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About the new version of maximum principle of Navier-Stokes equations

The below shows the links of the extreme values of the velocity vector, the kinetic energy density and pressure of nonlinear Navier-Stokes equations. The latter shows the validity of the maximum principle for nonlinear Navier-Stokes equations that from a mathematical perspective is fundamentally-key.

Key words: nonlinear Navier-Stokes equations system, the principle of maximum for Navier-Stokes equations, uniqueness of weak generalized solutions of Navier-Stokes equations, existence of strong solutions of Navier-Stokes equations.

1. Introduction

Academician O.A. Ladyzhenskaya in [1] has formed one of the unsolved problems of the theory of the Navier-Stokes' equations (NSE) as following: *«Do Navier-Stokes» equations with initial and boundary conditions give deterministic description of the dynamics of an incompressible fluid, or do not give?*

In solving problem 1. The choice of phase space and the class of generalized solutions are necessary to provide to the researcher, but not prescribe him in advance an infinitely smooth-bone or somewhat smoothness of solutions. It is necessary to require the only one? For a dedicated class of generalized solutions there is a place for a uniqueness theorem. It is advisable to begin the research of any initial-boundary value problem (as well as Cauchy's problem) with finding of classes of uniqueness».

The object of this work is the research of the problem and, in the end to prove the uniqueness, and the existence of solutions of Navier-Stokes' equations, respectively, from the class of functions

$$C\left(0,T;C(\Omega)\cap W_2^1(\Omega)\right) \text{ and } C\left(0,T;C(\Omega)\cap W_{2,0}^{2,1}(\Omega)\right).$$

In a number of papers [2–4] et al. are shown the results of exploratory research in order to study the maximum principle for NSE. The relation shows extreme knowledge of speed vector, kinetic energy density (in particular, the localmaximum) and pressure field in some points. The latter shows equity of the maximum principle for nonlinear system NSE that from a mathematical point of view, is the key. On the basis of which on the selected area, the uniqueness of the weak and the existence of strong solutions of the problem for the NSE for the whole time t is proven. Here the results are refined and details of the proof of some significant allegations brought to the mathematical rigor.

We consider the initial-boundary value problem for the NSE [1] with respect to the velocity vector $t\in[0,T],\forall T<\infty$.

We consider the initial-boundary value problem for the NSE [1] with respect to the velocity vector $U=(U_1,U_2,U_3)$ and the pressure P in domain $Q=(0,T]\times\Omega$:

$$\frac{\partial U}{\partial t}-\mu\Delta U+(U,\nabla)U+\nabla P=f(t,x); \tag{1a}$$

$$divU=0; \tag{1b}$$

$$U(0,x)=\Phi(x); \tag{1c}$$

$$U(t,x)\big|_{\partial\Omega}=0, \tag{1d}$$

where $x\in\Omega\subset R_3$; Ω — convex domain, filled with homogeneous fluid; $\partial\Omega$ — boundary of the domain, $t\in[0,T]$, $T<\infty$; μ — dynamic viscosity; Δ, ∇ — Laplace and Hamilton operators respectively; f and Φ — vectors of functions of external forces and initial data respectively, that satisfy the following conditions.

Let $\overset{0}{\mathrm{J}}(\Omega)$ — the space of solenoidal vectors, and $G(\Omega)$ consists of $\nabla\eta$, where η is a single-valued function in Ω, locally square integrable and has first derivatives from $L_2(\Omega)$. It is known from [1] that the orthogonal resolution, $L_2(Q)=G(Q)\oplus \overset{0}{\mathrm{J}}(Q)$, where elements of $\overset{0}{\mathrm{J}}(Q)$ in $\forall t$ belong to $\overset{0}{\mathrm{J}}(\Omega)$; f and Φ — vectors of functions of external forces and initial data respectively, that satisfy the following conditions:

i) $f(t,x)\in C(\bar{Q})\cap \overset{0}{\mathrm{J}}(Q)$; **ii)** $\Phi(x)\in C(\bar{\Omega})\cap W^1_{2,0}(\Omega)\cap \overset{0}{\mathrm{J}}(\Omega)$.

On the relationship between the extreme values of the velocity, the kinetic energy density and pressure

Definition 1. *We'll say that the vector function* $U(t,x)$ at point M_1 in *domain* Q *has a local extremum when each component of the vector function* $\{U_\alpha\}$ *at the same point* M_1 *reaches a local extremum*.

