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Generalization of Tsytovich strength conditions for soils of anisotropic structure

The problem of developing a new generalized strength (plasticity)condition is applied to soils of anisotropic, in particular a transversal-isotropic (trans-structural) structure. This condition is derived by generalizing the known Coulomb-Moore plasticity condition σ_c for two directions: along the layers parallel and crosswise to layers, that is, for the perpendicular directions to the layers, relative to the isotropy plane for the transtropic soil, systematized for the first time by A.K. Bugrov and A.I. Golubev. It is proposed to analyze the possibility of generalizing the conditions of plasticity (strength) in the principal stresses σ_1 and σ_2, as well as on the values of the critical principal stress σ_{1c} and σ_{2c} , proposed and developed in due time for soils of the isotropic structure N.A. Tsytovich and N.S. Bulychev. Following the approach of W. Witke, who proposed to apply such a criterion to rocks of an orthotropic structure, the formulated criterion is proposed, which allows to determine the moment of the onset of plastic destruction of anisotropic soil and the direction of its further spread from the initial point. The table contains critical values of anisotropic soil in two orthogonal directions of the coordinate axes, calculated with the help of the proposed new criterion, which allows us to solve a new class of problems of fracture mechanics.

Keywords: soil, loam, isotropy, anisotropy, plasticity, isotropy plane.

Introduction

Most soils in nature have an anisotropic structure. For example, cover soils of mountain slopes, which are accompanied by landslide processes or construction sites, leading to a heel of the foundations of a building, and so on. Existing methods and approaches in soil mechanics are based on the assumption that soils are inherently isotropic in structure. Mechanical structures of such soils are determined by two parameters: Young's modulus E and Poisson coefficient ν. Whereas the properties of soils of an anisotropic structure are determined by five parameters: two Young's moduli E_1, E_2, two Poisson's coefficients ν_1, ν_2 and a shear modulus G_2. This limits the ability to solve soil stability problems by analytical methods. Moreover, there are no criteria to determine the strength of the soil. Therefore, there the question of developing a criterion of strength for soils of an anisotropic structure arises.

Task 1. Below in Figure, on the left, the subsurface sandy sand (upper layer) and loamy structure (lower layer) is shown, and on the right, the denser one — solid pebble soil and anisotropic. Such structures have soil construction sites. And in the mountain slopes, where landslide processes are often observed, the cover soils have not only sandy-loamy and pebbly structures, but also have more complex, sloping structures. It is not difficult to see that if the soils on the left figure have horizontally layered structures, then on the right figure are visible the destroyed soils, under the road, which have sloping structures.

Figure. Natural soils layered anisotropic structure

Now for the development of the criterion of strength we start from the well-known classical Coulomb-Moore strength (plasticity)condition developed for soils and rocks of an isotropic structure [1, 2], which looks like:

$$\tau_c = C + \sigma_n tg\varphi, \tag{1}$$

where τ_c is the tangential component of the tear-off voltage, C is the adhesion force, σ_n is the normal stress at the slip site; ϕ is an angle of internal friction. Unaxial compression strength expressed by next formula:

$$\sigma_c = \frac{2C\cos\varphi}{1-\sin\varphi}. \tag{2}$$

The condition of plasticity (strength) in the principal stresses (σ_3-does not affect the strength) expressed by next formula:

$$\sigma_{1c} = \sigma_{maxc} = \sigma_c + \beta\sigma_{2c}, \tag{3}$$

where β is the bulk strength parameter [2]:

$$\beta = \frac{1+\sin\varphi}{1-\sin\varphi}. \tag{4}$$

The condition of limiting equilibrium for disconnected loose soils is written according to N.A. Tsytovich [1]:

$$\frac{\sigma_{c1}-\sigma_{c2}}{\sigma_{c1}+\sigma_{c2}} = \sin\varphi, \tag{5}$$

where σ_{c1} and σ_{c2} are the limiting principal stresses. Hence, to obtain the expressions for σ_{c2}, we transform the expression (5)

$$\sigma_{2c} = \xi\sigma_{1c}, \tag{6}$$

where

$$\xi = \frac{1}{\beta} = \frac{1-\sin\varphi}{1+\sin\varphi}. \tag{7}$$

Also, from (6) using (4) we get

$$\sigma_{1c} = \beta\sigma_{2c}. \tag{8}$$

Now expression (6) with regard for (7) can be represented as

$$\frac{\sigma_2}{\sigma_1} = tg^2(45^\circ \pm \frac{\varphi}{2}). \tag{9}$$

This expression is widely used in the theory of pressure of soils on fences. The minus sign in parentheses corresponds to the active pressure, and the plus sign indicates the passive resistance of loose soils. Now we write down the condition of maximum equilibrium for cohesive soils

$$\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2 + cctg\varphi} = \sin\varphi, \tag{10}$$

whence

$$\sigma_1 - \sigma_2 = 2\sin\varphi(\frac{\sigma_1 + \sigma_2}{2} + c\,\mathrm{ctg}\,\varphi, \tag{11}$$

or

$$\sigma_1 = \sigma_2 + 2\sin\varphi(\frac{\sigma_1 + \sigma_2}{2}) + c\,\mathrm{ctg}\,\varphi. \tag{12}$$

