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Boundary value problems for essentially-loaded parabolic equation

In this paper we investigate the first boundary value problem for essentially loaded equation of heat conduction, i.e. when laden terms are derivatives for any finite order. It is shown that if the point of load is fixed, this problem is uniquely solvable. The stated boundary problem is reduced to the Volterra integral equation of the second kind. Estimates of the kernel of the integral equation are made, which indicate a weak singularity of the kernel. It is shown that if the point of load is fixed, then the stated boundary problem is uniquely solvable.

Keywords: heat equation, boundary value problems, loaded equation, kernel, convolution theorem, eigenfunction.

Introduction

In work [1] the boundary value problems for the loaded parabolic equations are considered, and the loaded terms contain values of derivatives for only fixed points of their domain of definition. This paper argues that the corresponding boundary value problems are absolutely and uniquely solvable in the class of continuous functions, if the orders of the derivatives loaded terms are $\alpha < \frac{1}{2}$.

In works [2–5] it is shown that if the order of the derivative in the loaded term equals to the order of the differential part of the equation and the point of load moves at a constant or variable velocity, the corresponding boundary value problems are spectrally loaded,i.e. not uniquely solvable.

The purpose of this work is to show the unique solvability of the first boundary value problem for a loaded equation of heat conduction in the case, where the loaded terms are derivatives for any finite order and the point of load is fixed.

1 Statement of the problem

In the domain $Q = \{(x,t); 0 < x < l, t > 0\}$ we consider a loaded equation of heat conduction [2]:

$$\frac{\partial u}{\partial t} - a^2 \frac{\partial^2 u}{\partial x^2} + \lambda \frac{\partial^k u(x,t)}{\partial x^k}\bigg|_{x=\bar{x}} = f_0(x,t), \tag{1}$$

where $k > 0$ is an any integer number and $\bar{x}$ is fixed point, $0 < \bar{x} < l$.

Problem. Find in the domain Q a regular solution to equation (1) from the class $C(Q)$, satisfying the conditions:

$$u(x,0) = 0; \;\; u(0,t) = u(l,t) = 0. \tag{2}$$

2 Case $k = 1$

Let us consider in detail the case $k = 1$. We invert the differential operator of problem (4)–(5), considering temporarily that a loaded term is known, and we obtain:

$$u(x,t) = -\lambda \int\limits_0^t \frac{\partial u(\xi,\tau)}{\partial \tau}\bigg|_{\tau=\bar{x}} \int\limits_0^l G(x,\xi;t-\tau)d\xi d\tau + \tag{3}$$

$$+\int\limits_0^t\int\limits_0^l f_0(\xi,\tau)G(x,\xi;t-\tau)d\xi d\tau,$$

where the Green's function $G(x,\xi;t)$ has the form [6]:

$$G(x,\xi;t)=\frac{1}{2a\sqrt{\pi t}}\sum_{n=-\infty}^{\infty}\left[\exp\left\{-\frac{(x-\xi+2nl)^2}{4a^2t}\right\}-\right. \tag{4}$$

$$\left.-\exp\left\{-\frac{x+\xi+2nl}{4a^2t}\right\}\right].$$

We calculate the integral:

$$k(x,t-\tau)=\sum_{n=-\infty}^{\infty}\int\limits_0^l\frac{1}{2a\sqrt{\pi(t-\tau)}}\left[\exp\left\{-\frac{(x-\xi+2nl)^2}{4a^2(t-\tau)}\right\}-\right.$$

$$\left.-\exp\left\{-\frac{(x+\xi+2nl)^2}{4a^2(t-\tau)}\right\}\right]d\xi=$$

$$=\left\|z_1=\frac{x-\xi+2nl}{2a\sqrt{t-\tau}};\ z_2=\frac{x+\xi+2nl}{2a\sqrt{t-\tau}}\right\|=$$

$$=\frac{1}{\sqrt{\pi}}\sum_{n=-\infty}^{\infty}\left[\int\limits_{\frac{x+(2n-1)l}{2a\sqrt{t-\tau}}}^{\frac{x+2nl}{2a\sqrt{t-\tau}}}\exp\left\{-z_1^2\right\}dz_1-\int\limits_{\frac{x+2nl}{2a\sqrt{t-\tau}}}^{\frac{x+(2n+1)l}{2a\sqrt{t-\tau}}}\exp\left\{-z_2^2\right\}dz_2\right]=$$

