of reliable and economical methods for the numerical calculation of the corresponding dynamics problems. Prediction of the dynamic behavior of structural elements taking into account a number of weakening factors (holes, cavities, cutouts, etc.) has not only theoretical, but also applied value, determined by the demands of engineering practice [1–17]. The methodological apparatus of scientific research is based on finite-difference methods based on the use of characteristic surfaces and compatibility relations on them. The research methodology is confirmed by scientific and theoretical justification, the correctness and rigor of the mathematical formulation of the investigated problems.

Formulation of the problem. Let the rectangular cross section of the strip, weakened by two equal rectangular holes symmetrically spaced relative to the axis $x_2 = 0$ (Figure 1), be in an undisturbed state at $t \leq 0$, i.e.

$$v_1(x_1, x_2, 0) = v_2(x_1, x_2, 0) = p(x_1, x_2, 0) = q(x_1, x_2, 0) = \tau(x_1, x_2, 0) \tag{1}$$

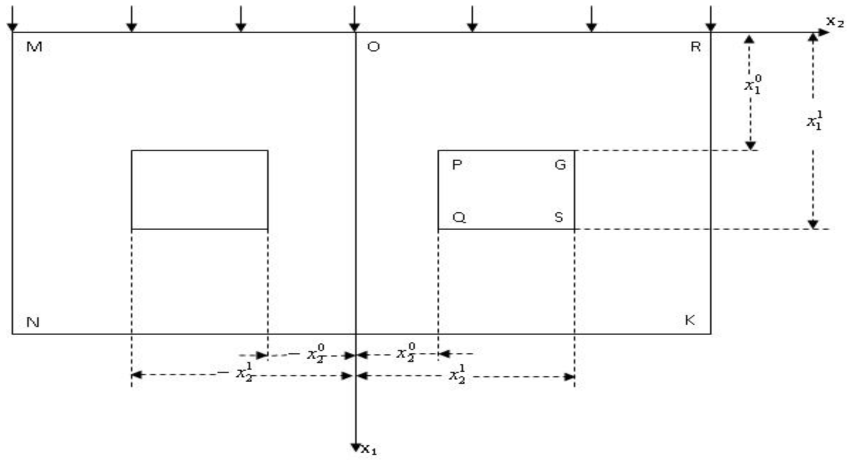

Figure 1. The study area

Wave processes in a strip with symmetrical holes are described by a system of differential equations of hyperbolic type, containing dimensionless stresses p, q, τ, displacement velocities v_1, v_2 as unknowns [1–2]:

$$\begin{gathered} v_{1,t} - p_{,1} - q_{,1} - \tau_{,2} = 0; \quad v_{2,t} - p_{,2} + q_{,2} - \tau_{,1} = 0, \\ \gamma^2(\gamma^2 - 1)^{-1} p_{,t} - v_{1,1} - v_{2,2} = 0; \quad \gamma^2 q_{,t} - v_{1,1} + v_{2,2} = 0, \\ \gamma^2 \tau_{,t} - v_{1,2} - v_{2,1} = 0. \end{gathered} \tag{2}$$

Dimensionless variables are introduced by the formulas [1–2]:

$$\begin{gathered} \overrightarrow{t} = \frac{tc_1}{b}; \quad \overrightarrow{x_i} = \frac{x_i}{b}; \ \ v_i = \frac{1}{c_1}\frac{\partial u_i}{\partial t}; \quad (i = 1, 2) \\ p = \frac{\sigma_{11} + \sigma_{22}}{2\rho c_1^2}; \quad q = \frac{\sigma_{11} - \sigma_{22}}{2\rho c_1^2}, \\ \tau = \frac{\sigma_{12}}{\rho c_1^2}; \quad \gamma = \frac{c_1}{c_2}, \end{gathered} \tag{3}$$

where b is the characteristic length, ρ is the density of material, c_1, c_2 are the velocities of waves of expansion and shear, $\sigma_{11}, \sigma_{22}, \sigma_{12}$ are the components of the stress tensor, γ is a constant parameter. In the future, the bar over dimensionless parameters is omitted.

