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Two-dimensional calculations of stratified turbulent flow in a pipe

In this paper, we consider the stratified turbulent flow of a two-phase medium in inclined pipes. Based on the new turbulence model [1], a program code for calculating two-dimensional flows for the study of two-phase stratified flows in pipes was developed, including taking into account the rough of the pipeline wall. The technique for calculating two-phase flows in extended pipelines is described. The problem of stationary stratified two-phase flow in a pipe of constant cross section in the case of turbulent regime is numerically solved. Calculations of the resistance of a rough pipe are carried out and the results on the influence of roughness on pipe resistance and velocity distribution are presented.

Keywords: stratified turbulent flow, resistance, two-dimensional calculations, rough surface.

Introduction

Calculations of two-phase flows in long pipelines remain relevant in our time [2]. In connection with the specific nature of the flow, the known turbulence models for such flows require an appropriate correction. Despite a satisfactory description with the proposed modification of the resistance of the pipeline [3], there remains some dissatisfaction with the description of velocity profiles. In the experiments, a small systematic deviation of the velocity profiles in the axial region of the pipe from the logarithmic law

$$\frac{u_m - u}{u_*} = f\left(\frac{r}{R}\right),$$

where u_* is the velocity in the laminar sublayer. This deviation obeys the so-called «speed defect law» in the form $\frac{u_m - u}{u_*} = f\left(\frac{r}{R}\right)$, where the function f reflects the speed excess over the logarithmic law calculated (u_m is the maximum speed on the pipe axis). Since the CAM-1 turbulence model described in [3] does not give such a deviation, a modification of the turbulence model, called CAM-2, was proposed in [1], where the logic of choosing the necessary dependencies is shown.

When calculating a two-phase stratified flow with a horizontal interfacial surface, we will use the simplest square grid in the cross section of the pipe, as was done in the case of laminar flow [4, 5]. Note that when approximating the equations of laminar flow on a square grid, the main error appeared near the walls of the pipe. In the case of turbulent flow, the situation becomes more complicated in connection with the special role of the wall laminar sublayer and the buffer region. Even in the one-dimensional problem, in connection with the singularity of the equations, it was necessary to use a grid of up to 5000 nodes with a condensation of the grid near the wall [1]. In the two-dimensional case, this would lead to an unjustified increase in the counting time and high demands on computational resources. A compromise solution in this situation is the use the «near – wall» functions, i.e. special approximations of the unknown functions in the near-wall region.

1 Distribution of speed

An example of such an approach is the method of determining frictional stress by measurements of velocity near the wall. Assuming that the velocity distribution obeys the logarithmic law:

$$\frac{u}{u_*} = C_1 \ln\left(\frac{yu_*}{\nu}\right) + C_2,$$

from this relationship, can be determined $u_* = \sqrt{\frac{\tau_w}{\rho}}$ by the measured values of y and u (τ_w is the frictional stress on the wall). To apply method the near-wall functions, it is necessary to have velocity approximations suitable at any point close to the wall, including in the laminar sublayer and in the buffer zone. As such, an approximation near a smooth wall, we can propose the following relationships:

$$\frac{u}{u_*} = C_1 \ln (y_*) + C_2 - \frac{C_3}{y_*} + \frac{C_4}{y_*^2}$$

and $\dfrac{u}{u_*} = y_*$ for $y_* < 5$. The values: C_3, C_4 are determined by the smooth conjugation of these formulas for $y_* = 5$. In the CAM-2 model [1] the following values of constants: $C_1 = 2.439$, $C_2 = 5.386$, $C_3 = 30.31$, $C_4 = 43.76$ are accepted.

Special attention was consider to velocity distributions near the rough wall and approximations were obtained

$$\frac{u}{u_*} = C_1 \ln \left(\frac{y_*}{k_c}\right) + B_2 - \frac{C_3}{y_*} + \frac{C_4}{y_*^2}, B_2 = 8.5 + 1.78b - 0.89b^2, \tag{1}$$

where $b = (\lg k_c - 2)^2$.