$$=\sum_{n=-\infty}^{\infty}\left[\operatorname{erfc}\left(\frac{x+2nl}{2a\sqrt{t-\tau}}\right)-\frac{1}{2}\operatorname{erfc}\left(\frac{x+(2n-1)l}{2a\sqrt{t-\tau}}\right)-\right.$$

$$\left.-\frac{1}{2}\operatorname{erfc}\left(\frac{x+(2n+1)l}{2a\sqrt{t-\tau}}\right)\right].$$

Equality (6) can be represented as:

$$u(x,t)=-\lambda\int_0^t k(x,t-\tau)\frac{\partial u(\xi,\tau)}{\partial\tau}\bigg|_{\xi=\bar{x}}d\tau+f(x,t), \tag{5}$$

where

$$f(x,t)=\int_0^t\int_0^l f_0(\xi,\tau)G(x,\xi;t-\tau)d\xi d\tau.$$

Differentiating both sides of (8) with respect to t, we assume $x=\bar{x}$, and introducing the notation

$$\varphi_1(t)=\frac{\partial u(x,t)}{\partial t}\bigg|_{x=\bar{x}},$$

we get an integral equation:

$$\varphi_1(t)+\lambda\int_0^t K_1(t,\tau,\bar{x})\varphi_1(\tau)d\tau=f_1(t), \tag{6}$$

where

$$K_1(t,\tau,\bar{x})=\frac{\partial k(t,\tau,x)}{\partial t}\bigg|_{x=\bar{x}},\quad f_1(t)=\frac{\partial f(x,t)}{\partial t}\bigg|_{x=\bar{x}}.$$

We find the explicit form of the kernel $K_1(t,\tau,\bar{x})$:

$$K_1(t,\tau,\bar{x}) =$$
$$= -\frac{2}{\sqrt{\pi}}\sum_{n=-\infty}^{\infty}\frac{1}{4a(t-\tau)^{3/2}}\left[(x+2nl)\exp\left\{-\frac{(x+2nl)^2}{4a^2(t-\tau)}\right\}-\right.$$
$$-\frac{x+(2n-1)l}{2}\exp\left\{-\frac{(x+(2n-1)l)^2}{4a^2(t-\tau)}\right\}-$$
$$\left.-\frac{x+(2n+1)l}{2}\exp\left\{-\frac{(x+(2n+1)l)^2}{4a^2(t-\tau)}\right\}\right]=$$
$$= K_1^{(0)}(t,\tau,\bar{x}) + K_1^{(+)}(t,\tau,\bar{x}) + K_1^{(-)}(t,\tau,\bar{x}),$$

where the designations are introduced

$$K_1^{(0)}(t,\tau,\bar{x}) = -\frac{1}{2a\sqrt{\pi}(t-\tau)^{3/2}}\left[\bar{x}\exp\left\{-\frac{\bar{x}^2}{4a^2(t-\tau)}\right\}+\right.$$
$$\left.+\frac{l-\bar{x}}{2}\exp\left\{-\frac{(l-\bar{x})^2}{4a^2(t-\tau)}\right\}-\frac{\bar{x}+l}{2}\exp\left\{-\frac{(l+\bar{x})^2}{4a^2(t-\tau)}\right\}\right];$$
$$K_1^{(+)}(t,\tau,\bar{x}) =$$
$$= -\frac{1}{2a\sqrt{\pi}(t-\tau)^{3/2}}\sum_{n=1}^{\infty}\left[(\bar{x}+2nl)\exp\left\{-\frac{(\bar{x}+2nl)^2}{4a^2(t-\tau)}\right\}-\right.$$
$$-\frac{\bar{x}+(2n-1)l}{2}\exp\left\{-\frac{[\bar{x}+(2n-1)l]^2}{4a^2(t-\tau)}\right\}-$$
$$\left.-\frac{\bar{x}+(2n+1)l}{2}\exp\left\{-\frac{[\bar{x}+(2n+1)l]^2}{4a^2(t-\tau)}\right\}\right];$$
$$K_1^{(-)}(t,\tau,\bar{x}) =$$
$$= \frac{1}{2a\sqrt{\pi}(t-\tau)^{3/2}}\sum_{m=1}^{\infty}\left[(2ml-\bar{x})\exp\left\{-\frac{2ml-(\bar{x})^2}{4a^2(t-\tau)}\right\}-\right.$$
$$-\frac{(2m+1)l-x}{2}\exp\left\{-\frac{((2m+1)l-\bar{x})^2}{4a^2(t-\tau)}\right\}-$$
$$\left.-\frac{(2m-1)l-\bar{x}}{2}\exp\left\{-\frac{[(2m-1)l-\bar{x}]^2}{4a^2(t-\tau)}\right\}\right].$$