This well-known N.A. Tsytovich criteria should be generalized taking into account the anisotropy of the soil shown in Figure. It should be noted that in recent years the issues of anisotropy of soils have been actively pursued by Kazakhstan scientists. For example, the authors of [5–10], in addition to researching the stressful-deformed state of soils and rocks of anisotropic structure, surface, underground and other engineering structures, develop and special criteria for destruction.

Results

Following V. Vitke's [10] approach, proposed in due time for rocks of an orthotropic structure, the criterion of strength (1)–(12) is extended for soils of a transtropically anisotropic structure. Here the plasticity in soils can develop along the isotropy plane (parallel) or (and) in directions crosswise to it (perpendicular). They will differ significantly from each other. Therefore, condition (1) is written separately for these two directions

$$\tau_{||c} = C_{||} + \sigma_{||n} tg\varphi_{||}; \tag{13}$$

$$\tau_{\perp c} = C_{\perp} + \sigma_{\perp n} tg\varphi_{\perp}, \tag{14}$$

where $\tau_{||c}$, $\tau_{\perp c}$ are the tangential stresses on slip sites; $C_{||}$, $C_{\perp}$ are cohesion forces, normal stresses on slip planes in parallel $\sigma_{||n}$, and perpendicular $\sigma_{\perp n}$ directions to the isotropy plane are determined from the experiment or removed from Mohr's circles, $\varphi_{||}$, $\varphi_{\perp}$ are internal friction angles. For oblique-laminated anisotropic materials, we denote the slope angle of the isotropy planes by $\overline{\overline{\varphi}}$. It should be recalled that the angles $\overline{\overline{\varphi}}$ and φ have completely different meanings that are not related to each other. The tensile strength for uniaxial compression is written in the form

$$\sigma_{||c} = \frac{2C_{||}\cos\varphi_{||}}{1 - \sin\varphi_{||}}; \tag{15}$$

$$\sigma_{\perp c} = \frac{2C_{\perp}\cos\varphi_{\perp}}{1 - \sin\varphi_{\perp}}. \tag{16}$$

Also, the plasticity(strength) condition in the principal stresses will have the form

$$\sigma_{||c} = \sigma_{max||c} = \sigma_{||c} + \beta_{||}\sigma_{2||c}; \tag{17}$$

$$\sigma_{\perp c} = \sigma_{max\perp c} = \sigma_{\perp c} + \beta_{\perp}\sigma_{2\perp c}, \tag{18}$$

where $\beta_{||}$, $\beta_{\perp}$ — are parameters of bulk strength:

$$\beta_{||} = \frac{1 + \sin\varphi_{||}}{1 - \sin\varphi_{||}}; \tag{19}$$

$$\beta_{\perp} = \frac{1 + \sin\varphi_{\perp}}{1 - \sin\varphi_{\perp}}. \tag{20}$$

The condition of limiting equilibrium for non-cohesive bulk solids written by next expressions:

$$\frac{\sigma_{c1||} - \sigma_{c2||}}{\sigma_{c1||} + \sigma_{c2||}} = \sin\varphi_{||}; \tag{21}$$

$$\frac{\sigma_{c1\perp} - \sigma_{c2\perp}}{\sigma_{c1\perp} + \sigma_{c2\perp}} = \sin\varphi_{\perp}, \tag{22}$$

where $\sigma_{c1\perp}$ and $\sigma_{c1||}$ are the limiting principal stresses. Similarly, we write expression (6) in the following form

$$\sigma_{2c||} = \xi_{||}\sigma_{1c||}; \tag{23}$$

$$\sigma_{2c\perp} = \xi_{\perp}\sigma_{1c\perp}, \tag{24}$$

where

$$\xi_{||} = \frac{1}{\beta_{||}} = \frac{1 - \sin\varphi_{||}}{1 + \sin\varphi_{||}}; \tag{25}$$

$$\xi_{\perp} = \frac{1}{\beta_{\perp}} = \frac{1 - \sin\varphi_{\perp}}{1 + \sin\varphi_{\perp}}, \tag{26}$$

or from the expressions (11), (12) using (7), (8) we obtain

$$\sigma_{1c||} = \beta_{||}\sigma_{2c||}; \tag{26}$$

$$\sigma_{1c\perp} = \beta_{\perp}\sigma_{2c\perp}. \tag{27}$$

Expressions (11) and (12) with allowance for (13) and (14) with respect to the isotropy plane can be represented in the form

$$\frac{\sigma_{2c||}}{\sigma_{1c||}} = tg^2(45^\circ \pm \frac{\varphi_{||}}{2}); \tag{28}$$

$$\frac{\sigma_{2c\perp}}{\sigma_{1c\perp}} = tg^2(45^\circ \pm \frac{\varphi_{\perp}}{2}). \tag{29}$$

