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The hybrids of the $\Delta - PJ$ theories

When studying Jonsson theories, which are a wide subclass of inductive theories, it becomes necessary to study the so-called Jonsson sets. Similar problems are considered both in model theory and in universal algebra. This topic is related to the study of model-theoretical properties of positive fragments. These fragments are a definable closure of special subsets of the semantic model of a fixed Jonsson theory. In this article are considered model-theoretical properties of a new class of theories, namely $\Delta - PJ$ theories of countable first-order language. These are theories that are obtained from $\Delta - PJ$ theories by replacing in the definition of $\Delta - PJ$ theories of morphisms (Δ-continuities) with morphisms (Δ-immersions). A number of results were obtained, $\Delta - PJ$ fragments, $\Delta - PJ$ sets, hybrids of $\Delta - PJ$ theories. All questions considered in this article are relevant in the study of Jonsson theories and their model classes.

Keywords: Jonsson theory, $\Delta - PJ$ theory, $\Delta - PJ$ fragment, semantic model, hybrid of the $\Delta - PJ$ theories.

In this article we want to define the concepts of a hybrid for a special positive case of Jonsson theories. Prior to this, we have defined the concepts of a hybrid of Jonsson theories, which are closely related to some fixed Jonsson theory. To familiarize ourselves with this material, we refer the reader to the following sources [1–5]. On the other hand, it is well known that the concept of Jonsson theory can be considered in a more general context, namely in the framework of the study of positive model theory. The main sources in this direction we would like to mention the following works: [6–8].

Further, as part of the study of the positive model theory, the study of Jonsson theories was begun [9]. In this paper, we consider a special case of a positive model theory and note that this particularity is related to the form of the formulas that are preserved during immersions; immersions, in turn, are a special case of homomorphisms.

About Jonsson theories, more detailed information can be extracted in the monograph [10] and in the works [11–15].

Let L be a first-order language. At is the set of atomic formulas of this language. $B^+(At)$ is a closed set of relatively positive Boolean combinations (conjunction and disjunction) of all atomic formulas, their subformulas, and variable substitutions. $Q(B^+(At))$ is the set of prenex normal formulas obtained by applying quantifiers ($\forall$ and $\exists$) to $B^+(At)$. A formula is called positive formula if it belongs to the set $Q(B^+(At)) = L^+$. A theory is called positively axiomatizable if its axioms are positive. $B(L^+)$ is an arbitrary Boolean combination of formulas from L^+. It is easy to see that $\Pi(\Delta) \subseteq B(L^+)$ for $\Delta = B^+(At)$, where $\Pi = \Pi(\Delta) = \{\forall y \neg\varphi(x, y) : \varphi \in \Delta\} = = \{\neg\psi : \psi \in \Delta\}$.

Following [6, 7] we define Δ - morphisms between structures. Let M and N be language structures, $\Delta \subseteq B(L^+)$. A mapping $h : M \to N$ is called a Δ - homomorphism (symbolically $h : M \to N$), if for any $\varphi(\overline{x}) \in \Delta$, $\forall \overline{a} \in M$ from the fact that $M \models \varphi(\overline{a})$, follows that $N \models \varphi(h(\overline{a}))$.

Following [6, 7], the M model is called the beginning in N and we say that M extends to N, and $h(M)$ is called the continuation of M. If the map h is injective, then it is said that the map h immerses M in N (symbolically $h : M \underset{\Delta}{\leftrightarrow} N$). In the future, we will use the term Δ-continuation and Δ-immersion. In the framework of this definition (Δ homomorphism), it is easy to notice that isomorphic embedding and elementary embedding are Δ-immersions when $\Delta = B(At)$ and $\Delta = L$, respectively.

Consider the following necessary definitions.

Definition 1. If K is a class of L-structures, then we say that an element M of K is Δ positively existentially closed in K, if every Δ is a homomorphism from M to any element of K is Δ-immersion. The class of all Δ - positively existentially closed models is denoted by $(E_K^{\Delta})^+$; if $K = ModT$ for some theory T, then by E_T, $(E_T^{\Delta})^+$ we mean, respectively, the class of existentially closed and Δ-positively existentially closed models of this theory.

Definition 2. We will say that the theory of T admits $\Delta - JEP$, if for any two $A, B \in ModT$ there is $C \in ModT$ and Δ-continuation $h_1 : A \underset{\Delta}{\to} C$, $h_2 : B \underset{\Delta}{\to} C$.