We estimate the kernel $K_1(t,\tau,\bar{x})$. For this we will estimate $K_1^{(0)}$, $K_1^{(+)}$, $K_1^{(-)}$ separately, in this case we use the inequality $z\cdot\exp\{-z\}\le\exp\{-1\}$, $z>0$:

$$|K_1^{(0)}(t,\tau,\bar{x})| \le \frac{a}{e\sqrt{\pi}}\left\{\frac{2}{\bar{x}}+\frac{1}{l-\bar{x}}+\frac{1}{l+\bar{x}}\right\}\frac{1}{\sqrt{t-\tau}} =$$
$$= \frac{2a}{e\sqrt{\pi}}\left\{\frac{1}{\bar{x}}+\frac{l}{l^2-\bar{x}^2}\right\}\frac{1}{\sqrt{t-\tau}}.$$

To estimate $|K_1^{(+)}(t,\tau,\bar{x})|$ and $|K_1^{(-)}(t,\tau,\bar{x})|$ the signs of the sum are replaced by the integrals in which we make replacements accordingly:

$$|K_1(t,\tau,\bar{x}) - K_1^{(0)}(t,\tau,\bar{x})| \le$$
$$\le \int_1^{\infty}|K_1^{(+)}(t,\tau,\bar{x})|dn + \int_1^{\infty}|K_1^{(-)}(t,\tau,\bar{x})|dn \le$$

$$\leq \left\| \begin{array}{l} \xi_1(n) = \dfrac{(\bar{x}+2nl)^2}{4a^2(t-\tau)}, \ \ \xi_2(n) = \dfrac{[\bar{x}+(2n-1)l]^2}{4a^2(t-\tau)}, \\ \xi_3(n) = \dfrac{[\bar{x}+(2n+1)l]^2}{4a^2(t-\tau)}, \ \ \eta_1(n) = \dfrac{(2ml-\bar{x})^2}{4a^2(t-\tau)}, \\ \eta_2(n) = \dfrac{[(2m+1)l-\bar{x}]^2}{4a^2(t-\tau)}, \ \ \eta_3(n) = \dfrac{[(2m-1)l-\bar{x}]^2}{4a^2(t-\tau)} \end{array} \right\| \leq$$

$$\leq \frac{1}{2l\sqrt{\pi(t-\tau)}} \left[\int_{\xi_1(1)}^{\infty} \exp\{-\xi_1\}\, d\xi_1 + \right.$$

$$+\frac{1}{2}\int_{\xi_2(1)}^{\infty} \exp\{-\xi_2\}\, d\xi_2 + \frac{1}{2}\int_{\xi_3(1)}^{\infty} \exp\{-\xi_3\}\, d\xi_3 +$$

$$+\int_{\eta_1(1)}^{\infty} \exp\{-\eta_1\}\, d\eta_1 + \frac{1}{2}\int_{\eta_2(1)}^{\infty} \exp\{-\eta_2\}\, d\eta_2 +$$

$$\left. +\frac{1}{2}\int_{\eta_3(1)}^{\infty} \exp\{-\eta_3\}\, d\eta_3 \right] = \frac{a}{4l\sqrt{\pi(t-\tau)}}\, [2\exp\{-\xi_1(1)\} +$$

$$+\exp\{-\xi_2(1)\} + \exp\{-\xi_3(1)\} + 2\exp\{-\eta_1(1)\} + \exp\{-\eta_2(1)\} +$$

$$+\exp\{-\eta_3(1)\}] \leq \frac{2a}{l\sqrt{\pi}} \frac{1}{\sqrt{t-\tau}} \exp\left\{ -\frac{(l-\bar{x})^2}{4a^2(t-\tau)} \right\}.$$

Thus, the kernel of integral equation (9) has a weak singularity, i.e. integral equation (9) is uniquely solvable. From relation (3) it follows that boundary value problem (1)–(2) has a unique solution.

