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Companions of fragments in admissible enrichments

In this paper, model-theoretic properties of companions of fragments of special subsets in admissible enrichments are considered. Admissible enrichments are understood as enrichments of a signature that preserve the basic syntactic properties of the Jonsson theory under consideration. The study of the properties of companions of the Jonsson theory is related to the classical problematics of studying inductive theories, which was determined at the time by one of the founders of the theory of models A. Robinson. In this paper, the main properties of companion fragments of definable subsets of the semantic model of a given Jonsson theory is categoricity and it is considered.

Keywords: Jonsson theory, semantic model,categoricity existentially prime, pregeometry, modular pregeometry, model companion.

Let us single out two directions in the development of model theory. In the well-known book H.J. Keisler they are called the western and eastern theory of models, since one of the founders of the theory of models A. Tarski lived on the west coast of the United States since 1940, and another founder A. Robinson - in the east. Western theory of models develops in the traditions of Skolem and Tarski. It was more motivated in theory, analysis and set theory, and it uses all formulas of first-order logic.

The Eastern theory of models develops in the traditions of Maltsev and Robinson. It was motivated by problems in abstract algebra, where the theory formulas usually have at most two blocks of quantifiers. It emphasizes the set of quantifier-free formulas and existential formulas. Unlike the Western theory of models, which studies complete theories, Eastern model theory generally deals with incomplete thorium. The class of incomplete theories is wide enough, so we can confine ourselves to inductive theories ($\forall\exists$-axiomatizable). In the sense of completeness of the theory considered, the maximum requirement, as a rule, is $\forall\exists$-completeness. All these conditions are satisfied by the Johnson theories. Thus, we conclude that the study of Johnson's theories refers to its essence to the problems of the «eastern» theory of models.

This article is related to one of the branches of Model Theory, and more precisely to studying of Jonsson sets. This part of Model Theory is concerned with the study of incomplete inductive theories and more precisely Jonsson theories and some of their generalizations [1]. Actually it examines the model-theoretical properties of Jonsson subsets of semantic model of some Jonsson theory. In particular the lattice of special formulas of such sets is considered.

In the study of complete theories, one of the main methods is to use the properties of a topological space. In the case of a fragment of a Jonsson set, one can consider a lattice of existential formulas, which is a sublattice of Boolean algebra. The main goal of this article is to develop the basic concepts and methods of that part of the theory of models, which will provide an opportunity for fruitful research of Jonsson theory and some of its generalizations, taking into account modern developments in model theory. Our technique is standard for studying incomplete theories. The method of research consists in translating the elementary properties of the center of Jonsson theory into the theory itself.

This article discusses the method of studying Jonsson theories, first proposed by T.G Mustafin in [2]. The basis of this method is the natural connection of the class of models of arbitrary Jonsson theory T with the class of models of the theory T^*, where $T^* = Th(C)$ and C are T — universal, T — homogeneous model of the theory T. The model C exists by the Morley-Vaught theorem [3], thereby C is semantic, and T^* is a syntactic invariant of the Jonsson theory of T. The weak point of such an approach to the study of Johnson theories is the presence in the proof of the Morley-Vaught theorem a hypothesis about the existence of a strongly unreachable cardinal. To avoid set-theoretic problems allows a change in the definition of the semantic model (T — universal, T — homogeneous model of the theory T). This was done in the work of E.T. Mustafin [4].

In the future, we will adhere to the method proposed by T. G. Musafin in [2] only with a change in the definition of the semantic model in [3]. Designations are standard as in [5]. All indefinable concepts here are considered known and can be found in [5]. We give the necessary definitions of the basic concepts of this article.

1 Preliminary Information on Jonsson theories

Consider the theory T of a countable language of first order L.

Definition 1.1. A theory T is Jonsson if:

1) theory T has infinite models;

2) theory T is inductive;

3) theory T has the joint embedding property (JEP);

4) theory T has the property of amalgam (AP).

