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## Mathematical layout model of coupling with tangentially located ropes

The article deals with the results of mathematical modeling of layout coupling with tangentially located ropes. Arrangement considered limiting geometric nature that must be considered when designing joints. These conditions take into account the possibility of tightening fasteners, bushings opportunity neighborhood inner coupling halves, the possibility of relative rotation of the coupling halves, the lack of interference of the outer and inner sleeves of the coupling halves, the absence of interference of adjacent ropes and sleeves inner halfcoupling. As a result of research obtained mathematical expressions to be used in the calculation of engineering couplings checking the basic conditions of their geometric existence. These have been tested according to the design of the coupling, and the calculation results are checked by comparing them with the results of construction and found a match. The results can be used in the design of the coupling with the end of direct installation of ropes tangential location.

Keywords: mathematical model, layout, coupling, rope, load, gap, torque.

#### Introduction

Most of the parts and components of machines are complex structures, and their successful design is possible only after a thorough study of the processes occurring in these units during their operation, and the development of models for justifying their parameters based on new knowledge. At the same time, for many assembly units, rational design is the result of a series of calculations performed by the method of successive approximation, an example of which are the layout calculations e.g. when «implementing» gears in predetermined dimensions, etc. [1]. The elastic couplings [2, 3], in particular those with rope elements, for which the design and verification calculation methods have not been developed are not an exception, therefore further development of such calculation methods and construction of appropriate mathematical models is an important task.

> Analysis of previous studies on this issue, highlighting the unsolved part of the problem

The authors of previous works [4, 5] proposed a new design of the coupling with an end-type installation of ropes of tangential location. The coupling (Fig. 1) consists of two half-couplings - the inner (1) and the outer one, which are connected by elastic elements (10), represented by ropes, each of being fixed by one end metricconverterProductID14 in (14) in the pin (8), installed in the outer half-coupling (11), and the other end metricconverterProductID15 in (15) in the pin (2) installed in the inner half coupling (1). The pins (8) and (2) are passed into the axial holes (7) of the sleeves (6) and the holes (9) of the flanges (5) of the half-couplings (11) and (1) and are tightened with the nuts (4) which are mounted on their threaded ends (3). The ropes (10) are passed into the transverse grooves (12) of the sleeves (6) and the grooves (13) of the pins (8) and (2). The pins (8) and (2) can be installed in their half-couplings (11) and (1) at different diameters which exclude interference of adjacent sleeves of the outer and inner half-couplings. This feature makes it possible for the coupling to perform safety functions, so that during overloading and pulling the ropes from the sleeves, the impact of the sleeves fixed on the half-couplings (11) and (1) is eliminated and the driving half-coupling can continue to rotate freely.



Figure 1. Scheme of the coupling with end-type installation of tangentially located straight ropes

As a result of theoretical studies, the basic geometric constraints are determined and five conditions for the geometric existence of couplings of the proposed design are formulated: the possibility of tightening fixture elements, the possibility of «proximity» of the sleeves of the inner half-coupling, the possibility of relative rotation of the half-couplings, the absence of interference between the sleeves of the outer and inner half-couplings, the absence of interference between the ropes and adjacent sleeves of the inner half-coupling. These geometric constraints also affect the loading of the coupling parts, but formulas for testing the last two conditions at the design stage and the mathematical model of the actual layout are not obtained, which complicates the process of designing couplings. Therefore, the purpose of this work is the theoretical justification of the features of geometric and power calculations, the design of couplings with end-type rope installation and obtaining a mathematical model that will simplify the work of the engineer during the layout of the couplings.

#### Statement of the main material

The design scheme of the coupling with end-type installation of tangentially located ropes is shown in Figure 2. In its half-couplings at different diameters, the outer  $D_1$  and the inner  $D_2$ , sleeves (1) and (2) are fixed, in which the ropes (3) are fixed, due to their tension the rotation from the driving half-coupling to the driven one is transmitted. The main initial data for checking the specified conditions for the existence of the coupling, in addition to the diameters of the sleeves location, are the diameters of the sleeves  $d_1$  and the ropes  $d_2$ , as well as the angle of the installation displacement of the half-couplings  $\xi$  - the angle between the radii on which the sleeves of the outer and inner half-couplings that hold one rope are fixed. This angle can be adjusted when mounting the coupling to the required limits. The value of the angle  $\xi$  also affects the force parameters of the coupling, therefore, due to the rational choice of the angle  $\xi$ , it is necessary to ensure minimum loading of the coupling parts and its geometric existence within the limits of the indicated constraints.