3 Case $k = 2$

Now let $k = 2$. Equality (6) in this case will have the form:

$$u(x,t) = -\lambda \int_0^t \left. \frac{\partial^2 u(\xi,\tau)}{\partial \tau^2} \right|_{x=\bar{x}} \int_0^l G(x,\xi;t-\tau)d\xi d\tau + \qquad (7)$$

$$+\int_0^t \int_0^l f_0(\xi,\tau) G(x,\xi;t-\tau)d\xi d\tau.$$

If we differentiate both sides of this equality with respect to t twice and we introduce the notation

$$\varphi_2(t) = \left. \frac{\partial^2 u(x,t)}{\partial t^2} \right|_{x=\bar{x}},$$

we will have the following Volterra integral equation:

$$\varphi_2(t) + \lambda \int_0^t K_2(t,\tau,\bar{x})\varphi_2(\tau)d\tau = f_2(t); \qquad (8)$$

where

$$K_2(t,\tau,\bar{x}) = \left. \frac{\partial^2 k(t,\tau,x)}{\partial t^2} \right|_{x=\bar{x}}, \ \ f_2(t) = \left. \frac{\partial^2 f(x,t)}{\partial t^2} \right|_{x=\bar{x}};$$

$$f(x,t) = \int_0^t \int_0^l f_0(\xi,\tau) G(x,\xi;t-\tau)d\xi d\tau.$$

Similarly to the case $k = 1$, we estimate the kernel of integral equation (8), for that we find the explicit form of the kernel $K_2(t,\tau,\bar{x})$:

$$K_2(t,\tau,\bar{x}) = \left. \frac{\partial^2 k(t,\tau,x)}{\partial t^2} \right|_{x=\bar{x}} = \frac{1}{8a\sqrt{\pi}(t-\tau)^{1/2}} \sum_{n=-\infty}^{\infty} \left[6\frac{\bar{x}+2nl}{(t-\tau)^2} \times \right.$$

$$\times \exp\left\{-\frac{(\bar{x}+2nl)^2}{4a^2(t-\tau)}\right\} - \frac{(\bar{x}+2nl)^3}{2a^2(t-\tau)^3}\exp\left\{-\frac{(\bar{x}+2nl)^2}{4a^2(t-\tau)}\right\} -$$

$$-\frac{3(\bar{x}+(2n-1)l)}{8a\sqrt{\pi}(t-\tau)^{5/2}}\exp\left\{-\frac{(\bar{x}+(2n-1)l)^2}{4a^2(t-\tau)}\right\}+$$

$$+\frac{(\bar{x}+(2n-1)l)^3}{8a^3\sqrt{\pi}(t-\tau)^{7/2}}\exp\left\{-\frac{(\bar{x}+(2n-1)l)^2}{4a^2(t-\tau)}\right\}-$$

$$-\frac{3(\bar{x}+(2n+1)l)}{8a\sqrt{\pi}(t-\tau)^{5/2}}\exp\left\{-\frac{(\bar{x}+(2n+1)l)^2}{4a^2(t-\tau)}\right\}+$$

$$\left.+\frac{(\bar{x}+(2n+1)l)^3}{8a^3\sqrt{\pi}(t-\tau)^{7/2}}\exp\left\{-\frac{(\bar{x}+(2n+1)l)^2}{4a^2(t-\tau)}\right\}\right] =$$

$$= K_2^{(0)}(t,\tau,\bar{x}) + K_2^{(+)}(t,\tau,\bar{x}) + K_2^{(-)}(t,\tau,\bar{x}).$$