Definition 3. We will say that the theory T admits $\Delta - AP$, if for any $A, B, C \in ModT$ such that $h_1 : A \underset{\Delta}{\to} C$, $g_1 : A \underset{\Delta}{\to} B$, where h_1, g_1 is a Δ-continuation, there is $D \in ModT$ and $h_2 : C \underset{\Delta}{\to} D$, $g_2 : B \underset{\Delta}{\to} D$, where h_2, g_2 – Δ-continuation such that $h_2 \circ h_1 = g_2 \circ g_1$.

Definition 4. A theory T is called a Δ-positive Jonsson ($\Delta - PJ$) theory if it satisfies the following conditions:

1) T has an infinite model;
2) is positively $\forall\exists$-axiomatizable;
3) admits $\Delta - JEP$;
4) admits $\Delta - AP$.

Let C be a semantic model of some fixed Jonsson theory T.

Definition 5. Let $cl : P(C) \to P(C)$ be an operator on the power set of C. We say that (C, cl) is a Jonsson pregeometry if the following conditions are satisfied.

If $A \subseteq C$, then $A \subseteq cl(A)$ and $cl(cl(A)) = cl(A)$.

If $A \subseteq B \subseteq C$, then $cl(A) \subseteq cl(B)$.

(Exchange). $A \subseteq C$, $a, b \in C$, and $a \in cl(A \cup \{B\})$, then $a \in cl(A), b \in cl(A \cup \{a\})$.

(Finite character). If $A \subseteq C \bigvee$ and $a \in cl(A)$, then there is a finite $A_0 \subseteq A$ such that a $a \in cl(A_0)$.

We say that $A \subseteq C$ is closed if $cl(A) = A$.

Definition 6. If (C, cl) is a Jonsson pregeometry, we say that A is Jonsson independent (J-independent) subset in C, if $a \notin cl(A \setminus \{a\})$ for all $a \in A$ and B is J-basis for Y, $Y \subseteq C$, if B-J-independent and $Y \subseteq acl(B)$.

Lemma 1. If (C, cl) is a J-pregeometry, $Y \subseteq C$, $B_1, B_2 \subseteq Y$ and each B_i is a J-basis for Y, then $| B_1 |=| B_2 |$.

We call $| B_i |$ the J-dimension of Y and write $Jdim(Y) =| B_i |$.

If$A \subseteq C$, we also consider the localization $cl_A(B) = cl(A \cup B)$.

Lemma 2. If (C, cl) is a J-pregeometry, then (C, cl_A) is a J-pregeometry.

If (C, cl) is a J-pregeometry, we say that $Y \subseteq C$ is J-independent over A if Y is J-independent in (C, cl_A). We let $Jdim(Y/A)$ be the J-dimension of Y in the localization (C, cl_A). We call $Jdim(Y/A)$ the J-dimension of Y over A.

Definition 7. We say that (C, cl) is a modular pregeometry if, for any finite-dimensional closed $A, B \subseteq C$ the following is true

$$dim(A \cup B) = dimA + dimB - dim(A \cap B).$$

Definition 8. If (C, cl) is modular, then the Jonsson theory T s called modular, where C is a semantic model of theory T.

Definition 9. Let $X \subseteq C$. We will say that a set X is Δ-positive Jonsson subset of C, if X satisfies the following conditions:

1) X is Δ-definable set (this means that there is a formula from Δ, the solution of which in the C is the set X, where $\Delta \subseteq B(L^+)$, that is Δ is a view of formula, for example $\exists^+, \forall^+, \forall\exists^+$ и тд.);

2) $dcl(X) = M$, $M \in (E_T^{\Delta})^+$, where dcl is a definable closure operator and cl is an operator defining pregeometry over C.

All morphisms which we are considering below will be Δ-immersions.

Lemma 3. Let T be a $\Delta - PJ$-positive perfect Jonsson theory, $(E_T^{\Delta})^+$ is class of its existentially closed models. Then for any model $A \in (E_T^{\Delta})^+$ theory $Th_{\forall\exists^+}(A)$ is a $\Delta - PJ$ theory.

Definition 10. The inductive theory T is called the existentially prime if: it has a algebraically prime model, the class of its AP (algebraically prime models) denote by AP_T; class E_T non trivial intersects with class AP_T, i.e. $AP_T \bigcap E_T \neq 0$.