Thus, for a point, spaced by a distance y along the normal from the surface, the velocity of the fluid will be known. These formulas allow us to determine the frictional stress, which is considered at this point equal to friction on the wall. After this, the differentiation of the approximation formulas determines the viscosity, from which the boundary condition for the turbulent viscosity transport equation can be obtained. This solves the problem of the boundary condition for $y \geq h_s$ (h_s is the height of roughness), which was mentioned in the consideration of roughness [6].

2 Calculation of two-dimensional two-phase flows in a pipe

Consider in more detail the application of the CAM-2 turbulence model in the two-dimensional approximation, changing the notation somewhat. Now let x, y be the Cartesian coordinates in the cross section of the pipe, and the z coordinate is directed along the pipe axis. Accordingly, the longitudinal velocity is denoted by w, ν_t is the turbulent viscosity, the total kinematic viscosity $\nu_\Sigma = \nu_t + \nu$ (in the section of logarithmic velocity distribution $\tau_\Sigma \approx \tau_w = u_*^2 \rho$), on the interface ($h$ is the depth of the lower layer of the liquid), R is the radius of the pipe.

For stationary problems for a circular pipe with allowance for the axial symmetry, the balance equation for the turbulent viscosity is written in the form

$$\frac{k_y \bar{C}_b \nu_\Sigma}{\tau_m^2} \left(\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 \right)^3 + \frac{\partial}{\partial x} \left(\nu_t \frac{\partial \nu_t}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu_t \frac{\partial \nu_t}{\partial y} \right) =$$

$$= \bar{C}_d \left(\left(\frac{\partial \nu_t}{\partial x}\right)^2 + \left(\frac{\partial \nu_t}{\partial y}\right)^2 \right) + \bar{C}_w \left(\frac{\nu_t}{R - r} \right)^2. \tag{2}$$

For speed, we have

$$\frac{1}{\rho} \frac{dp}{dz} = \frac{\partial}{\partial x} \left(\nu_\Sigma \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu_\Sigma \frac{\partial w}{\partial y} \right).$$

To approximate these equations on a square grid, the simplest approximation of the second order of accuracy on a five-point template is used. It is important that equation (2) in the near-wall nodes of the grid is not approximated, and the turbulent viscosity in them is determined with help of the near-wall functions, as was described [1].

To organize iterations for solving difference equations with respect to the value of the desired function, in the central node of the grid template it is possible. In addition, for convergence it is necessary to apply lower relaxation [7]. The convergence turns out to be very slow, but due to the not too shallow grid, the counting time is quite acceptable.

The program code of this solution by on the modified CAM-2 turbulence model for the two-dimensional approximation made it possible to obtain the following results.