At first we estimate the terms of this sum when $n = 0$:

$$|K_2^{(0)}(t,\tau,\bar{x})| = \frac{1}{\sqrt{t-\tau}}\left[\frac{3\bar{x}}{4a\sqrt{\pi}(t-\tau)^2}\exp\left\{-\frac{\bar{x}^2}{4a^2(t-\tau)}\right\}+\right.$$

$$+\frac{\bar{x}^3}{8a^3\sqrt{\pi}(t-\tau)^3}\exp\left\{-\frac{\bar{x}^2}{4a^2(t-\tau)}\right\}+$$

$$+\frac{3(l-\bar{x})}{8a\sqrt{\pi}(t-\tau)^2}\exp\left\{-\frac{(l-\bar{x})^2}{4a^2(t-\tau)}\right\}+$$

$$+\frac{(l-\bar{x})^3}{8a^3\sqrt{\pi}(t-\tau)^3}\exp\left\{-\frac{(l-\bar{x})^2}{4a^2(t-\tau)}\right\}+$$

$$+\frac{3(l+\bar{x})}{8a\sqrt{\pi}(t-\tau)^2}\exp\left\{-\frac{(l+\bar{x})^2}{4a^2(t-\tau)}\right\}+$$

$$\left.+\frac{(l+\bar{x})^3}{8a^3\sqrt{\pi}(t-\tau)^3}\exp\left\{-\frac{(l+\bar{x})^2}{4a^2(t-\tau)}\right\}\right].$$

Further we use estimates of the following form:

$$J_1 = \frac{b}{(t-\tau)^2}\exp\left\{-\frac{b^2}{4a^2(t-\tau)}\right\} \le \frac{64\,a^4}{b^3}\exp\{-2\}$$

and

$$J_2 = \frac{b^3}{(t-\tau)^3}\exp\left\{-\frac{b^2}{4a^2(t-\tau)}\right\} \le \frac{1728\,a^6}{b^3}\exp\{-3\}, (b>0),$$

here we take into account the validity of inequality: $(z > 0)$

$$z^n e^{-z} \le n^n e^{-n},\ n = 0, 1, 2, ...$$

Thus we finally have:

$$|K_2^{(0)}(t,\tau,\bar{x})| \le C\cdot\frac{1}{\sqrt{\pi(t-\tau)}}\cdot\frac{a^3}{d^3(\bar{x})},$$

where $d(\bar{x}) = \min\{\bar{x}; l-\bar{x}\}$.

Similarly to the case $k = 1$, we estimate $K_2^{(+)}(t,\tau,\bar{x})$ and $K_2^{(-)}(t,\tau,\bar{x})$. It is enough to consider the following integral and its estimate:

$$\frac{1}{\sqrt{\pi}(t-\tau)^{3/2}}\int\limits_1^{\infty}\left[b_1\frac{\bar{x}+2nl}{(t-\tau)} + b_2\frac{(\bar{x}+2nl)^3}{(t-\tau)^2}\right]\exp\left\{-\frac{(\bar{x}+2nl)^2}{4a^2(t-\tau)}\right\}dn =$$

$$= \left\|\frac{(\bar{x}+2nl)^2}{4a^2(t-\tau)} = \xi; \quad \frac{4l(\bar{x}+2nl)}{4a^2(t-\tau)}dn = d\xi\right\| =$$

$$= \frac{1}{\sqrt{\pi}(t-\tau)^{3/2}} \int_{\frac{(\bar{x}+2l)^2}{4a^2(t-\tau)}}^{\infty} \left[\frac{a^2}{l} \cdot b_1 + \frac{4a^2 \cdot a^2}{l} \cdot \xi\right] \exp\{-\xi\}\, d\xi =$$

$$= \frac{1}{l\sqrt{\pi}(t-\tau)^{3/2}} \left[\frac{a^2 b_1}{l} \exp\{-\xi\} - \frac{4a^4}{l}(\xi+1)\exp\{-\xi\}\right]\Bigg|_{\frac{(\bar{x}+2l)^2}{4a^2(t-\tau)}}^{\infty} =$$

$$= \frac{1}{l\sqrt{\pi}(t-\tau)^{3/2}} \exp\left\{-\frac{(\bar{x}+2l)^2}{4a^2(t-\tau)}\right\} \times$$

$$\times \left[(a^2 b_1 + 4a^4) + \frac{(\bar{x}+2l)^2}{4a^2(t-\tau)} \cdot 4a^4\right] =$$

$$= \frac{a^2 b_1 + 4a^4}{l\sqrt{\pi(t-\tau)}} \cdot \frac{1}{(t-\tau)} \exp\left\{-\frac{(\bar{x}+2l)^2}{4a^2(t-\tau)}\right\} +$$

