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The energy method for solving a nonlinear problem of thermoelasticity for arod of variable cross section

A horizontal rod of limited length is considered. Radius of the rod varies linearly along its length. The cross-sectional area of the left end is larger than the cross-sectional area of the right end. The lateral surface of the test rod is completely insulated. The heat flow is fed to the cross-sectional area of the left end. Through the cross-sectional area of the right end of the rod, heat exchange takes place with the surrounding medium. The field of distribution of temperature, displacement, three components of deformation and stresses are determined in the work, provided that both ends of the rod are rigidly fixed. And also, the magnitude of the elongation of the rod is determined when one end of the rod is fixed and when the other is free.In the case of fixing the two ends of the rod, the magnitude of the resulting axial compressive force is also calculated. When studying the rod, the fundamental laws of conservation of energy were used.

Keywords: elongation, axial force, cross-section, temperature, displacement, deformation, stress.

Introduction

Many load-bearing elements of gas-generator, nuclear and thermal power stations, jet engines and the processing industry are rods of variable cross-section. To ensure reliable operation of these equipments, it is necessary to provide the thermal strength of load-bearing elements in the form of variable-section rods that operate with the simultaneous action of dissimilar kinds of heat sources. Because of the variability of the cross section, nonlinear thermomechanical processes appear in such rods.

To study the nature of such processes, consider a horizontal rod of limited length, of variable cross-section. In this case, the radius of the section varies linearly along the length of the investigated rod, i.e. $r = ax + b$, $0 \le x \le l$), where is the l-length of the rod, $a, b - const$. The cross-sectional area of the rod varies nonlinearly along the length of the rod in the following manner $F(x) = \pi(ax + b)^2[m^2]$. The lateral surface of the test rod along the entire length is heat-insulated. On the cross-sectional area of the left end of the rod $F(x = 0) = \pi b^2$, a heat flux with a constant intensity $q\left[\frac{watt}{cm^2}\right]$. Through the cross-sectional area of the right end of the rod $F(x = l) = \pi(al + b)^2$, heat exchange takes place with the surrounding medium. At the same time, the heat transfer coefficient $h\left[\frac{watt}{cm^2\ {}^0C}\right]$, ambient temperature $T_{oc}[K]$, the physical and mechanical properties of the core material is characterized by the coefficient of thermal expansion $\alpha\left[\frac{1}{K}\right]$, thermal conductivity $K_{xx}\left[\frac{watt}{cm \cdot K}\right]$ and modulus of elasticity $E\left[\frac{kg}{cm^2}\right]$. The scheme of the investigated rod is shown in Figure 1.

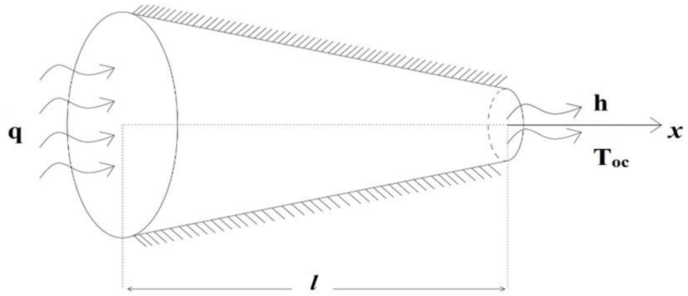

Figure 1. Scheme of the investigated rod

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In the presence of heat flow, heat insulation and heat transfer, the functional of the total thermal energy for the investigated rod has the form [1]:

$$J = \int_{F(x=0)} qTds + \int_V \frac{K_{xx}}{2}(\frac{\partial T}{\partial x})^2 dv + \int_{F(x=l)} \frac{h}{2}(T - T_{oc})^2 ds, \tag{1}$$

where $T = T(x)$ the field of distribution of temperatures along the length of the rod, which is approximated by a complete polynomial of the fourth order

$$T(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 = \varphi_i(x)T_i + \varphi_j(x)T_j + \varphi_k(x)T_k + \varphi_m(x)T_m + \varphi_n(x)T_n, \tag{2}$$

where $\varphi(x)$ — are spline functions:

$$\begin{aligned} \varphi_i(x) &= \frac{(3l^4 - 25l^3x + 70l^2x^2 - 80lx^3 + 32x^4)}{3l^4}; \\ \varphi_j(x) &= \frac{(48l^3x - 208l^2x^2 + 288lx^3 - 128x^4)}{3l^4}; \\ \varphi_k(x) &= \frac{(-36l^3x + 228l^2x^2 - 384lx^3 + 192x^4)}{3l^4}; \\ \varphi_m(x) &= \frac{(16l^3x - 112l^2x^2 + 224lx^3 - 128x^4)}{3l^4}; \\ \varphi_n(x) &= \frac{-3l^3x + 22l^2x^2 - 48lx^3 + 32x^4}{3l^4}, \end{aligned} \tag{3}$$