Explanation to the definition. Let the vector function $U(t,x)$ has at local $M_1(t',x')$ extrema, then (as in the case of a scalar function) all the partial derivative of its first in order to point $M_1(t',x')$ must be vanished, ie.

$$\frac{\partial U}{\partial x_\beta}(M_1)=0,\ \beta=\overline{1,3}. \tag{2}$$

Equality (2) has the place if and only if all the partial derivative of the first order of components of the vector function at the point $M_1(t',x')$ are vanished, i.e.

$$\frac{\partial U_\alpha}{\partial x_\beta}(M_1)=0,\ \alpha,\ \beta=\overline{1,3};\ \text{или}\ \nabla U_\alpha(M_1)=0,\ \alpha=\overline{1,3}. \tag{3}$$

The presence of an extremum in vector function $U(t,x)$ at $M_1(t',x')$ means that there are sufficient conditions for a local extremum for each U_α. Thus, the matrix of the quadratic form (Hessian) corresponding to each component is a fixed sign, i.e. if the Hessian

$$G_\alpha(M_1)=\left\|\frac{\partial^2 U_\alpha}{\partial x_\beta \partial x_\gamma}\right\|,\ \ \alpha=\overline{1,3};\ \ \beta,\gamma=\overline{1,3},$$

negative definite for some α, then U_α local maxi mum at $M_1(t',x')$ (positive definite - a local minimum).

Next we will prove some statements in the class of smooth functions, which are related to the following question:

Are there any direct link between extreme values of the vector function U at M_1 with extreme values of the kinetic energy (k. e.) density $E(t,x)=0.5(U_1^2+U_2^2+U_3^2)$ (in particular, local maximum) and the pressure P at the same point M_1?

A positive answer are stated in Theorem 1, 2.

Theorem 1. If the vector function $U(t,x)$ *reaches an extremum at some point* $M_1(t',x')\in Q$, *and at least one of the components reaches its maximum positive (minimum negative), then at the same point k. e. density to* E *takes the local maximum.*

Proof. Let the vector function U reach an extremum at the point M_1. We verify that the necessary and sufficient conditions for a local maximum of k. e. density E point M_1.

We write the necessary conditions for a local maximum k. e. density E at:

$$\frac{\partial E}{\partial x_\beta}(M_1)\equiv \sum_{\alpha=1}^{3} U_\alpha \frac{\partial U_\alpha}{\partial x_\beta}(M_1)=0,\ \ \beta=\overline{1,3}. \tag{4}$$

These conditions at point M_1 are performed at certain conditions (3), where vector-function $U(t,x)$ has at point M_1 local *extremum.*

We introduce the Hessian matrix to ensure sufficient conditions for the implementation of local maximum k. e. density E at point M_1:

$$\mathrm{B}(M_1)=\left\|\frac{\partial^2 E}{\partial x_\beta \partial x_\gamma}\right\|,\ \ \beta,\gamma=\overline{1,3}.$$

Differentiating k. e. density to E we find elements of the matrix B

$$\frac{\partial^2 E}{\partial x_\beta \partial x_\gamma} = \sum_{\alpha=1}^{3} U_\alpha \frac{\partial^2 U_\alpha}{\partial x_\beta \partial x_\gamma} + \sum_{\alpha=1}^{3} \frac{\partial U_\alpha}{\partial x_\beta} \frac{\partial U_\alpha}{\partial x_\gamma}, \ \beta = \overline{1,3};\ \gamma = \overline{1,3}. \qquad (5)$$

Sufficient conditions for a local maximum of k. e. density at point M_1, with respect to the principal minor of the matrix B can be written: $B_1(M_1) < 0;\ B_2(M_1) > 0;\ \mathrm{B}_3(M_1) < 0.$

For the first minor B_1 taking account of (5), (3) we obtain the inequality

$$B_1(M_1) = \sum_{\alpha=1}^{3} U_\alpha \frac{\partial^2 U_\alpha}{\partial x_1^2} < 0. \qquad (6)$$

Consider the inequality for the principal minor of the second order at M_1,

$$B_2(M_1) = \begin{vmatrix} B_1 & \dfrac{\partial^2 E}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 E}{\partial x_2 \partial x_1} & \dfrac{\partial^2 E}{\partial x_2^2} \end{vmatrix} > 0.$$