Figure 2. Design scheme of the coupling with end-type installation of tangentially located ropes

In order to obtain formulas that will test the two conditions mentioned above-the absence of interference between the sleeves of the outer and inner half-couplings, as well as the absence of interference between the ropes and adjacent sleeves of the inner half-coupling, it is necessary to consider a number of simple drawings shown in Figure 2. Thus, the first of above mentioned conditions is the condition that there is no interference between the sleeve of the outer half-coupling and the corresponding sleeve of the inner half-coupling, provided there is a gap  $k_3$  between them. The condition is tested to ensure the shock-free operation of the coupling and is described by the expression (1). Thus, the provision of this condition actually reduces itself to the calculation of the distance  $h_1$  between the axes A and C of the adjacent sleeves of the outer and inner half-couplings.

$$k_3 = h_1 - d_1, \quad k_3 \ge [\Delta_r],$$
 (1)

where  $[\Delta_r]$  is expected radial misalignment, at which the coupling will be operable.

The second of these conditions - the absence of interference between the ropes and adjacent sleeves of the inner half-coupling, is met when there is a gap  $k_4$  between them. The condition is tested to ensure the shock-free operation of the coupling and is described by the expression (2). Thus, the verification of the provision of this condition reduces itself to determining the distance  $h_2$  between the axis C of the sleeve of the inner half-coupling and the axis AB of the rope.

$$k_4 = h_2 - 0.5(d_1 + d_2), \quad k_4(2...4)mm.$$
 (2)

To achieve this goal, let us first consider a rectangular triangle OPB for which we can write as follows:

$$PB = OB \times \sin \xi = \frac{D_2}{2} \sin \xi; \tag{3}$$

$$OP = OB \times \cos \xi = \frac{D_2}{2} \cos \xi. \tag{4}$$

It follows from the rectangular triangle APB:

$$tg\alpha_2 = \frac{PB}{AP} = \frac{PB}{OA - OP} = \frac{0, 5D_2 \sin\xi}{0, 5D_1 - 0, 5D_2 \cos\xi} = \frac{D_1 \sin\xi}{D_1 - D_2 \cos\xi} = X;$$
(5)

$$\lambda = \frac{2\pi}{z} - \xi. \tag{6}$$

It follows from the rectangular triangle OSC:

$$CS = OC \times \sin \lambda = \frac{D_2}{2} \sin \lambda; \tag{7}$$

$$CS = OC \times \cos \lambda = \frac{D_2}{2} \cos \lambda.$$
(8)

From the rectangular triangle ASC we can derive as follows:

$$tg\alpha_3 = \frac{CS}{AS} = \frac{CS}{OA - OS} = \frac{0, 5D_2 \sin \lambda}{0, 5D_1 - 0, 5D_2 \cos \lambda} = \frac{D_2 \sin \lambda}{D_1 - D_2 \cos \lambda} = Y;$$
(9)

$$\cos \alpha_3 = \frac{AS}{CA} = \frac{OA - OS}{CA} = \frac{0, 5D_1 - 0, 5D_2 \cos \lambda}{CA}.$$
 (10)

We introduce the substitution (11)

$$\cos \alpha_3 = \frac{1}{\sqrt{1 + tg^2 \alpha_3}} = \frac{1}{\sqrt{1 + Y^2}},$$
(11)

then

$$h_1 = CA = \frac{AS}{\cos\alpha_3} = (0, 5D_1 - 0, 5D_2\cos\lambda) \times \sqrt{1 + tg^2\alpha_3} = 0, 5(D_1 - D_2\cos\lambda) \times \sqrt{1 + Y^2}.$$
 (12)

From the rectangular triangle *CDA*, we can derive as follows:

$$\alpha_4 = \alpha_2 + \alpha_3. \tag{13}$$

Серия «Математика». № 1(89)/2018

We introduce the substitutions  $(14) \ \mu \ (15)$ :

$$tg\alpha_4 = \frac{tg\alpha_2 + tg\alpha_3}{1 - tg\alpha_2 \times tg\alpha_3} = \frac{X + Y}{1 - X \times Y} = Z;$$
(14)

$$\sin \alpha_4 = \frac{tg\alpha_4}{\sqrt{1 + tg^2\alpha_4}} = \frac{Z}{\sqrt{1 + Z^2}}.$$
(15)

Then

$$h_2 = CD = CA\sin\alpha_4 = CA\frac{tg\alpha_4}{\sqrt{1 + tg^2\alpha_4}} = h_1\frac{Z}{\sqrt{1 + Z^2}},$$
(16)

or finally

$$h_2 = \frac{0, 5(D_1 - D_2 \cos \lambda) \times \sqrt{1 + Y^2} \times Z}{\sqrt{1 + Z^2}}.$$
(17)

The obtained formulas (3)-(15) made it possible to derive the dependences (12) and (17), which can be used in the design and layout of couplings in order to test the possibility of their geometric existence according to the conditions mentioned above (1) and (2).