3 Results

Figure 1 shows the results of a two-dimensional calculation of drag and velocity profiles in a smooth pipe on a square grid with 50 cells per radius (the fragment of the grid is shown in the left part of the Fig.)
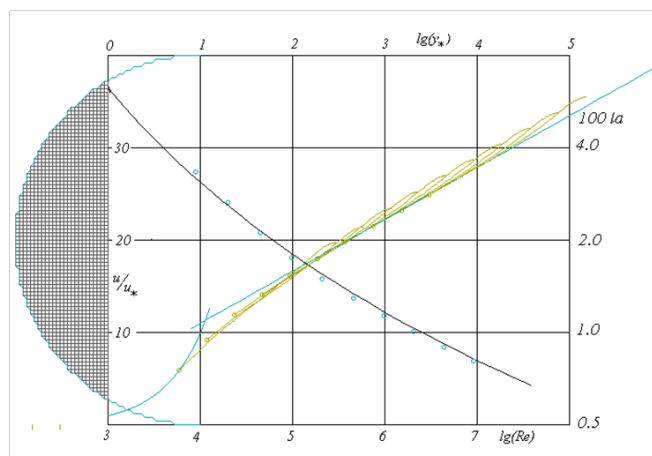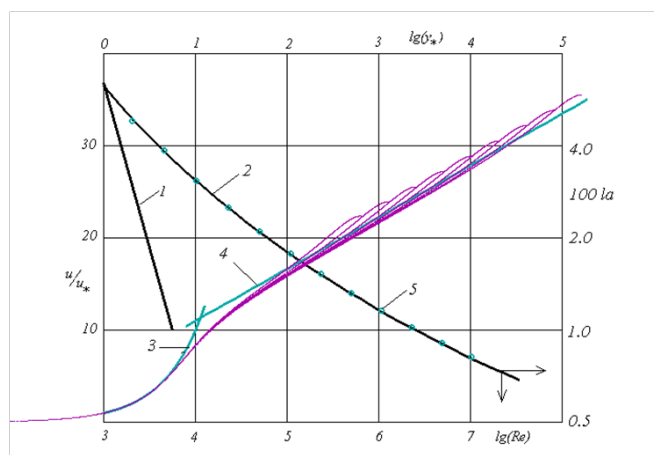

Figure 1. Results of a two-dimensional flow calculation
in a smooth pipe on the grid $h/R = 0.02$

A comparison with a more accurate one-dimensional calculation, the results of which are shown in [1]. For convenience of comparison, we represent this result in Figure 2.

1 — theoretical in the laminar regime; 2 — experimental Prandtl under turbulent regime; 3 — the velocity distribution in the laminar sublayer; 4 — in the turbulent core of the current; 5 — coefficients of resistance over the CAM-2

Figure 2. Coefficient of resistance and velocity distribution in a smooth
pipe (the CAM-2 turbulence model) for the one-dimensional case

As a result of the numerical solution to the modified turbulence model (CAM-2), a graph of the dependence of the drag coefficient on Re in logarithmic coordinates is constructed for the one-dimensional case and is shown in blue circles in Figure 2. The results shown in Figure 2 give the best agreement with the experimental data on frictional resistance given in [8].

In Figure 1, you can see that the use the near-wall functions provides acceptable accuracy. Let is attention that in yellow circles speed values are marked in the grid node closest to the surface.

To judge the influence of the grid step on the accuracy of the calculation, in Figure 3 the results obtained for 20 cells per radius are given. In this figure, you can see the principle of approximation of the boundary of the calculated area, shown by a blue broken line.
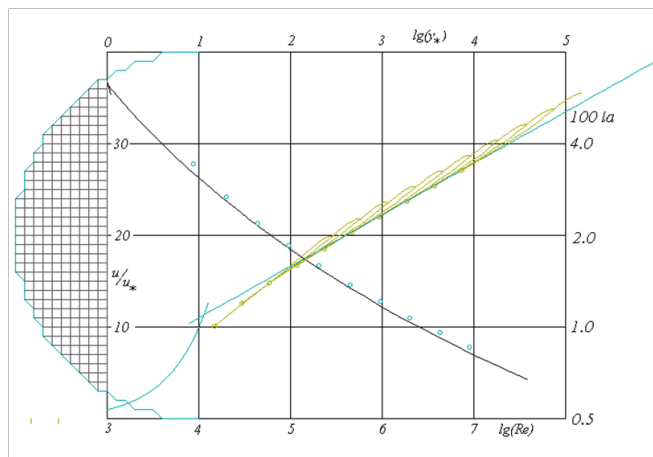
Figure 3. Results of two-dimensional flow computation
in a smooth pipe on a grid $h/R = 0.05$

As mentioned above, the near-wall functions can also be used for rough surfaces. The most famous results on the influence of rough on pipe resistance and velocity distribution were obtained by I. Nikuradze [9] and are given in many monographs and textbooks, for example [8, 10].