It is known that, in the isotropic version, these expressions in the form (9) are used in the theory of pressure of soils on fences. And here, in expressions (28) and (29) the minus sign in parentheses corresponds to the active pressures, and the plus sign to the passive resistances of free-flowing soils.If the barrier is a retaining wall, then the pressure on this wall according to expression (29) acts either perpendicularly or at an angle, depending on the slope of the layers of the isotropy planes of soils with an inclined anisotropic structure.Since the pressure in expression (28) acts across the layers of isotropy, then relative to the wall, they act parallel to the wall at the points of adhesion, that is, on the boundary layer. Now we write the condition of limiting equilibrium for connected soils by analogy with N.A. Tsytovich

$$\frac{\sigma_{1c||} - \sigma_{2c||}}{\sigma_{1c||} + \sigma_{2c||} + C_{||}ctg\varphi_{||}} = \sin\varphi_{||}; \tag{30}$$

$$\frac{\sigma_{1c\perp} - \sigma_{2c\perp}}{\sigma_{1c\perp} + \sigma_{2c\perp} + C_{\perp}ctg\varphi_{\perp}} = \sin\varphi_{\perp}. \tag{31}$$

We transform these expressions to the next form:

$$\sigma_{1||} - \sigma_{2||} = 2\sin\varphi_{||}(\frac{\sigma_{1||} + \sigma_{2||}}{2} + C_{||}ctg\varphi_{||}); \tag{32}$$

$$\sigma_{1\perp} - \sigma_{2\perp} = 2\sin\varphi_{\perp}(\frac{\sigma_{1\perp} + \sigma_{2\perp}}{2} + C_{\perp}ctg\varphi_{\perp}), \tag{33}$$

or to calculate the largest principal stresses, we represent them in the form

$$\sigma_{1||} = \sigma_{2||} + 2\sin\varphi_{||}(\frac{\sigma_{1||} + \sigma_{2||}}{2} + C_{||}ctg\varphi_{||}); \tag{34}$$

$$\sigma_{1\perp} = \sigma_{2\perp} + 2\sin\varphi_{\perp}(\frac{\sigma_{1\perp} + \sigma_{2\perp}}{2} + C_{\perp}ctg\varphi_{\perp}). \tag{35}$$

The need to develop such generalized criteria, allowing to determine not only the state of pre-failure, but also the direction of damage propagation, confirms Figure. There are their experimentally determined values. Such single data is available, for example, in [3]. The following table shows the critical values β for some of the main types of surface soils. For comparison, limit values β for limestone and concrete are also given. These data also confirm the need to develop a criterion for destruction for soils of an anisotropic structure.

Table

Critical parameters of anisotropic plasticity for soils of an anisotropic structure

No	Soils	Volume weight	Strength parameter	Young's Modules		Shear modulus	Poisson's coefficients		Clutch Strength		Internal friction		Main stresses	
		γ, kH $/m^3$	ξ_0	$E_\parallel$, Мпа	$E_\perp$, Мпа	$G_{\parallel\perp}$, Мпа	$\nu_\parallel$	$\nu_\perp$	$C_\parallel$, Мпа	$C_\perp$, Мпа	$\varphi^0_\parallel$	$\varphi^0_\perp$	$\sigma_{c\parallel}$	$\sigma_{c\perp}$
1	Loam	17.0	0.60	13.4	26.4	7.6	0.16	0.24	0.025	0.050	26	26	0.80	0.160
2	Sand	17.0	0.43	23.0	16.0	7.0	0.30	0.30	0.005	0.005	27	33	0.16	0.180
3	Loam saturated	20.0	0.48	30.0	15.0	7.6	0.36	0.24	0.030	0.060	19	23	0.084	0.197
4	Priming	19.0	1.00	10.0	20.0	7.4	0.30	0.40	0.080	0.120	20	24	0.230	0.370
5	Loam	9.4	0.65	12.0	8.0	3.4	0.39	0.35	0.010	0.014	20	24	0.029	0.043
6	Loam	8.0	0.65	6.0	4.0	1.7	0.39	0.35	0.050	0.007	15	18	0.130	0.019
7	Loam	9.2	0.65	9.0	6.0	2.5	0.39	0.35	0.006	0.008	18	22	0.017	0.024
8	Sandy loam	19.8	0.53	19.6	18.4	7.1	0.31	0.30	0.003	0.003	18	21	0.008	0.009
9	Loam is hard	19.9	0.58	39.8	27.0	10.0	0.36	0.35	0.02	0.02	13	17	0.005	0.054
10	Sand fine	21.1	0.25	81.3	85.0	32.7	0.28	0.30	0.002	0.002	35	37	0.008	0.008
11	Rock Limestone	2.5	0.33	3200	1600	1185	0.38	0.32	47	0.25	31	29	116.1	0.849
12	Concrete BIT-PE polyester	1.65		4941	4941	1930	0.28	0.28					201.4	0.849