Where from, taking into account (5) and (3), we have $B_2(M_1) = \mathrm{B}_1 \sum_{\alpha=1}^{3} U_\alpha \frac{\partial^2 U_\alpha}{\partial x_2^2} - \left(\frac{\partial^2 E}{\partial x_1 \partial x_2} \right)^2 > 0.$

Hence, using (6), we conclude that $B_2(M_1)$ is positive if and only if

$$\sum_{\alpha=1}^{3} U_\alpha \frac{\partial^2 U_\alpha}{\partial x_2^2}(M_1) < 0. \qquad (7)$$

Now for the minor third order and the inequality of B_3 can be written as

$$B_3(M_1) = \begin{vmatrix} B_1 & \dfrac{\partial^2 E}{\partial x_1 \partial x_2} & \dfrac{\partial^2 E}{\partial x_1 \partial x_3} \\ \dfrac{\partial^2 E}{\partial x_2 \partial x_1} & \dfrac{\partial^2 E}{\partial x_2^2} & \dfrac{\partial^2 E}{\partial x_2 \partial x_3} \\ \dfrac{\partial^2 E}{\partial x_3 \partial x_1} & \dfrac{\partial^2 E}{\partial x_3 \partial x_2} & \dfrac{\partial^2 E}{\partial x_3^2} \end{vmatrix} \equiv \frac{1}{B_1} \begin{vmatrix} B_1 & \dfrac{\partial^2 E}{\partial x_1 \partial x_3} \\ \dfrac{\partial^2 E}{\partial x_3 \partial x_1} & \dfrac{\partial^2 E}{\partial x_3^2} \end{vmatrix} B_2 - \frac{1}{B_1} \begin{vmatrix} B_1 & \dfrac{\partial^2 E}{\partial x_1 \partial x_3} \\ \dfrac{\partial^2 E}{\partial x_2 \partial x_1} & \dfrac{\partial^2 E}{\partial x_2 \partial x_3} \end{vmatrix}^2 < 0. \qquad (8)$$

The validity of (8) can be verified by direct computation of determinants. Taking into account signs B_1 and B_2 from (8) we conclude that B_3 will be negative if and only if, when

$$\begin{vmatrix} B_1 & \dfrac{\partial^2 E}{\partial x_1 \partial x_3} \\ \dfrac{\partial^2 E}{\partial x_3 \partial x_1} & \dfrac{\partial^2 E}{\partial x_3^2} \end{vmatrix} > 0.$$

Where, taking into account (5), (6) and (3), we have

$$B_1 \sum_{\alpha=1}^{3} U_\alpha \frac{\partial^2 U_\alpha}{\partial x_3^2} - \left(\frac{\partial^2 E}{\partial x_1 \partial x_3} \right)^2 > 0.$$

This inequality will be positive if and only if, when

$$\sum_{\alpha=1}^{3} U_\alpha \frac{\partial^2 U_\alpha}{\partial x_3^2}(M_1) < 0. \qquad (9)$$

As a result, the sufficient conditions for the implementation of the local maximum k. e. density to E at the point M_1 have a system of inequalities (6), (7) and (9) in the components of the vector function U:

$$\sum_{\alpha=1}^{3} U_\alpha \frac{\partial^2 U_\alpha}{\partial x_\beta^2}(M_1) < 0.\ \ \beta = \overline{1,3}. \qquad (10)$$

System of inequalities (10) is possible if and only if the vector function U has an extremum at the point M_1 wherein at least one of the components of the vector function satisfies the sufficient conditions for a positive maximum or a negative minimum at the point M_1, and the rest — a local extremum.

Indeed, let's at some α' component $U_{\alpha'}$ satisfies the sufficient conditions for maximum positive (minimum negative) at M_1, then

$$U_{\alpha'}(M_1)>0\wedge\frac{\partial^2U_{\alpha'}}{\partial x_\beta^2}(M_1)<0,\left(U_{\alpha'}(M_1)<0\wedge\frac{\partial^2U_{\alpha'}}{\partial x_\beta^2}(M_1)>0\right),\ \beta=\overline{1,3},$$

thus

$$U_{\alpha'}(M_1)\frac{\partial^2U_{\alpha'}}{\partial x_\beta^2}(M_1)<0,\ \forall\beta=\overline{1,3}. \tag{11}$$

When the system of inequalities (11) holds for $\forall\alpha$, then all of the components of the vector function U_α satisfy the sufficient conditions for positive maximum (negative minimum) at M_1 и and takes place (10).