As a demonstration of this fact, and to check the accuracy of the approximations (1), we give in Figures 4–7 the results of two-dimensional calculations of flows in rough pipes. We note that, with a significant rough and a fine grid, several layers of grid cells can enter the zone of near – wall approximation, which complicates the calculations. For this reason, the results of calculations with a strong rough are not given here. The results shown in Figures 4–7 show that even calculations on a coarse grid have satisfactory accuracy. In these figures, the blue circles are the results of calculating the resistance at a fixed roughness, the horizontal line to the right is an experimentally determined resistance with full roughness, and the dark blue circle is the experimental value of the minimum coefficient of resistance.
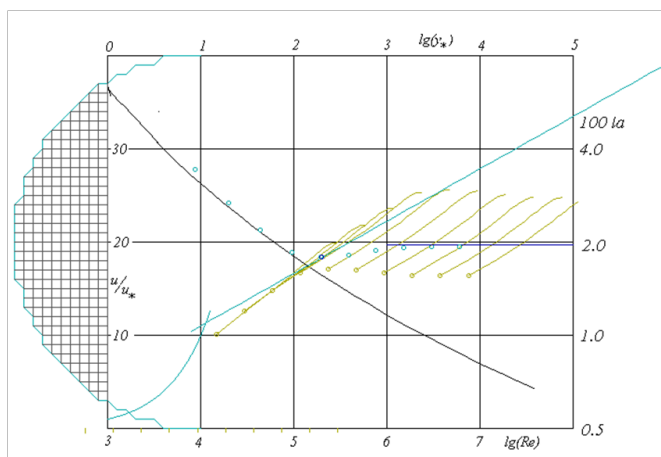

Figure 4. Results of two-dimensional calculation of flow in a pipe
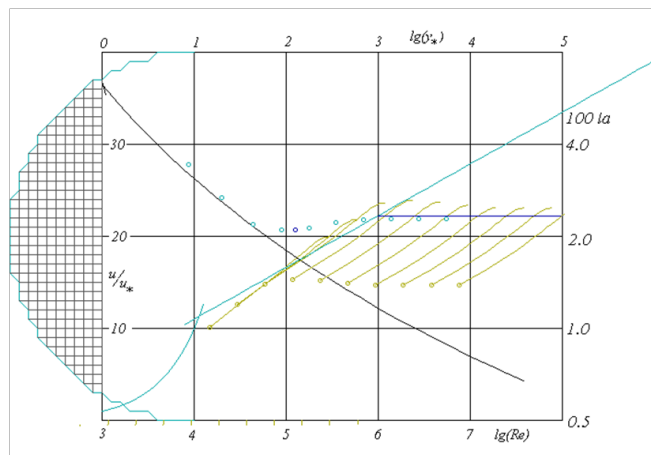with the roughness $R/h_s = 507$, on a grid $h/R = 0.05$

Figure 5. Results of two-dimensional calculation of flow in a pipe
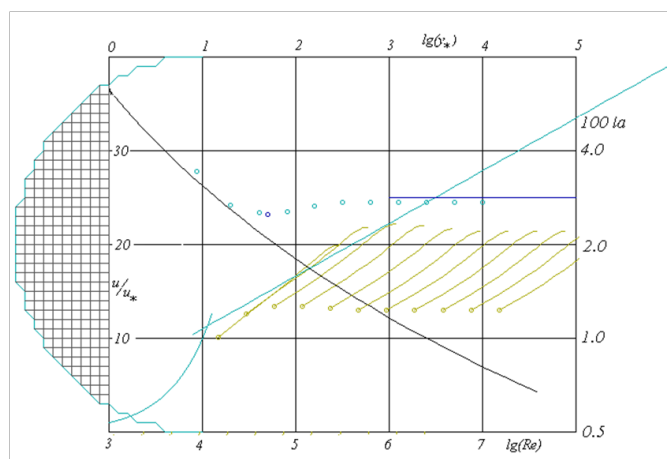with the roughness $R/h_s = 252$, on a grid $h/R = 0.05$