Definition 1.2. Let $\kappa \geq \omega$. Model M of theory T is called κ-universal for T, if each model T with the power strictly less κ isomorphically imbedded in M; κ-homogeneous for T, if for any two models A and A_1 of theory T, which are submodels of M with the power strictly less then κ and for isomorphism $f : A \to A_1$ for each extension B of model A, wich is a submodel of M and is model of T with the power strictly less then κ there is exist the extension B_1 of model A_1, which is a submodel of M and an isomorphism $g : B \to B_1$ which extends f.

Definition 1.3. A homogeneous-universal for T model is called κ — homogeneous-universal for T power model κ, where $\kappa \geq \omega$.

The following sentences can be found in [6]:

Fact 1. [6; 0.1]. Each Jonsson theory T has κ^+ — homogeneous-universal model of power 2^κ. Conversely, if the theory T is inductive, has an infinite model, and has a ω^+ — homogeneous-universal model, then the theory T is a Jonsson theory.

Fact 2. [6; 0.2].

1. Let T be a Jonsson theory. Two models M and M_1 κ-homogeneous-universal for T are elementarily equivalent.

2. If there exists a model M κ-homogeneous-universal for T of cardinality κ, then it is unique up to isomorphism. In addition, the model is M κ-homogeneous, i.e. any isomorphism between two submodels A and B of a model M of cardinality is strictly less than κ, which are models of the theory T, extends to an automorphism M.

We show that in the framework of the definition of homogeneity and universality from [6] the following is true:

Definition 1.4. A model C of a Jonsson theory T is called a semantic model of the theory T, if it is ω^+-homogeneous-universal in the sense of [6].

Definition 1.5. A model $\mathcal{A}$ of the theory T is called T-existentially closed if for any model $\mathcal{B}$ of the theory T and any existential formula $\varphi(\overline{x})$ with constants from $\mathcal{A}$ is $\mathcal{A} \models \exists \overline{x} \varphi(\overline{x})$ provided that, $\mathcal{A}$ is a submodel $\mathcal{B}$ and $\mathcal{B} \models \exists \overline{x} \varphi(\overline{x})$.

In connection with the definition 1.4, the following fact is true.

Lemma 1.1. The semantic model C of the Jonsson theory T is T-existentially closed.

Proof. Let C have cardinality $\kappa \geq \omega$. Let M be some extension of C cardinality 2^{κ_1}, which exists by virtue to the fact [6; 0.1]. The model M is isomorphically embedded in M_1 by virtue of κ_1^+-universality of M_1. Then C is isomorphically embedded in M_1. Let $m \in C$ and $M \models \exists x \varphi(x, m)$. Then $M_1 \models \exists x \varphi(x, m)$. therefore, by virtue of [6; 0.2]. (1), M_1 and C are elementarily equivalent. So, $C \models \exists x \varphi(x, m)$. Thus, C - T-exsentially closed.

The following fact was considered in [6].

Fact 3. [6; 0.3]. Let T be a Jonsson theory. If T^* is model complete and $\kappa \geq \omega$, then κ is homogeneous - universal for T models κ-saturated; if T^* is not model-complete, then no model is ω^*-saturated.

In the framework of the new definition of the semantic model of the Jonsson theory we give the following definition.

Definition 1.6. A Jonsson theory of T is called perfect if each semantic model of T is ω^*-a saturated model of T^*.

Theorem 1.1 [6]. Let T is a Jonsson theory. Then the following conditions are equivalent:

– theory T is perfect;

– theory T^* is a model companion of theory T.

Consequence 1.1. Let T be a Jonsson theory. Then the following conditions are equivalent:

1. $ModT^* = E_T$.

2. $T^* = T^f$, where E_T is a class of T-existentially closed models, $T^f = Th(F_T)$, where F_T is a class of generic models T (in the sense of Robinson finite forcing).

Moreover, you can notice the following:

Remark 1.1. Perfectness of the Jonsson theory is equivalent to the model completeness T^*.