However, this can't be in a the best accident only one of the components can be satisfied the sufficient condition for of maximum positive (minimum negative), but the rest components can be satisfied the sufficient condition of maximum negative or minimum positive and inequality (10) be fulfilled, there is a system of inequality

$$\left|U_{\alpha'}\frac{\partial^2U_{\alpha'}}{\partial x_\beta^2}(M)_1\right|>\sum_{\alpha\neq\alpha'}U_\alpha\frac{\partial U_\alpha}{\partial x_\beta^2}(M_1),\quad\forall\beta=\overline{1,3},$$

i.e. the module of left-hand side of inequality (11) must exceed the sum of the remaining two terms in (10).

If this requirement does not take place, then together with this an inequality too (10). Then there remains the case that any two velocity components satisfy the sufficient condition for a maximum positive or minimum negative, and the last component satisfies the sufficient condition for the maximum negative (positive low) and probably the system of inequality (10). As a result you showed the necessary and sufficient conditions of a local maximum for k. e. density to E at M_1, where it reaches the vector function $U(t,x)$ of local extremum. Theorem 1 is proved.

For a function of pressure P holds a similar statement.

Theorem 2. If the k. e. density to $E(t,x)$ *reaches a local maximum at a point* $M_1(t',x')$ *domain* $Q=(0,T]\times\Omega$, *the point* M_1 *is a stationary point for a function of pressure* P, *that is the equality* $\nabla P(M_1)=0$.

Proof. We write the well-known formula of vector analysis

$$(U,\nabla)U=\nabla E-[U,\omega], \tag{12}$$

where $[\cdot,\cdot]$ — vector product, $\omega=rotU$.

Let the vector function $I\equiv[U,\omega]$ is continuous in a bounded domain Ω at $\forall t\in[0,T]$, thus $I\in L_2(\Omega)$, $\forall t\in[0,T]$. Then, following [1] the vector function I we imagine as the orthogonal sum

$$I=\nabla R+V^{(J)},\text{ where }\nabla R\in G(\Omega),\ V^{(J)}\in\overset{0}{\mathrm{J}}(Q). \tag{13}$$

Where, at the same time, we calculate the boundary values of ∇R:

$$\left.\frac{\partial R}{\partial n}\right|_{\partial\Omega}=0, \tag{14}$$

since $V^{(J)}n|_{\partial\Omega}=0$ and $[U,\omega]n|_{\partial\Omega}=0$ respectively, into force $V^{(J)}\in\overset{0}{J}$ and (1d), where n is the unit outward normal vector at x boundaries $\partial\Omega$.

Legality ratio (13) follows from the solvability of the Neumann's problem for the following Poisson equation

$$\Delta R=divI, \tag{15}$$

with the boundary condition (14).

Remark. The boundary condition (14) is discharged to mention in passing the solvability of the Neumann problem, because for the maximum principle, this fact will not be used.
Applying with the operator div on the vector function $(U,\nabla)U$, we find

$$div\{(U,\nabla)U\}=\sum_{\alpha,\beta=1}^{3}\frac{\partial U_\alpha}{\partial x_\beta}\frac{\partial U_\beta}{\partial x_\alpha}. \tag{16}$$

Formula (12) taking into account the representation (13) can be rewritten

$$(U,\nabla)U=\nabla E-\nabla R\text{-}V^{(J)}, \tag{17}$$

we apply to both sides of this relation operation div and using (16), we obtain

$$\sum_{\alpha,\,\beta=1}^{3}\frac{\partial U_\alpha}{\partial x_\beta}\frac{\partial U_\beta}{\partial x_\alpha}=\Delta E-\Delta R. \tag{18}$$

From this point of M_1 to a maximum of E we find

$$\Delta E(M_1)-\Delta R(M_1)=0,$$

since the left side of the formula (18) vanishes at the point M_1 based on (3).

From where $\Delta R(M_1)\le 0,$ since by hypothesis point M_1 is the maximum point E and there is a necessary condition for the local maximum $\Delta E(M_1)\le 0.$ Thus, for the function R at the point $M_1\in Q$ is also the necessary condition for a local maximum.