The solution of the system of equations (2) with respect to the desired quantities v_1, v_2, p, q, τ constructed for zero initial (1) and the following boundary conditions ($t \geq 0$):

$$v_1 = f(t), \quad v_2 = 0 \quad when \quad x_1 = 0, \quad -L \leq x_2 \leq L. \tag{4}$$

$$p - q = 0, \quad \tau = 0 \quad when \ |x_2| = L, \quad 0 \leq x_1 \leq \ell, \tag{5}$$

$$v_1 = v_2 = 0 \quad when \ |x_1| = \ell, \quad |x_2| \leq L, \tag{6}$$

$$p+q=0,\quad \tau=0 \quad when \quad x_1=x_1^0,\quad x_2^0\leq|x_2|\leq x_2^1 \quad and \quad x_1=x_1^1,\;\; x_2^0\leq|x_2|\leq x_2^1x_1^1, \quad (7)$$

$$p-q=0,\quad \tau=0 \quad when \;\; |x_2|=x_2^0 \quad x_1^0\leq x_1\leq x_1^1 \quad and \quad |x_2|=x_2^1,\;\; x_1^0\leq x_1\leq x_1^1. \quad (8)$$

Analysis of the calculation results. Numerical results are given for the rectangular region $0\leq x_1\leq 100\cdot h_1$, $|x2|\leq 100h_2$. Moreover, the accepted values of the steps along the coordinate are the same $h_1=h_2=h=0.05$. The body material has the following characteristics: elastic modulus $E=200GPa$, Poisson's ratio $\nu=0.3$, density $\rho=7.9\cdot 10^3 kg/m^3$, $c_1=5817m/sec$, $c_2=3109m/sec$, $\gamma=1.87$. The dimensions of the holes are taken as follows: $x_1^0=25\cdot h$, $x_1^1=75\cdot h$, $|x_2^0|=25\cdot h$, $|x_2^1|=75\cdot h$. The wave field parameters were obtained with the following initial data

$$f(t)=A\cdot t\cdot e^{-st}, A=1, cs=0.2, k=0.025, h=0.05.$$

Here A is a constant factor, the parameter s characterizes the rate of change of the external load. Since the body under study has free boundaries $x_2=\pm 100\cdot h$ and contains rectangular holes inside itself, then over time reflection waves (diffracted) superimposed on each other determine the complex nature of the manifestation of displacement, strain, and stress velocities in it. The corner points of the rectangular region and the corner points of rectangular holes are sources of disturbance, causing both longitudinal and transverse waves.

The study of the stability showed that a grid ratio k/h of 0.5 provides the stable results for a sufficiently large period of time, with multiple reflections and diffraction waves. In fact, the calculation was performed up to $t=1000\cdot k$. In the calculations at any time t, all boundary conditions are exactly satisfied both at the corner points of the strip and at the corner points of rectangular holes. This circumstance, unlike many approximate methods, ensures the reliability of the obtained solutions and the corresponding results.

The solution of the system of equations (2) under initial (1) and boundary (4)–(8) conditions is found by the method of spatial characteristics at the nodal points into which the entire studied area is divided [1–2]. A feature of the body under consideration is that at the corner points (P,G,Q,S) of the rectangular hole, the "smoothness" of functions that is "familiar" to dynamic problems is violated. It was precisely such features that were not extended, or in general, as we know, there was no method for solving such problems. In addition to the known relations [1–2], the calculated relations are obtained at the internal corner points (P,G,Q,S) of the rectangular hole [6].

The calculation results are presented by graphs in figures 2–3. Due to the symmetry of the location of the rectangular holes and the nature of the loading, the desired parameters v_1,p,q, are even, and v_2,τ are the odd functions relative to the axis $x_2=0$ of the strip.

The construction of dynamic stress fields generated from the interaction of symmetrically located holes will allow, using various strength criteria, to judge the shape and possible size of the fracture zones. In Figs. 2–3, in the plane $x_1/h\cdot x_2/h$ for the time instant $t=400\cdot k$, the contours of normal stresses $p+q=const$, $p-q=const$ are shown, respectively. The stress state due to the symmetry of the problem relative to the axis $x_2=0$ of the strip is shown only for positive values $x_2(x_2\geq 0)$ near a symmetrically located hole.

The presented graphs clearly illustrate the strong interaction of wave fields on the stress distribution in the vicinity of the holes, which leads to the formation of a number of local extrema. The location of the latter changes in time due to repeatedly reflected and diffracted waves from the boundaries of the holes and strip.