2 Countable and uncountable categoricity

The purpose of this section is to give proof of two results (Theorems 2.5 and 2.9) related to the countable and uncountable categorical nature of the center of some classes of Jonsson theories. We define the concepts and related results necessary for the proof of Theorem 2.5.

Definition 2.1. The inductive theory T is called the existentially prime if: it has a algebraically prime model, the class of its AP (algebraically prime models) denote by AP_T; class E_T non trivial intersects with class AP_T, i.e. $AP_T \bigcap E_T \neq 0$.

Definition 2.2. The theory is called convex if for any its model A and for any family $\{B_i \mid i \in I\}$ of substructures of A, which are models of the theory T, the intersection $\bigcap_{i \in I} B_i$ is a model of T.

Definition 2.3. Let X be $\Delta - cl$-Jonsson subset of semantic model of fixed Jonsson theory and let $cl : P(X) \to P(X)$ be an operator on the power set of X. We say that (X, cl) is a Jonsson pregeometry if the following conditions are satisfied.

If $A \subseteq X$, then $A \subseteq cl(A)$ and $cl(cl(A)) = cl(A)$.

If $A \subseteq B \subseteq X$, then $cl(A) \subseteq cl(B)$.

(exchange) $A \subseteq X$, $a, b \in X$, and $a \in cl(A \cup \{B\})$, then $a \in cl(A), b \in cl(A \cup \{a\})$.

(finite character) If $A \subseteq X$ and $a \in cl(A)$, then there is a finite $A_0 \subseteq A$ such that a $a \in cl(A_0)$.

We say that $A \subseteq X$ is closed if $cl(A) = A$.

Definition 2.4. If (X, cl) is a Jonsson pregeometry, we say that A is Jonsson independent if $a \notin cl(A \setminus \{a\})$ for all $a \in A$ and that B is a J-basis for Y if is J-independent and $Y \subseteq acl(B)$.

Lemma 2.1. If (X, cl) is a J — pregeometry, $Y \subseteq X$, $B_1, B_2 \subseteq Y$ and each B_i is a J — basis for Y, then $\mid B_1 \mid = \mid B_2 \mid$.

We call $\mid B_i \mid$ the J-dimension of Y and write $Jdim(Y) = \mid B_i \mid$.

If$A \subseteq X$, we also consider the localization $cl_A(B) = cl(A \cup B)$.

Lemma 2.2. If (X, cl) is a J-pregeometry, then (X, cl_A) is a J-pregeometry.

If (X, cl) is a J-pregeometry, we say that $Y \subseteq X$ is J-independent over A if Y is J-independent in (X, cl_A). We let $Jdim(Y/A)$ be the J-dimension of Y in the localization (X, cl_A). We call $Jdim(Y/A)$ the J-dimension of Y over A.

Definition 2.5. We say that a J-pregeometry (X, cl) is J-geometry if $cl(\emptyset) = \emptyset$ and $cl(\{x\}) = \{x\}$ for any $x \in X$.

If (X, cl) is a J-pregeometry, then we can naturally define a J-geometry. Let $X_0 = X \setminus cl(\emptyset)$. Consider the relation $\sim$ on X_0 given by $a \sim b$ if and only if $cl(\{a\}) = cl(\{b\})$. By exchange, $\sim$ is an equivalence relation. Let $\widehat{X}$ be $X_0/\sim$. Define $\widehat{cl}$ on $\widehat{X}$ by $\widehat{cl}(A/\sim) = \{b/\sim : b \in cl(A)\}$.

Lemma 2.3. If (X, cl) is a J-pregeometry, then $(\widehat{X}, \widehat{cl})$ is a J-geometry.