$0 \leq x \leq l$, where the nodal temperature values are determined by the formulas

$$T_j = T(x = 0); \quad T_i = T\left(x = \frac{l}{4}\right); \quad T_k = T\left(x = \frac{l}{2}\right); \quad T_m = T\left(x = \frac{3l}{4}\right); \quad T_n = T(x = l). \tag{4}$$

Taking into account (2)–(4), minimizing (1) with T_j, T_i, T_k, T_m and T_n we obtain a resolving system of algebraic equations taking into account existing natural boundary conditions.Solving the system we determine the nodal values of temperature (4), and by (2) we construct the field of temperature distribution along the length of the rod. If one end of the rod is fixed and the other end is free, then the length of the rod Δl_T[cm] is determined according to the general law of thermophysics [1]

$$\Delta l_T = \int_0^l \alpha T(x)dx.$$

If both ends of the rod are rigidly fixed, then an axial compressive force R[kG] arises in the rod, which is determined from the compatibility condition of the deformation [1]

$$R = \frac{\Delta l_T \cdot E \int_0^l F(x)dx}{l^2}.$$

In this case, a distribution field of the thermo-elastic component of the voltage $\sigma(x)\left[\frac{kg}{cm^2}\right]$ arises in the rod:

$$\sigma(x) = \frac{R}{F(x)}, 0 \le x \le l.$$

Then, according to Hooke's law, we can determine the distribution field of the thermoelastic deformation component $\varepsilon(x)$ [dimensionless]:

$$\varepsilon(x) = \frac{\sigma(x)}{E}.$$

The temperature component of deformations $\varepsilon_{T(x)}$ [dimensionless] is determined according to the general law of thermophysics [1]:

$$\varepsilon_{T(x)} = -\alpha T(x).$$

Then, according to Hooke's law, the field of distribution of the temperature component of the stress $\sigma_T(x)\left[\frac{kg}{cm^2}\right]$:

$$\sigma_T(x) = E \cdot \varepsilon_T(x) = -\alpha E \cdot T(x).$$

According to the theory of thermoelasticity, the laws of distribution of elastic components of deformations $\varepsilon_{x(x)}$ [dimensionless] and stresses $\sigma_T(x)\left[\frac{kg}{cm^2}\right]$:

$$\varepsilon_x(x) = \varepsilon(x) - \varepsilon_T(x);$$

$$\sigma_x(x) = E \cdot \varepsilon_x(x) = \sigma(x) - \sigma_T(x).$$

The potential energy of elastic deformations is used to determine the displacement field [2]:

$$\Pi = \int_V \frac{\sigma_x(x)}{2}\varepsilon_x(x)dv - \int_V \alpha E \cdot T(x) \cdot \varepsilon_x(x)dv.$$

According to the Cauchy relation [2], we have:

$$\varepsilon_x(x) = \frac{\partial U}{\partial x};$$

$$U = U(x) = \varphi_i(x)U_i + \varphi_j(x)U_j + \varphi_k(x)U_k + \varphi_m(x)U_m + \varphi_n(x)U_n,$$

where U is the displacement field. Minimizing Π from the nodal values of the displacement, a system of linear algebraic equations is constructed. To solve this system, it is necessary to specify the conditions for securing the two ends of the rod, i.e. $U_i = U(x=0) = 0$ and $U_n = U(x=l) = 0$. Further, defining U_i, U_j, U_k, U_m, U_n, a displacement field is constructed. For practical application of the above method and algorithm, we take the following initial data $l = 20$ см, $a = \frac{1}{10}$, $b = 4$ см, $\alpha = 0{,}0000125\frac{1}{K}$, $E = 2 \cdot 10^6\frac{kG}{cm^2}$, $K_{xx} = 100\frac{watt}{cm \cdot K}$, $h = 10\frac{watt}{cm^2 \cdot K}$, $T_{oc} = 40^0 K$, $q = -500\frac{watt}{cm^2}$.