To illustrate the possibilities of applying the obtained formulas, let us consider an example of the layout simulation for the following coupling data: the torque transmitted by the coupling is T = 500 Nm, the diameters of the sleeves in the half-coupling  $D_1 = 145 \text{ mm}$ ,  $D_2 = 110 \text{ mm}$ , number of ropes z = 8 pcs. In the design calculation, the angle of the installation displacement of the half couplings, which ensures the minimum tension of the ropes, was calculated from the previously [6] obtained formula (18).

$$\xi_F = \arccos \frac{D_1}{D_2} = \arccos \frac{110}{145} = 40,66^0.$$
 (18)

After calculating the tension of the ropes according to the formula (19), the ropes were selected with a dimmer  $d_2 = 3.8mm$  GOST 2688, with a breaking force of 8400 N, marking group 1170 MPa, the diameter of the sleeves is taken to be  $d_1 = 24mm$ .

$$F_H = \frac{4T\sqrt{0,25(D_1^2 + D_2^2) - 0,5D_1D_2\cos\xi}}{zD_1D_2\sin\xi}.$$
(19)

The possibility of the coupling layout with accepted and calculated parameters is illustrated by the graph (Fig. 3). With the use of the obtained formulas (1)–(19), the graph shows the influence of the angle of the mounting displacement of the half-couplings  $\xi$  for the gaps  $k_3$  and  $k_4$ , as well as the tension of the ropes  $F_H$ . Other conditions for the geometric existence of the coupling are met for any  $\xi$ , therefore they are not shown on the graph.



Figure 3. Graph of the mutual influence of design and force parameters of the coupling

The analysis of the graphs presented in Fig. 3 makes it possible to state that the interference between the adjacent sleeves, as well as that between the ropes and the sleeves according to the conditions (1) and (2) is absent at angles  $\xi = 32^{0}$ . The limiting layout parameter is the design gap  $k_{3}$ , which only at the angle value  $\xi < 32^{0}$ , has the value which is greater than zero (this position corresponds to the vertical line of the border of the layout limits on the graph).

Conclusions. To set in the model coupling the value of the mounting displacement angle  $\xi$ , calculated according to the formula (18), which is optimal from the point of view of ensuring minimum rope tension, is not possible due to interference between the adjacent sleeves of the outer and inner half-couplings. Therefore, for a successful layout, based on the graph data (Fig. 3), it is possible to reduce the angle of the mounting displacement of the half-couplings e.g. to a value of  $25^{0}$ . With this change, the load of the coupling parts will change insignificantly, since the force of the rope tension when the angle  $\xi$  changes from 40.66<sup>0</sup> to  $25^{0}$ , which is acceptable from the viewpoint of the layout, will increase by about 6%. Consequently, the research performed and the model obtained make it possible to simplify the coupling layout characterized by the compactness and the possibility of calculation automation.

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# Арқандары тангенциалды орналасқан муфта бөлшектерін құрастырудың математикалық моделі

Мақалада тангенс бойынша орналасқан беттік құрылымды канат пен муфта құрылымының математикалық модельдеу нәтижесі ұсынылған. Зерттеу нәтижесінде осы әсердің ықпалы ашылған және олардың геометриялық бар болуының негізгі шарттарын тексеру барысында муфталарды жобалап есептеп қолдану үшін математикалық өрнектер алынған. Сыртқы және ішкі жартылай муфталардың тығындарының интерференцияларының болмайтыны, сонымен қатар ішкі жартылай муфталардың аралық тығындылар мен канаттардың интерференцияларының болмайтын жағдайы қарастырылған. Муфтаны жобалаудан кейін тәуелділіктер алынған, сондай-ақ есептеу нәтижелері құру нәтижелерімен салыстырылып дұрыс есептелгеніне көз жеткізілген. Алынған нәтижелер тангенс бойынша орналасқан беттік құрылымды түзу канатты муфталарды жобалау үшін қолданымын таба алады.

*Кілт сөздер:* математикалық модель, құрастыру, сыртқы және ішкі жартылай муфталар, канат, жүк, саңылау, мезет.

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# Математическая модель компоновки муфты с канатами тангенциального расположения

В статье представлены результаты математического моделирования компоновки муфт с торцевой установкой канатов тангенциального расположения. В результате выполнения исследований раскрыто это влияние и получены математические выражения для использования при проектировочном расчете муфт и проверке основных условий их геометрического существования. Рассмотрены случаи отсутствия интерференции втулок внешней и внутренней полумуфт, а также отсутствия интерференции канатов и смежных втулок внутренней полумуфты. Полученные зависимости апробированы при проектировании муфты, а результаты расчета по ним сравнены с результатами построения и показали совпадение. Данные результаты могут быть использованы при проектировании муфт с торцевой установкой прямых канатов тангенциального расположения.

*Ключевые слова:* математическая модель, компоновка, внешняя и внутренняя полумуфты, канат, нагрузки, зазор, момент.

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