Definition 2.6. Let (X, cl) be J-pregeometry. We say that (X, cl) is trivial if $cl(A) = Y_{a \in A} cl\{a\}$ for any $A \subseteq X$. We say that (X, cl) is modular if for any finite-dimensional closed $Jdim(A \cup B) = Jdim(A) + Jdim(B) -$ $- Jdim(A \cap B)$

We say that (X, cl) is locally modular if (X, cl_a) is modular for some $a \in X$.

Definition 2.7. We say that (X, cl) is modular if for any finite-dimensional closed $A, B \subseteq X$

$$dim(A \cup B) = dimA + dimB - dim(A \cap B).$$

Definition 2.8. If $X = C$, then the Jonsson theory of T is called modular.

Theorem 2.1. Let (X, cl) be J- pregeometry. The following are equivalent.

1) (X, cl) is modular;

2) if $A \subseteq X$ is closed and nonempty, $b \in X$, and $x \in cl(A, b)$, then there is $a \in A$ such that $x \in cl(a, b)$;

3) if $A, B \subseteq X$ are closed and nonempty, and $x \in cl(A, B)$, then there are $a \in A$ and $b \in B$, such that $x \in cl(a, b)$.

Definition 2.9. [7]. $\mathfrak{A}$ is called (Γ_1, Γ_2)-the atomic model of T theory, if $\mathfrak{A}$ model T and for each n, each n-k elements from $\mathfrak{A}$ satisfies some formula from Γ_1, which is complete for Γ_2 - formulas.

Theorem 2.2. If L is a countable language and T is a complete ω-categorical theory, then T has a ω-categorical model companion.

Definition 2.10. Let $X \subseteq C$. We will say that a set X is $\nabla - cl$-Jonsson subset of C, if X satisfies the following conditions:

1) X is ∇-definable set (this means that there is a formula from ∇, the solution of which in the C is the set X, where $\nabla \subseteq L$, that is ∇ is a type of formula, for example $\exists, \forall, \forall\exists$ and so on);

2) $cl(X) = M$, $M \in E_T$, where cl is some closure operator defining a pregeometry [8; 289] over C (for example $cl = acl$ or $cl = dcl$).

Definition 2.11 [7].

1. $\mathcal{A}, a_0, ..., a_{n-1} \Rightarrow_\Gamma (\mathcal{B}, b_0, ..., b_{n-1})$ means that for each formula $\varphi(x_0, x_2, ..., x_{n-1})$ of Γ, if $\mathcal{A} \models \varphi(\bar{a})$, then $\mathcal{B} \models \varphi(\bar{b})$.

2. $(\mathcal{A}, \bar{a}) \equiv_\Gamma (\mathcal{B}, \bar{b})$ means $(\mathcal{A}, \bar{a}) \Rightarrow_\Gamma (\mathcal{B}, \bar{b})$ and $(\mathcal{B}, \bar{b}) \Rightarrow_\Gamma (\mathcal{A}, \bar{a})$.

Definition 2.12.

1. $\mathcal{A}$ is called a Σ-nice-algebraically prime model of T, if $\mathcal{A}$ is a countable model of T and for each model of $\mathcal{B}$ theory of T, each $n \in \omega$ and for all $a_0, a_2, ..., a_{n-1} \in A$, $b_0, b_2, ..., b_{n-1} \in B$, if $\mathcal{A}, a_0, ..., a_{n-1} \Rightarrow_\exists (\mathcal{B}, b_0, ..., b_{n-1})$, then for each $a_n \in A$ there exists some $b_n \in B$ such that $\mathcal{A}, a_0, ..., _n \Rightarrow_\exists (\mathcal{B}, b_0, ..., b_n)$.

2. $\mathcal{A}$ is called a Σ^*-nice-algebraically prime model of the theory T, if $\mathcal{A}$ is a countable model of T and for each model $\mathcal{B}$ theory T, each $n \in \omega$ and for all $a_0, a_2, ..., a_{n-1} \in A$, $b_0, b_2, ..., b_{n-1} \in B$, if $\mathcal{A}, _0, ..., _{n-1} \equiv_\exists (\mathcal{B}, b_0, ..., b_{n-1})$, then for each $a_n \in A$ there exists some $b_n \in B$ such that $\mathcal{A}, _0, ..., _n \equiv_\exists (\mathcal{B}, b_0, ..., b_n)$.