Figure 2 shows that the temperature is higher near the left end of the rod, where the heat flow is supplied. Due to the thermal insulation of the lateral surface, heat is lost minimally, so that the temperature at the right end of the rod is maintained at 2400 K.

With these initial data, the obtained solutions are shown in Figures 2–5.

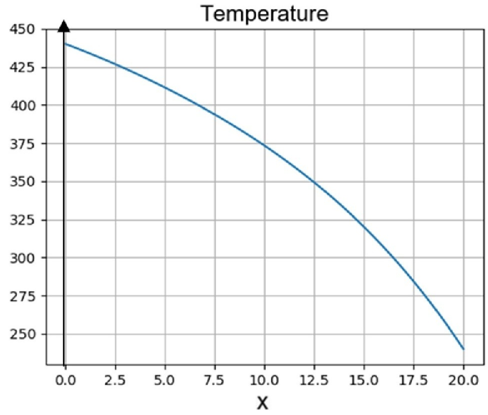

Figure 2. Dependences of the temperature T along the length of the rod

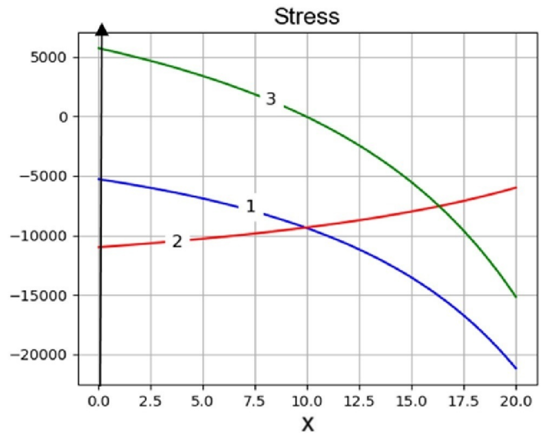

Figure 3. Stress Dependencies along the length of the rod

Decision

The stresses along the length of the rod are shown in Figure 3 ($1 - \sigma(x)$ is the thermoelastic, $2 - \sigma_{(x)}$ is the temperature, $3 - \sigma_E(x)$ is the elastic component of the stress). It can be seen from the figure that the thermoelastic $-\sigma(x)$ and temperature $-\sigma_{(x)}$ are the components of the stress along the entire length of the rod are of a compressive nature. While the elastic $-\sigma_E(x)$ component of the stress in the area $0 \leq E \leq \frac{l}{2}$ has a tensile character, and in the area $\frac{l}{2} \leq E \leq l$ it is compressive.

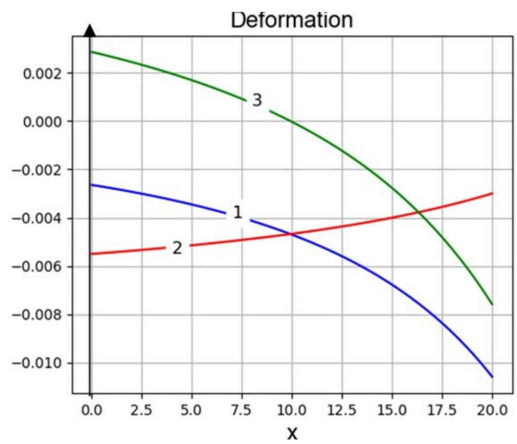

Figure 4. Dependence of the deformation along the length of the rod

Dependences of deformations along the length of the rod are shown in Figure 4 ($1-\sigma(x)-$thermoelastic, $2-\sigma_{(x)}-$ temperature, $3-\sigma_E(x)-$ elastic component deformation). The distribution field of the deformation components is proportional to the corresponding stresses.