$$+ \frac{(\bar{x}+2l)^2 a^2}{l\sqrt{\pi(t-\tau)}} \cdot \frac{1}{(t-\tau)^2} \exp\left\{-\frac{(\bar{x}+2l)^2}{4a^2(t-\tau)}\right\} \le$$

$$\le \frac{a^2 b_1 + 4a^4}{\sqrt{\pi(t-\tau)}} \cdot \frac{4a^2}{(\bar{x}+2l)^2} e^{-1} + \frac{(4a^2)^2 \cdot (\bar{x}+2l)^2 \cdot a^2 \cdot 4e^{-2}}{(\bar{x}+2l)^4 \cdot l\sqrt{\pi(t-\tau)}} \le$$

$$\le \frac{C}{(\bar{x}+2l)^2} \cdot \frac{1}{\sqrt{\pi(t-\tau)}}.$$

Using this estimate for $K^{(+)}$ and $K^{(-)}$, we get:

$$|K_2^{(+)}(t,\tau,\bar{x})| + |K_2^{(-)}(t,\tau,\bar{x})| \le \frac{C}{(l-\bar{x})^2} \cdot \frac{1}{\sqrt{\pi(t-\tau)}}.$$

Thus, for kernel of integral equation (8) the following estimate is valid:

$$|K_2(t,\tau,\bar{x})| \le C \frac{1}{d_2(\bar{x})} \frac{1}{\sqrt{t-\tau}},$$

where $d_2(\bar{x}) = \min\{\bar{x}, l-\bar{x}, (l-\bar{x})^2\}$.

From this estimate it follows that the Volterra integral equation (8) is uniquely solvable, and from relation (7) we obtain the unique solvability of problem (1)–(2) for the case of $k = 2$.

4 Case $\forall k \in \mathbb{N}$. *Main result*

Carrying out similar arguments for any finite value k, we can show that the kernel of the corresponding integral equation has estimate of the form:

$$|K_k(t,\tau,\bar{x})| \le \frac{C}{d_k(\bar{x})} \frac{1}{\sqrt{t-\tau}},$$

where $d_k(\bar{x}) = \min\{\bar{x}, l-\bar{x}, (l-\bar{x})^k\}$, i.e. $K_k(t,\tau,\bar{x})$ has also a weak singularity.

Thus it is proved the validity of the following theorem.

Theorem. The problem (4)–(5) is absolutely and uniquely solvable for every $f(x,t)$ such that

$$\left\{\frac{\partial^k}{\partial t^k} \int_0^t \int_0^l G(x,\xi;t-\tau) f(\xi,\tau) d\xi d\tau\right\}_{x=\bar{x}} \in C(0,\infty).$$

Remark. Articles [3–5] are closer to the subject of this work.

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Айтарлықтай-жүктемелі параболалық теңдеулер үшін шекаралық есептер

Көрсетілген жұмыста айтарлықтай-жүктемелі параболалық теңдеулер үшін бірінші шекаралық есептер зерттелді және олардың жүктемелі мүшелері кез келген ақырғы реттік туындысы болып табылады. Қойылған шекаралық есеп Вольтерра теңдеуінің екінші түріне келеді. Интегралдық теңдеуінің ядросынының ерекшелігі әлсіз екенін көрсететін бағалау жасалды. Егер жүктемелі нүкте белгіленген болса, онда қойылған есеп бірмәнді шешіледі.

Кілт сөздер: жылуөткізгіштік теңдеу, шекаралық есептер, жүктемелі теңдеу, ядро, орама теоремасы, меншікті функция.

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Краевые задачи для существенно-нагруженного параболического уравнения

В статье исследована в полуполосе первая краевая задача для существенно-нагруженного уравнения теплопроводности, причем нагруженные члены являются производными любого конечного порядка. Поставленная граничная задача сведена к интегральному уравнению Вольтерра второго рода. Произведены оценки ядра интегрального уравнения, которые указывают на слабую особенность ядра. Показано, что если точка нагрузки фиксирована, то поставленная краевая задача однозначно разрешима.

Ключевые слова: уравнение теплопроводности, краевые задачи, нагруженное уравнение, ядро, теорема свертки, собственная функция.

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