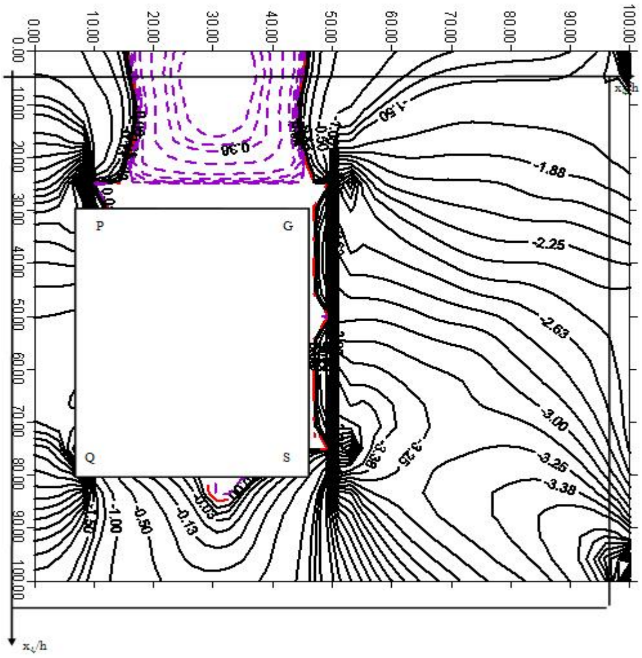

Figure 2. Isolines of normal stresses $p+q=const$ at time $t=400\cdot k$

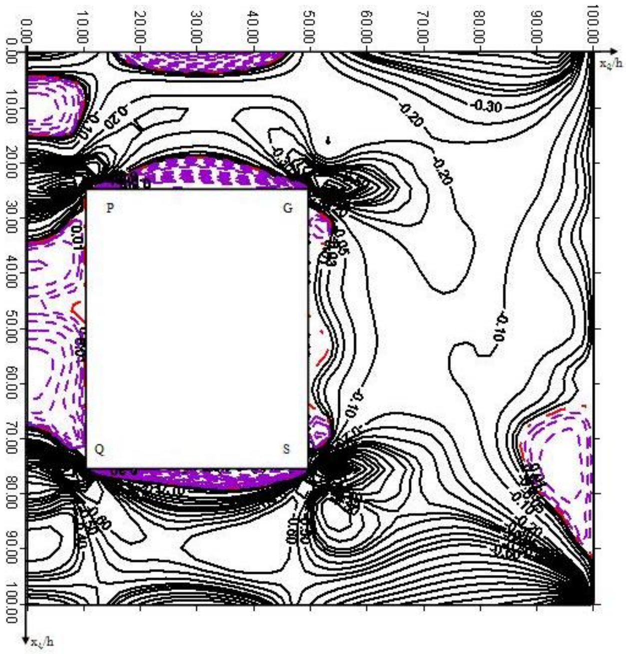

Figure 3. Isolines of normal stresses $p-q=const$ at time $t=400\cdot k$

It should be noted (Figure 2) the appearance of an extensive zone of tensile stresses $p+q=const$ in front of the hole. In the region behind the hole, the stress level is extremely low. Although the maximum values of tensile stresses $p+q=const$ turned out to be lower by an order of magnitude than the maximum absolute values of compressive stresses, nevertheless, for a number of materials (such as soils, bulk material, etc.) they can pose a serious danger. Therefore, a detailed study of the appearance and development of extension zones is an important applied problem. In the vicinity of the front corner points (P,G) of the hole relative to the loading surface, the concentration of compressive stresses $p+q=const$ at a given time is maximum. In the vicinity of the lower corner points (Q,S), where the stress amplitude is less, they remain compressive throughout the entire considered time.

Slightly smaller stress gradients than in the vicinity of the corner points are observed near the lateral free edges (PQ, GS) of the hole. The maximum concentration of compressive stresses $p + q = const$ is formed near the corner points (R, K) of the strip, in the vicinity of which the stress fields have the greatest gradients.

Analysis of normal stress isolines $p - q$ (Figure 3) shows that the resulting zone of tensile stresses on the lower free face (QS) of the hole is less extensive than on the front free face (PG). This is due to the fact that a perturbation is specified on the surface $x_1 = 0$, and the strip surface $x_1 = 100 \cdot h$ is pinched.