Figure 6. Results of two-dimensional calculation of flow in a pipe
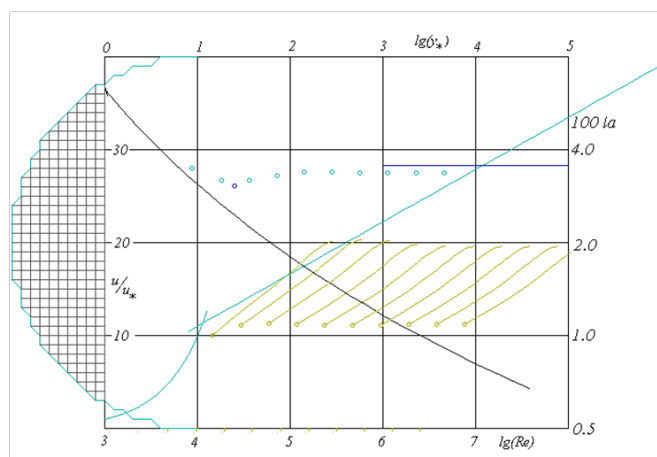with the roughness $R/h_s = 126$, on a grid $h/R = 0.05$

Figure 7. Results of two-dimensional calculation of flow in a pipe
with the roughness $R/h_s = 60$, on a grid $h/R = 0.05$

4 Conclusion

In this paper, we present the results of a two-dimensional calculation of the flow in a pipe based on the CAM-2 turbulence model, where the turbulent viscosity balance equations are used. The technique for calculating two-phase flows in long pipelines is described, and with roughness. In the formulation of the corresponding conditions at the level of the maximum roughness height (or higher), this method is combined with the method of applying the near-wall functions describing the distribution of parameters near the wall and resting on experimental data. The results of calculations with this roughness description are given in the form of dependences of laminar and turbulent resistances on the Reynolds number, and in the form of a velocity distribution calculated from the equations of the model.

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Құбырдағы стратифицирланған турбуленттік ағымның екіөлшемді есептеулері

Мақалада қосфазалы ортада көлбеу құбырлардағы стратифицирланған турбуленттік ағым қарастырылды. Жаңа турбуленттік модель [1] негізінде қосфазалы стратифицирланған, сонымен қатар құбырдың беті кедір-бұдыр жағдайдағы, турбуленттік ағымдарды зерттеу мақсатында, екіөлшемді ағымдар үшін программалық код жазылды. Созылған құбырдағы қосфазалы ағымдарды есептеудің әдістемесі ұсынылды. Турбулентті режимдегі тұрақты қима жағдайда стационарлы стратифицирланған қосфазалы ағымның есебі шешілді. Кедір-бұдырлы құбырлардың кедергісі және кедір-бұдырлықтың құбырдың кедергісіне әсері және жылдамдықтардың таратылуы есептелінді.

Кілт сөздер: стратифицирланған турбуленттік ағым, кедергі, екіөлшемді есептеулер, кедір-бұдыр бет.

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Двумерные расчеты стратифицированного турбулентного потока в трубе

В статье рассмотрено стратифицированное турбулентное течение двухфазной среды в наклонных трубах. На основе новой модели турбулентности [1] был разработан программный код для расчета двумерных течений для изучения двухфазных стратифицированных течений в трубах, в том числе с учетом шероховатости стенки трубопровода. Описана методика расчета двухфазных течений в протяженных трубопроводах. Численно решена проблема стационарного стратифицированного двухфазного потока в трубе постоянного сечения в случае турбулентного режима. Проведены расчеты сопротивления шероховатой трубы, и представлены результаты по влиянию шероховатости на сопротивление трубы и распределение скоростей.

Ключевые слова: стратифицированный турбулентный поток, сопротивление, двумерные расчеты, шероховатая поверхность.

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