Definition 11. The theory T is called convex if for any its model A and for any family $\{B_i \mid i \in I\}$ of substructures of A, which are models of the theory T, the intersection $\bigcap_{i\in I} B_i$ is a model of T.

Let T be a $\Delta - PJ$ theory and C is a semantic model of $\Delta - PJ$ theory. Let X_1, X_2 be a $\Delta - PJ$ subsets of C.

$Fr(X_1), Fr(X_2)$ are $\Delta - PJ$ fragments.

Let $M_1 = dcl(X_1)$, $M_2 = dcl(X_2)$, where $M_1, M_2 \in (E_T^{\Delta})^+$.

$Th_{\forall\exists^+}(M_1) = T_1$, $Th_{\forall\exists^+}(M_2) = T_2$,

C_1 is the semantic model of $\Delta - PJ$ theory of T_1, C_2 is the semantic model of $\Delta - PJ$ theory of T_2.

$T_1 = Th_{\forall\exists^+}(M_1) = Fr^+(X_1)$, $T_2 = Th_{\forall\exists^+}(M_2) = Fr^+(X_2)$,

We define the essence of the operation of an algebraic construction.

Let $\boxdot \in \{\cup, \cap, \times, +, \oplus, \prod\limits_F, \prod\limits_U\}$, where $\cup$-union, $\cap$-intersection, $\times$-Cartesian product, $+$-sum and $\oplus$-direct sum, $\prod\limits_F$-filtered and $\prod\limits_U$-ultra-production.

The following definition gives a hybrid of two $\Delta - PJ$ fragments of the same signature.

Definition 12. A hybrid $H(Fr^+(X_1), Fr^+(X_2))$ of $\Delta - PJ$ fragments $Fr^+(X_1)$, $Fr^+(X_2)$ is called the theory $Th_{\forall\exists^+}(C_1 \boxdot C_2)$, if it is $\Delta - PJ$ theory, where C_i are the semantic models of $Fr^+(X_i)$, $i = 1, 2$.

Note the following fact:

Fact. For the theory $H(Fr^+(X_1), Fr^+(X_2))$ to be a $\Delta - PJ$ theory enough to $(C_1 \boxdot C_2) \in (E_T^{\Delta})^+$.

The following examples will be examples of hybrids of $\Delta - PJ$ theories:

Let $\Delta = B^+(At)$.

1) Let T be a $\Delta - PJ$ theory, C be a semantic model of $\Delta - PJ$ theory of T. A, B are the $\Delta - PJ$ subsets, $A, B \subseteq C$. $dcl(A) = M_1$, $dcl(B) = M_2$, where $M_1, M_2 \in (E_T^{\Delta})^+$. Then $Th_{\forall\exists^+}(M_1 \times M_2)$ will be a hybrid of $\Delta - PJ$ theories.

2) Let T_1, T_2 be the $\Delta - PJ$ theories of Abelian groups, C_1, C_2 be the semantic models of $\Delta - PJ$ theories of T_1, T_2, respectively. Then $Th_{\forall\exists^+}(C_1 \times C_2) = H(T_1, T_2)$ will be a hybrid of $\Delta - PJ$ theories.

3) Let V be a linear space, V_1, V_2 be the linear subspaces,$V_1, V_2 \subseteq V$. Then $Th_{\forall\exists^+}(V_1 \oplus V_2)$ will be a hybrid of $\Delta - PJ$ theories.

And also, there are a number of tasks that will be examples of hybrids of $\Delta - PJ$ theories.

1) Let G be a group, $T = Th(G)$, H_1, H_2 are normal divisors of the group G. $X_1, X_2 \subseteq C$, C is the semantic model of $\Delta - PJ$ theory of T. Let $H_1 = cl(X_1)$, $H_2 = cl(X_2)$, where $H_1, H_2 \in (E_T^{\Delta})^+$. $Th_{\forall\exists^+}(H_1) = T_1$, $Th_{\forall\exists^+}(H_2) = T_2$, T_1, T_2 are a $\Delta - PJ$ theory. Then their hybrid will be $H(T_1, T_2) = Th_{\forall\exists^+}(C_1 \boxdot C_2)$, where C_1 is the semantic model of $\Delta - PJ$ theory of T_1, C_2 is the semantic model of $\Delta - PJ$ theory of T_2,respectively. Then is there such a theory T_3, $T_3 = H(T_1, T_2) = Th_{\forall\exists^+}(C_1 \boxdot C_2)$ and if there is a theory of T_3, then which $H(T_1, T_2)$ satisfy these conditions? Here for the place of algebraic construction will be direct sum: $\boxdot = \oplus$.