We introduce an auxiliary function $D(t,x)=C(t)+\dfrac{K}{6}\sum\limits_{\alpha=1}^{3}(x_\alpha-x'_\alpha)^2$ with properties:

$$a)\ \nabla D(M_1)=0,\qquad b)\ \Delta D(M_1)=K,$$

where $C(t)=const(t)>0,$ $\{x'_\alpha\}$ — coordinates at the point M_1 and the constant $K=\|divI\|_{C(Q)}$.

Putting b) and (15), we write the Poisson's equation

$$\Delta R_s=K+divI,\ x\in\Omega \text{ где } R_s=D+R. \tag{19}$$

We construct the ball $B_\varepsilon^{M_1}$ with center at M_1 a radius ε so small enough that the ball $B_\varepsilon^{M_1}$ together with its boundary S_ε — lies entirely inside $\Omega.$ From (19) we have

$$\Delta R_s=K+divI\ge 0,\ \ \forall x\in\overline{B}_\varepsilon^{M_1}.$$

Thus [5] the function R_s is sub harmonic in the ball $B_\varepsilon^{M_1}$, since it satisfies ratio

$$\Delta R_s\ge 0,\ \forall x\in\overline{B}_\varepsilon^{M_1}.$$

If such a function R_s in the closed domain $\overline{B}_\varepsilon^{M_1}$ has a maximum point in $B_\varepsilon^{M_1}$, then $R_s=const$ in the $B_\varepsilon^{M_1}$.
Where, taking into account the fact that $R_s(x)=D(x)+R(x)$ we have

$$D(x)+R(x)=const \text{ или } \nabla D(x)+\nabla R(x)=0,\ \ \forall x\in B_\varepsilon^{M_1}.$$

From which it follows that $\nabla R(M_1)=0,$ because $\nabla D(M_1)=0.$ Thus, the point M_1 is a stationary point of the function $R(t,x).$

Next, equation (1a), we multiply the gradient of an arbitrary single-valued function $\eta(t,x)\in L_\infty(0,T;C^\infty(\Omega)),$ satisfying condition $\eta\big|_{\partial\Omega}=\dfrac{\partial\eta}{\partial n}\Big|_{\partial\Omega}=0.$ And then, using the orthogonality of subspaces $G(\Omega),$ $\overset{0}{J}(\Omega),$ we integrate over the domain $\Omega,$ and as a result we obtain

$$\int_\Omega\big(\nabla P+(U,\nabla)\,U\big)\nabla\eta\,dx=0,\quad \forall t\in[0,T].$$

Hence, by replacing the integrand $(U,\nabla)U$ corresponding value of formula (17) and given the orthogonality of subspaces $G(\Omega)$, $\overset{0}{J}(\Omega)$, we find

$$\int_{\Omega}\nabla(\mathrm{P}+E-R)\nabla\eta\,dx=0,\quad \forall t\in[0,T].$$

Where, due to arbitrariness $\nabla\eta$, we have

$$\nabla P(t,x)+\nabla E(t,x)-\nabla R(t,x)=0,\ \forall(t,x)\in Q.$$

This identity can be written the point $M_1\in Q$ maximum of the function E

$$\nabla P(M_1)+\nabla E(M_1)-\nabla R(M_1)=0.$$

Whence $\nabla P(M_1)=0$, since $\nabla E(M_1)=0$ and $\nabla R(M_1)=0$ at the point $M_1\in Q$ of maximum of the function E or extreme of vector of the velocity U. Theorem 2 is proved.

The following is after Theorem 2.

Corollary 1. If the vector function U reaches an extremum at some point M_1 and at least one of the components reaches its maximum positive (minimum negative) the point M_1 is a stationary point of the function of the pressure $P(t,x)$, i.e. the equality $\nabla P(M_1)=0$.