Теорема 2.3. Let T $\forall\exists$ theory be complete for existential sentences, and let $\mathcal{A}$ be a countable model of T. Then (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3), where:

1. $\mathcal{A}$ - (Σ, Σ) is atomic;
2. $\mathcal{A}$ - Σ^*-nice;
3. $\mathcal{A}$ is existentially closed and Σ-nice.

Theorem 2.4. Let T - is a theory complete for existential sentences. Then any two countable (Σ, Σ)-atomic models of the theory T are isomorphic.

Now we consider the admissible enrichment. An enrichment is called admissible if it preserves a definability of the type in any existentially closed extension. The following $\{P\} \cup \{c\}$ - enrichment is admissible.

Let T be an arbitrary Jonsson theory in the first-order language of the signature σ. Let $\mathcal{C}$ be a semantic model of the theory T. Let $A \subset C$ be a $\nabla - cl$- a subset in T theory, where $\nabla = \forall\exists$, $cl = acl$ and at the same time $acl = dcl$. Let $\sigma_\Gamma(A) = \sigma \cup \{c_a | a \in A\} \cup \Gamma$, $\Gamma = \{P\} \cup \{c\}$. Let $T_A^C = T \cup Th_{\forall\exists}(C, a)_a \in A \cup \{P(c_a) | a \in \in A\} \cup \{P(c)\} \cup \{''P \subseteq''\}$, where $\{''P \subseteq''\}$ is an infinite number of sentences expressing the fact that the interpretation of the P symbol is existentially closed submodel in the language of the signature $\sigma_\Gamma(A)$ and this model is the definable closure of the set A. It is clear that the considered set of sentences is not necessarily a Jonsson theory, and this theory, generally speaking, is not complete. The reason for the no jonssoness is the lack of amalgam in some cases, i.e. there are counterexamples of enrichment by the predicate of the Jonsson theory, which do not allow amalgam. In the case of modularity, there is amalgam. Therefore, in the future, the considered theories are modular.Let T^* be the center of the Jonsson theory T_A^C and $T^* = Th(C')$, where C' is the semantic model of the theory T_A^C.

Below we give the results of [Theorem 2.5, 2.9] on countable and uncountable categoricity in the framework of the above enrichment of the signature of the Jonsson theory. Relatively prime Jonsson theory without enrichment, the first author had previously obtained similar results [9; 104].

Theorem 2.5. Let T be a modular convex Jonsson theory complete for $\forall\exists$-sentences. Then the following conditions are equivalent:

1. T^* ω-categorical;
2. T_A^C ω-categorical.

Proof. 1) $\Rightarrow$ 2) Let T^* ω — categorical. Then, by virtue of the theorem 2.2, T^* has a ω — categorical model companion $T^{*\prime}$. With the virtue of consistency the model T_A^C и T^*, T^* и $T^{*\prime}$, $T^{*\prime}$ the model of consistency with T_A^C, therefore, $T^{*\prime}$ is a model companion of T_A^C, in particular, $T^{*\prime}$ is model complete. By virtue of the model completeness of $T^{*\prime}$ any formula in the language $T^{*\prime}$ is equivalent to some existential formula. Then, by the Robinson theorem on the uniqueness of a model companion and by the Jonsson theory 1.1 criterion of perfectness, it follows that $T^* = T^{*\prime}$. Since $T^{*\prime}$ ω-categorical, its only countable model is countably saturated and belongs to $ModT$, since $ModT^* \subseteq ModT_A^C$. By virtue of 1.1 $ModT^{*\prime} = E_T$ and in E_T there is one up to isomorphism a countable model $\mathcal{A}$, where (L, L) is atomic (in the sense of 2.2), where L is the whole language. With the virtue of convexity of T hence $\mathcal{A}$ is (Σ, Σ)-the atomic model T^*, because of its model completeness $(T^* = T^{*\prime}$. Due to $\forall\exists$-completeness of the theory T_A^C, for any n $E_n(T_A^C) = E_n(T^*)$, where $E_n(T_A^C)$ - is a lattice of existential formulas of the theory T_A^C of n free variables. Then $\mathcal{A}$ - (Σ, Σ)-atomic model of the theory T_A^C. Then, by Theorem 2 the mode $\mathcal{A}$ является Σ^*- a nice-model. Let $\mathcal{B} \in ModT_A^C$, $|\mathcal{B}| = \omega$. We show that $\mathcal{B}$ is a (Σ, Σ)-atomic model of the theory T_A^C. We prove by induction. By virtue of $\forall\exists$-completeness of the theory T_A^C we have the theory T_A^C $\exists$-complete, and therefore $\mathcal{A} \equiv_\exists \mathcal{B}$ (induction basis). Suppose $(\mathcal{A}, a_1, ..., a_{n-1})_{a_1,...,a_{n-1} \in A} \equiv_\exists (\mathcal{B}, f(a_1), ..., f(a_{n-1}))_{a_1,...,a_{n-1} \in A}$, where f - is an isomorphic embedding from $\mathcal{A}$ to model $\mathcal{B}$. This isomorphic embedding exists because $\mathcal{A}$, being (Σ, Σ)-atomic, is algebraically prime, that is, is isomorphically embedded in any model of the theory T_A^C. This fact follows from [9]. And since $\mathcal{A}$ is Σ^*-a nice model of the theory of T_A^C, for any $a_n \in A$ there is such $b \in B$, such that $(\mathcal{A}, a_1, ..., a_{n-1}, a_n)_{a_1,...,a_n \in A} \equiv_\exists (\mathcal{B}, f(a_1), ..., f(a_{n-1}), b)_{a_1,...,a_n \in A}$. Let $f(a_n) = b$. Hence $\mathcal{A} \preceq_\exists \mathcal{B}$, т.е. $\mathcal{B}$ is an elementary extension with respect to existential formulas. Therefore, $\mathcal{B}$ - (Σ, Σ) is the atomic model T_A^C. Otherwise, since $\mathcal{A}$ belongs to E_T, $\mathcal{A}$ will not be (Σ, Σ)-the atomic model of T_A^C. Then, by Theorem 2.4 we have $\mathcal{B} \cong \mathcal{A}$. Since the model is $\mathcal{B}$ arbitrary, the theory T_A^C ω-categorical.

2) $\Rightarrow$ 1) Let a theory T_A^C ω-categorical. Suppose that the theory T^* not ω-categorical, then there exist non-isomorphic countable models $\mathcal{A}$ and $\mathcal{B}$ of the theory T^*. But, since $T_A^C \subseteq T^*$, then $ModT^* \subseteq ModT_A^C$. Therefore, $\mathcal{A}$ и $\mathcal{B}$ belong to $ModT_A^C$. We obtain a contradiction with the ω-categorical of T_A^C.

We define the concepts and related results necessary for the proof of Theorem 2.9 [9].

Definition 2.13. The formula $\varphi(\overline{x})$ is called a Δ-formula with respect to the theory T, if there are existential formulas $\psi_1(\overline{x})$ and $\psi_2(\overline{x})$ such that $T \models (\varphi \leftrightarrow \psi_1)$ and $T \models (\neg\varphi \leftrightarrow \psi_1)$

Definition 2.14. We will say that the theory T admits R_1, if for any existential formula $\varphi(\overline{x})$ consistency with T there is a formula $\psi(\overline{x}) \in \Delta$ consistency with T such that $T \models \psi \to \varphi$.

Definition 2.15. A countable model of the theory T is called a countably algebraic universal model, if all countable models of a given theory is isomorphically embedded into it.