2) Let T_1, T_2 be a $\Delta - PJ$ theory and T_3, T_4 be a $\Delta - PJ$ theory. Then C_1, C_2 are the semantic models of $\Delta - PJ$ theory of T_1, T_2, C_3, C_4 are the semantic models of $\Delta - PJ$ theory of T_3, T_4, respectively.

If $C_1 \equiv C_2$, $C_3 \equiv C_4$, то $C_1 \times C_3 \equiv C_2 \times C_4$, then are there such theories $\exists T_5 : T_5 = H(T_1, T_3) = Th_{\forall\exists^+}(C_1 \times C_3)$, $\exists T_6 : T_6 = H(T_2, T_4) = Th_{\forall\exists^+}(C_2 \times C_4)$, which will be hybrids of $\Delta - PJ$ theories?

In the study of this class of theories, we obtained the following results:

Let $\Delta = B^+(At)$.

Theorem 1. Let $Fr^+(X)$ be perfect convex existentially prime complete for $\forall\exists^+$-sentences a $\Delta - PJ$ fragment. X_1, X_2 are the $\Delta - PJ$-sets of the theory $Th_{\forall\exists^+}(C)$, where M_i=$dcl(X_i) \in E_{Fr(Th_{\forall\exists^+}(C))}$, $Fr^+(X_i) = Th_{\forall\exists^+}(M_i)$ are also perfect convex existentially prime complete for $\forall\exists^+$-sentences a $\Delta - PJ$ fragments. C_1, C_2 are their semantic models, respectively. Then, if their hybrid $H(Fr^+(X_1),\ \ Fr^+(X_2))$ is a model consistent with $Fr^+(X_i)$, then $H(Fr^+(X_1), Fr^+(X_2))$ is a perfect $\Delta - PJ$ theories for $i = 1, 2$.

Proof. Suppose the contrary. Then, since the hybrid $H(Fr^+(X_1), Fr^+(X_2))$ is a $\Delta - PJ$ theories and has a semantic model M, by the assumption not perfectness of this hybrid $H(Fr^+(X_1), Fr^+(X_2))$, the considered semantic model M will not be saturated in its power. And this means that there is such $X \subseteq M$ and such type $p \in S_1(X)$, which is not realized in M, more precisely in $(M, m)_{m \in X}$. By virtue of the consistency of type p, this type is realized in some elementary extension $M' \succ M$. y virtue of the Jonssonness of hybrid $\Delta - PJ$ fragments $H(Fr^+(X_1), Fr^+(X_2))$ and model consistency with $Fr^+(X_i)$, $i = 1, 2$ there is a model $A_i \in ModFr^+(X_i)$, $i = 1, 2$ such that M' is a submodel of A. A in turn, is embedded in the semantic model C_i, $i \in 1, 2$, but C_i is a saturated model of the theory $Fr^+(X_i)$, $i \in 1, 2$. By virtue of the Δ-immersion, suppose h from M' in A, $h(X) \subseteq A$ and since the type of p is realized in M' it will be realized in C_i. But $C_i \in E_{Fr^+(X_i)}$ and since $Fr^+(X_i)$ are existentially prime convex theories, there exists a countable model $N_i \in E_{Fr^+(X_i)}$, in which the type p will be realized. By virtue of convexity, the model N_i will be a nuclear model, i.e. it is algebraically prime embedded in other models from $Mod(Fr^+(X_i))$ exactly one time. But by virtue of the model consistency of $Fr^+(X_i)$ with the hybrid $H(Fr^+(X_i))$, N_i Δ-immerse oneself in some model from $ModH(Fr^+(X_i))$. Since $Fr^+(X_i)$ are perfect theories, their center is model-complete, i.e. any monomorphism is elementary between the models of this center. And such, by virtue of perfection, are all the models from $E_{Fr^+(X_i)}$. Then the above Δ-immersion will be elementary, i.e. type p is realized in a countable submodel of model M. We got a contradiction with the assumption of imperfection.

Theorem 2. Let $Fr^+(X), Fr^+(X_1), Fr^+(X_2)$ satisfy the conditions of Theorem 1 and $Fr^+(X_1)$, $Fr^+(X_2)$ be ω-categorical $\Delta - PJ$ fragments. Then their hybrid $H(Fr^+(X_1), Fr^+(X_2))$ is also a ω-categorical $\Delta - PJ$ theory.