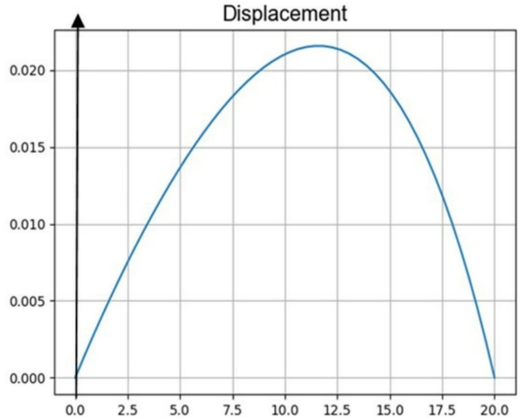

Figure 5. Dependences of displacement along the length of the rod

Figure 5 shows the field of distribution of displacements of a rod fixed at two ends. From this it can be seen that all sections (except for exceptions) move in the direction of the x axis. The greatest amplitude of displacement corresponds to the coordinate of$x \approx \frac{3l}{5}$.

Conclution

A numerical model of nonlinear thermomechanical processes in a rod of variable cross-section is developed, based on the fundamental law of conservation of energy. This allows to obtain reliable numerical results taking into account all natural boundary conditions. The results obtained are consistent with the corresponding laws of physics. This method can be used for the numerical solution of a class of problems determined by the steady-state thermomechanical state of load-bearing structural elements operating under the influence of dissimilar kinds of heat sources.

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Айнымалы көлденең қимасы бар сырықтың сызықты емес термоэластикалық есебін шешуде энергетикалық әдісті қолдану

Газ генераторларының, ядролық және жылу электр станцияларының, реактивті қозғалтқыштарының және өңдеу өнеркәсібінің көптеген элементтері айнымалы көлденең қимасы бар сырық болып табылады. Осы жабдықтардың сенімді жұмыс істеуін қамтамасыз ету үшін, жылу көздерінің әртүрлі түрлерінің бір уақытта әсер етуімен жұмыс істейтін айнымалы көлденең қимасы бар сырықты қарастырып, мойынтіректер элементтерінің жылу берілуін қамтамасыз ету қажет. Мақалада айнымалы көлденең қимадағы шектеулі ұзындықтағы сырық қарастырылды. Көлденең қимасы дөңгелек, оның радиусы ұзындығы бойымен сызықты түрде өзгереді. Сол жақтың көлденең қимасы оң жақтан үлкен. Зерттелген сырықтың бүйір беті толығымен термалды түрде оқшауланған. Жылу ағыны сол жақтың көлденең қимасына қолданылады. Сырықтың оң жақ шетінің көлденең қимасы арқылы қоршаған ортаға жылу алмасуы жүргізіледі. Жұмыста температура, ығысу, сырықтың екі жағы қатаң бекітілген жағдайдағы деформация және стрестің үш құрамдас бөліктері анықталған. Сондай-ақ бір шетіне бекітіп, екіншісі еркін болған кезде сырықтың ұзарту шамасы анықталды. Сырықтың екі ұшын бекіту нәтижесінде алынған осьтік қысымды күштің мәні есептелді. Сырықты зерттеу кезінде энергияны сақтаудың іргелі заңы пайдаланылды.

Кілт сөздер: ұзару, осьтік күш, қима, температура, жылжу, деформация, стресс, энергетикалық әдіс, сырық.

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Энергетический метод для решения нелинейной задачи термоэластичности для стержня переменного поперечного сечения

Многие несущие элементы реактивных двигателей, оборудования а также газогенераторных, атомных и тепловых электростанций и перерабатывающей промышленности являются стержнями переменного сечения. Для обеспечения надежной работы этих оборудований необходимо обеспечить термопрочность несущих элементов в виде стержней переменного сечения, которые работают при одновременном воздействии разнородных видов источников тепла. В статье рассмотрен горизонтальный стержень ограниченной длины переменного поперечного сечения. Радиус стержня меняется линейно по его длине. Площадь поперечного сечения левого конца больше площади поперечного сечения правого конца. Боковая поверхность исследуемого стержня полностью теплоизолирована. На площадь поперечного сечения левого конца подводится тепловой поток. Через площадь поперечного сечения правого конца стержня происходит теплообмен с окружающей средой. В работе определены поле распределения температуры, перемещения, три составляющие деформации и напряжения при условии, что оба конца стержня жестко закреплены. А также определена величина удлинения стержня, когда один конец стержня закреплен, а другой — свободен. В случае закрепления двух концов стержня вычислена величина возникающего осевого сжимающего усилия. При исследовании стержня использовался фундаментальный закон сохранения энергии.

Ключевые слова: удлинение, осевая сила, сечение, температура, перемещение, деформация, напряжение.