Conclusion

A justified criterion of strength is proposed, which makes it possible to determine the direction of propagation of a fracture of earth fractures relative to the isotropy plane of inhomogeneous layers. They are directions parallel to the layers and perpendicular.The developed parameters of strength (plasticity) for soils of natural anisotropic structure allows to solve the class of geomechanics tasks associated with the definition of stress-deformed condition of the covering soils of mountain slopes necessary for the prediction of landslide processes and the state of stability of under foundation soils of construction sites.

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Анизотропты құрылымды топырақтарға арналған Цытовичтың қатаңдық шарттарын жалпылауға арналған трифотикалық күшейтулердің синтезі

Тасымалдаудың трансверсалды-изотоптық (транстроптык) құрылыстарына қатысты жаңа анизотроптық грунтты қолдануға арналған жаңа беріктік (пластикалық) жалпыланған шартын құру мәселесі талқыланды. Мұндай шарт белгілі Кулон-Мор σ_c беріктік шартын екі бағытта жалпылаудан шығады: қабаттарға параллель және қабаттарға қарсы, яғни, А.К. Бугров және А.И. Голубев алғаш рет жүйелеген транстропты құрылымы бар топырақтардың изотропия жазықтығына қатысты қабаттарға перпендикуляр бағыттар үшін. Негізгі σ_1 және σ_2, сонымен қатар кезінде Цытович және Н.С. Булычевпен ұсынылып дамытылған σ_{1c} және σ_{2c} негізгі критикалық кернеулер мәні бойынша иілгіштіктің (беріктіктің) шарттарын жалпылау мүмкіндігіне талдау ұсынылды. В.Витке ұсынысы бойынша, осы критерийді ортотропты құрылымды тау жыныстарына қолдануды ескере отырып, анизотропты құрылымы бар топырақтардың пластикалық қирау моментін және бастапқы нүктеден ары қарай таралу бағытын анықтауға мүмкіндік беретін критерий құру ұсынылды. Қирау механикасы есептерінің жаңа класын шешуге мүмкіндік туғызатын жаңа критериймен есептелген екі ортогоналды осьтер бағытында анизотропты құрылымы бар топырақтардың критикалық мәндерін қамтитын кесте берілген.

Кілт сөздер: топырақ, изотроптық, анизотропиялық, иілгіштік, изотропты жазықтығы.

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Обобщение условий прочности Цытовича для грунтов анизотропного строения

Рассмотрен вопрос разработки нового обобщенного условия прочности (пластичности) применительно к грунтам анизотропного, в частности, трансверсально-изотропного (транстропного) строения. Такое условие выводится обобщением известного условия пластичности Кулона-Мора σ_c для двух направлений: вдоль слоев параллельно и вкрест слоям, т.е. для направлений, перпендикулярных к слоям, относительно плоскости изотропии для грунтов транстропного строения, систематизированного впервые А.К. Бугровым и А.И. Голубевым. Сделан анализ возможности обобщения условий пластичности (прочности) по главным напряжениям σ_1 и σ_2, а также по значениям критических главных напряжений σ_{1c} и σ_{2c}, предложенный и развитый в свое время для грунтов изотропного строения Н.А. Цытовичем и Н.С. Булычевым. Следуя подходу В.Витке, который предложил применить такой критерий к горным породам ортотропного строения, предложен сформулированный критерий, который позволяет определить момент наступления пластического разрушения грунтов анизотропного строения и направление его дальнейшего распространения от начальной точки. Приведена таблица, содержащая критические значения грунтов анизотропного строения в двух ортогональных направлениях координатных осей, вычисленные с помощью предложенного нового критерия, который позволяет решить новый класс задач механики разрушения.

Ключевые слова: грунт, изотропия, анизотропия, пластичность, плоскость изотропии.

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