Proof. We note that, by virtue of the above Theorem 1, the hybrid $H(Fr^+(X_1), Fr^+(X_2))$ will be a perfect $\Delta - PJ$ theory. Suppose the contrary, i.e. the hybrid $H(Fr(X_i))$ is not a ω-categorical Jonsson theory. Let A and B be two countable models from $ModH(Fr^+(X_i))$. Then there are A' and B' countable models from $E_{H(Fr^+(X_i))}$ such that A is isomorphically embedded into A', and B is isomorphically embedded into B'. This follows from the fact that in any inductive theory any model is isomorphically embedded in some existentially closed model of this theory. But fragments of $Fr^+(X_i)$ are mutually model consistent with $H(Fr^+(X_i))$ by virtue of the condition of the theorem. Then A' and B' are Δ-immerse oneself in some countable model $D \in E_{Fr^+(X_i)}$, но but as $Fr^+(X_i)$ are convex fragments, then the image of A' and the image of B' in the model D intersects non-empty. Let this intersection be a model R. y virtue of the above existential primeness and countable categoricity of $Fr^+(X_i)$, since $R \in E_{T_i}$ it follows that in $R \models \varphi(x) \wedge \neg\varphi(x)$, where in $A' \models \varphi(x)$, and in $B' \models \neg\varphi(x)$. But this is not true, as T_i are ω-categorical by condition. Consequently, we obtain a contradiction with the assumption of non-ω-categoricity $H(Fr^+(X_i))$.

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Δ-PJ теориялардың гибридтері

Индуктивті теориялардың кең класы болып табылатын йонсондық теорияларды зерттегенде, йонсондық ішкі жиындарды зерттеу қажет болады. Осыған ұқсас мәселелер модельдер теориясында да, әмбебап алгебрада да қарастырылған. Бұл тақырып позитивті фрагменттердің модельді-теоретикалық қасиеттерін зерттеумен байланысты, яғни фрагменттер белгілі бір йонсондық теорияның семантикалық моделінің арнайы жиындарының тұйықтамасы болып табылады. Мақалада теориялардың жаңа класының модельді-теоретикалық қасиеттері, атап айтқанда бірінші реттегі тілдегі Δ-PJ теориялары қарастырылған. Бұл Δ-PJ теориясынан алынған Δ-PJ морфизмдер теориясын (Δ-жалғасы) морфизмдермен (Δ - бату) алмастыру арқылы алынған теориялар. Сонымен қатар берілген жұмыста бірқатар нәтижелер алынды, олар: Δ-PJ фрагменттер, Δ-PJ жиындар, Δ-PJ теориялардың гибридтері. Мақалада қарастырылған барлық сұрақтар йонсондық теорияларды және олардың модельдер кластарын зерттеуде өзекті болып табылады.

Кілт сөздер: йонсондық теория, Δ-PJ теория, Δ-PJ фрагмент, семантикалық модель, Δ-PJ-теориялардың гибриді.

А.Р. Ешкеев, Н.М. Мусина

Гибриды Δ-PJ-теорий

При изучении йонсоновских теорий, которые являются широким подклассом индуктивных теорий, возникает необходимость изучения так называемых йонсоновских множеств. Подобные задачи рассматриваются как в теории моделей, так и в универсальной алгебре. Данная тематика связана с изучением теоретико-модельных свойств позитивных фрагментов, которые являются определимым замыканием специальных подмножеств семантической модели фиксированной йонсоновской теории. В статье рассмотрены теоретико-модельные свойства нового класса теорий, а именно Δ-PJ-теорий счетного языка первого порядка. Это теории, которые получаются из Δ-PJ-теорий заменой в определении Δ-PJ-теорий морфизмов (Δ-продолжений) на морфизмы (Δ-погружения). При этом получен ряд результатов, Δ-PJ-фрагменты, Δ-PJ-множества, гибриды Δ-PJ-теорий. Следует заметить, что основные синтаксические и семантические атрибуты этих новых классов являются новыми понятиями, и они появились при изучении позитивных йонсоновских классов теорий. Все вопросы, рассматриваемые в статье, являются актуальными в области изучения йонсоновских теорий и их классов моделей.

Ключевые слова: йонсоновская теория, Δ-PJ-теория, Δ-PJ-фрагмент, семантическая модель, гибрид Δ-PJ-теорий.

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