A small stretch zone is localized near the lateral face (PQ) of symmetrically located holes, as well as near the front (MR) and lateral (RK) faces of a rectangular strip. Moreover, the zone of tensile stresses is bordered by an isoline of zero stresses. The appearance of a large number of local maximums of compressive stresses in the vicinity of the corner points of the symmetric hole is due to the complex interaction of reflected, diffraction, and interference phenomena, which is characteristic of the stress state. As a result of the reflection of the waves, a region of compressive stresses is formed near the upper (MR), lower (NK), and lateral (RK) surfaces of the strip. From an engineering point of view, when calculating and constructing structural elements with several holes, it must be borne in mind that under dynamic loads, the stress concentration does not change monotonically when the holes approach each other, as is the case in static, but is determined by a more complex dependence and interaction of holes, corner points. In some cases, the stress concentration can even decrease as the holes approach each other, which leads to the appearance of a class of problems for optimizing the design with holes. When carrying out the calculations, it is necessary to take into account in advance the possible frequency ranges and disturbances in which the structure will subsequently operate.

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Симметриялы орналасқан тесіктері бар денедегі толқындық өріс

Жұмыста сызықты жағдайда берілген симметриялы орналасқан тіктөртбұрышты тесіктері бар серпімді денедегі стационар емес толқындық процестің таралу есебі қарастырылған. Кернеулер мен жылдамдықтар терминінде қойылған аралас есеп айқын айырымдық схема, атап айтқанда сандық кеңістіктік сипаттамалар әдісімен шешілген. Толқындық процесс тіктөртбұрышты дененің беттік шекаралық нүктелерінде сырттай динамикалық күштің берілуінен пайда болады, ал дененің бүйір қабырғаларында кернеулік нөлге тең. Тіктөртбұрышты дененің төменгі шекаралық нүктелері қатаң бекітілген. Симметриялы орналасқан тіктөртбұрышты тесіктердің контурында кернеуліктер нөлге тең. Осы жұмыста әзірленген сандық техникалық негізінде динамикалық есептердің ақырғыайырымдық қатынастары динамикалық есептерге «әдеттегі» шекті функциялардың бұзылған тіктөртбұрышты тесіктің бұрыштарында алынған. Бұл бұрыштық нүктелерде ізделінді, функциялардың бірінші және екінші ретті туындылары бірінші текті үзілісті. Симметриялы орналасқан тіктөртбұрышты тесіктері бар серпімді денедегі толқындық өріс өзгерісінің нәтижелері изосызық түрінде келтірілген. Тік бұрышты тесіктің бұрыштық нүктелерінің маңайында кернеуліктің динамикалық концентрациясы зерттелген. Сандық әдісті қолдану нәтижесінде есептеу алгоритмдерінің жеткілікті үлкен уақытқа тұрақтылығы анықталған.

Кілт сөздер: серпімді, толқындық процесс, кернеу, жылдамдық, жазықтық деформациясы, сандық шешім.

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Волновое поле в полосе с симметрично-расположенными отверстиями

В линейной постановке рассмотрена задача о распространении нестационарных волн напряжений в упругом теле с симметрично-расположенными прямоугольными отверстиями. Сформулированная в терминах напряжений и скоростей смешанная задача моделируется численно с помощью явной разностной схемы сквозного счета, основанной на методе пространственных характеристик. Волновой процесс вызывается прикладыванием внешней динамической нагрузки на лицевой границе прямоугольной области, а боковые границы области свободны от напряжений. Нижняя граница прямоугольной области жестко закреплена. Контур симметрично-расположенных прямоугольных отверстий свободен от напряжений. На основе разработанной в работе численной методики получены расчетные конечно-разностные соотношения динамических задач в угловых точках прямоугольного отверстия, где нарушается «привычная» для динамических задач гладкость функций. В этих угловых точках первые и вторые производные искомых функций терпят разрыв первого рода. В виде изолинии представлены результаты изменения волновых полей в упругом теле с симметрично-расположенными прямоугольными отверстиями. Исследована концентрация динамических напряжений в окрестности угловых точек прямоугольного отверстия. Путем численной реализации установлена устойчивость расчетных алгоритмов для достаточно большого времени.

Ключевые слова: упругость, волновой процесс, напряжение, скорость, плоская деформация, численное решение.

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