Theorem 2.6. Let T be a universal theory, complete for existential sentences, having a countably algebraically universal model. Then T has an algebraically prime model, which (Σ, Δ) is atomic.

Theorem 2.7. Let T $\forall\exists$ theory be complete for existential sentences, admitting R_1. Then the following conditions are equivalent:

(1) T has an algebraically prime model;

(2) T has (Σ, Δ) is an atomic model;

(3) T has (Δ, Σ) is an atomic model;

(4) T has Δ-nice algebraically prime model;

(5) T has a unique algebraically prime model.

Definition 2.16. Let $\mathcal{A}, \mathcal{B} \in E_T$ and $\mathcal{A} \subsetneqq \mathcal{B}$. Then a $\mathcal{B}$ is called an algebraically prime model extension of $\mathcal{A}$ в E_T, if for any model $\mathcal{C} \in E_T$ from the fact that $\mathcal{A}$ isomorphically embedded in $\mathcal{C}$ in its own way, it follows that $\mathcal{B}$ is isomorphically embedded in $\mathcal{C}$.

Definition 2.17. [10]. A model $\mathcal{A}$ is called a prime own elementary extension $\mathcal{B}$, if $\mathcal{A} \supsetneqq \mathcal{B}$ and for any model $\mathcal{C}$ such that $\mathcal{C} \supsetneqq \mathcal{B}$ $\mathcal{A}$ is elementary embedded in $\mathcal{C}$.

Theorem 2.8. [11]. A complete theory T ω_1-categorical if and only if any of its countable models has a prime own elementary extension.

The following theorem describes the properties of T_A^C in the above enrichment under the following conditions.

Theorem 2.9. Let T be a modular convex Jonsson theory complete for $\exists$-sentences, for which R_1 is executed. Then the following conditions are equivalent:

1) theory of T^* ω_1-categorical;

2) any countable model in $E_{T_A^C}$ has an algebraically prime model extension in $E_{T_A^C}$.

Proof. 1) $\Rightarrow$ 2) If the theory is T^* ω_1-categorical, then it is perfect by the Morley theorem on uncountable categorical. Then, by virtue of the criterion of perfectness of the 1.1 Jonsson theory, we have that the theory T^* is model complete and $ModT^* = E_{T_A^C}$. If the theory T^* s model-complete, then any isomorphic embedding is elementary. Since T^* is a complete theory, then applying the theorem 2.8 to it, we obtain the required one.

2) $\Rightarrow$ 1) Applying the 1.1 lemma to the $\mathcal{C}$ semantic model of the T_A^C theory (it exists because T_A^C - Jonsson theory), we obtain that the model $\mathcal{C}$ is ω-universal. Its power, generally speaking, is greater than the counting one. Therefore, we consider its countable elementary submodel $\mathcal{D}$. Since T is a convex and the model $\mathcal{C}$ is existentially closed, its elementary submodel $\mathcal{D}$ is also existentially closed. Hence we have that it is countably algebraically universal. It now remains to apply Theorem 2.6, according to which the theory T_A^C has an algebraically prime model $\mathcal{A}_0$. We define by induction $\mathcal{A}_{\delta+1}$, which is an algebraically prime model extension of the model $\mathcal{A}_\delta$ and $\mathcal{A}_\lambda = \bigcup\{\mathcal{A}_\delta | \delta < \lambda\}$. Then let $\mathcal{A} = \bigcup\{\mathcal{A}_\delta | \delta < \omega_1\}$. Suppose that $\mathcal{B} \models T_A^C$ and $|\mathcal{B}| = \omega_1$. To show that $\mathcal{B} \approx \mathcal{A}$, decompose $\mathcal{B}$ into a chain $\{\mathcal{B}_\delta | \delta < \omega_1\}$ countable models. By virtue of the jonssoness theory of T_A^C this is possible. Define the function $g : \omega_1 \to \omega_1$ and the chain $\{f_\delta : \mathcal{A}_{g\delta} \to \mathcal{B} | 0 \leq \delta < \omega_1\}$ isomorphisms by induction on δ:

1. $g_0 = 0$ and $f_0 : \mathcal{A}_0 \to \mathcal{B}_0$.
2. $g_\lambda = \bigcup\{g_\delta | \delta < \lambda\}$ and $f_\lambda = \bigcup\{f_\delta | \delta < \lambda\}$.
3. $f_{\delta+1}$ equals the union of the chain $\{f_\delta^\gamma | \gamma \leq \rho\}$, which is defined by induction on γ.
4. $f_{\delta+1}^0 = f_\delta$, $f_{\delta+1}^\lambda = f_\delta = \bigcup\{f_{\delta+1}^\lambda | \delta < \lambda\}$.

5. Suppose that $f_0^\gamma : \mathcal{A}_{g\delta+\gamma} \to \mathcal{B}_{\delta+1}$. If $f_{\delta+1}^\gamma$ is a mapping on, then $\rho = \gamma$. Otherwise, by the algebraic simplicity of $\mathcal{A}_{g\delta+\gamma+1}$ you can continue $f_{\delta+1}^\gamma$ to $f_{\delta+1}^{\gamma+1} : \mathcal{A}_{g\delta+\gamma+1} \to \mathcal{B}_{\delta+1}$.

6. $g(\delta+1) = g\delta + \rho$. Clearly, $f = \bigcup\{f_\delta | \delta < \omega_1\}$ maps $\mathcal{A}$ isomorphically to $\mathcal{B}$. Now it remains to apply the theorem.

7. By virtue of the convexity of the theory T and since $\mathcal{B}$ is an arbitrary model of the theory T_A^C, and $\mathcal{A}$ is the only algebraic prime and existentially closed model By virtue of the condition and construction, then it follows that $E_{T_A^C}$ in uncountable cardinality has a single model, which means the semantic model of the theory T_A^C is saturated, i.e. the Jonsson theory of T_A^C is perfect. It follows that $ModT^* = E_{T_A^C}$. Therefore, T^*-ω_1 — categorical.

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Байытуға рұқсаттылығы бар фрагменттердің компаньондары

Мақалада рұқсат етілген байытулардағы арнайы ішкі жиындардың фрагменттерінің модельді-теориялық қасиеттері қарастырылды. Рұқсат етілген байытулар йонсон теориясының негізгі синтаксистік қасиетін сақтайтын сигнатурадағы байыту ретінде түсіндірілді. Йонсондар теориясындағы компаньондар қасиетін зерттеу индуктивті теориялардың классикалық мәселесін зерттеумен байланысты, яғни, өз уақытында модельдер теориясының негізін қалаушылардың бірі — А. Робинсон анықтаған. Авторлар қарастырылған йонсон теориясының анықталған жиындардағы компаньон фрагменттерінің қасиеті қатаң болады деген қорытынды жасады.

Кілт сөздер: йонсондық теория, семантикалық модель, экзистенциалды жай, предгеометрия, модулярлы предгеометрия, модельді компаньон.

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Компаньоны фрагментов в допустимых обогащениях

В статье рассмотрены теоретико-модельные свойства компаньонов фрагментов специальных подмножеств в допустимых обогащениях. Под допустимыми обогащениями понимаются обогащения сигнатуры, которые сохраняют основные синтаксические свойства рассматриваемой йонсоновской теории. Изучение свойств компаньонов йонсоновской теории относится к классической проблематике изучения индуктивных теорий, которую определил в свое время один из основателей теории моделей А. Робинсон. Авторы статьи пришли к выводу, что основные свойства компаньонных фрагментов определяемых подмножеств семантической модели йонсоновской теории категоричны.

Ключевые слова: йонсоновская теория, семантическая модель, экзистенциально простой, предгеометрия, модулярная предгеометрия, модельный компаньон.

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