The maximum principle

The vector equation (1a) can be rewritten in the form of system of scalar equations:

$$\frac{\partial U_\alpha}{\partial t}-\mu\Delta U_\alpha+(U,\nabla U_\alpha)+\frac{\partial P}{\partial x_\alpha}=f_\alpha,\ \alpha=\overline{1,3},\qquad (1\,a')$$

Theorem 3. Let Ω be a closed bounded domain in R_3 with boundary $\partial\Omega$ and $\overline{Q}=(0,T]\times\overline{\Omega}$ *cylindrical domain in the space of variables $t,x.$ We will assume that the functions $U\in C(\overline{Q})\cap C^2(Q)\wedge P\in C^2(Q)$ and satisfy the equations* (1a). Then, if for some α' function $f_{\alpha'}(t,x)\le 0\ (f_{\alpha'}(t,x)\ge 0)$ in Q, *the function $U_{\alpha'}$ takes its maximum positive (minimum negative) in Q on the lower base $\{0\}\times\overline{\Omega}$ or on its lateral surface* $[0,T]\times\partial\Omega$, i. e.

$$U_{\alpha'}(t,x)\le\max\{\sup_{t=0\wedge x\in\overline{\Omega}}U_{\alpha'}(t,x),\ \sup_{t\in[0,T]\wedge x\in\partial\Omega}U_{\alpha'}(t,x)\},\ (t,x)\in\overline{Q};\qquad (20a)$$

$$\left(U_{\alpha'}(t,x)\ge\min\{\inf_{t=0\wedge x\in\overline{\Omega}}U_{\alpha'}(t,x),\ \inf_{t\in[0,T]\wedge x\in\partial\Omega}U_{\alpha'}(t,x)\},\ (t,x)\in\overline{Q}\right).\qquad (20b)$$

Proof. For this we use the well-known method [6; 510]. We will assume the contrary, i.e, vector function $U(t,x)$— is a solution of the Navier-Stokes equations (1) reaches an extremum in the domain $Q=(0,T]\times\overline{\Omega}$, with one components of the solution $U_{\alpha'}$ reaches an extremum in the domain, with one components of the solution reaches its maximum positive value at some point $M_0(t^0,x^0)$.

$$U_{\alpha'}(M_0)>\max\left\{\sup_{t=0\wedge x\in\overline{\Omega}}U_{\alpha'}(t,x),\ \sup_{t=[0,T]\wedge x\in\partial\Omega}U_{\alpha'}(t,x)\right\}=C\ge 0.\qquad (21)$$

We will designate $m=U_{\alpha'}(M_0)-C>0$ 0 and introduce

$$H_{\alpha'}(t,x)=U_{\alpha'}(t,x)+\frac{m}{2}\left(1-\frac{t}{T}\right).$$

Hence, for all (t,x) of $\partial\Omega\times[0,T]$ or $\{0\}\times\overline{\Omega}$ have

$$H_\alpha(t^0,x^0)\ge H_\alpha(t,x)+\frac{m}{2}.$$

The function $H_{\alpha'}(t,x)$ also takes its maximum value at some point $M_1(t',x')\in Q,$ and $H_{\alpha'}(M_1)\ge H_{\alpha'}(M_0)\ge m.$ Also on the basis of Corollary 1 of Theorem 2 $\nabla P(M_1)=0.$

We write down all the necessary conditions maximum of function $H_{\alpha'}$ at the point M_1:

$$\frac{\partial H_{\alpha'}}{\partial t} \geq 0;\ \Delta H_{\alpha'} \leq 0;\ \nabla H_{\alpha'} = 0;\ \frac{\partial P}{\partial x_{\alpha'}} = 0. \qquad (22)$$

From equation ($1a'$) taking in to account the conditions (22) we find for the points M_1 chain of inequalities

$$LH_{\alpha'} \equiv \frac{\partial H_{\alpha'}}{\partial t} - \mu \Delta H_{\alpha'} + \left(H, \nabla H_{\alpha'}\right) + \frac{\partial P}{\partial x_{\alpha'}}\left(M_1\right) - f_{\alpha'} + \frac{m}{2T} \geq \frac{m}{2T} > 0.$$

This means that inequality (21) is incorrect. Consequently, the true (20a). Theorem 3 is proved

From Theorem 3, following [6], it is easy to obtain a proof of the following proposition:

Corollary 2. If the vector functions f, Φ *satisfy the conditions* ***i)*** *and* ***ii),*** then the solution $U\left(t,x\right)$ of the problem *(1)* the following estimate holds:

$$\|U\|_{C(Q)} \leq \|\Phi\|_{C(Q)} + T\|f\|_{C(Q)} = \mathrm{A}_1,\ \ \forall T < \infty,\ \text{где}\ \|U\|_{C(\overline{Q})} = \max_{1\leq\alpha\leq 3} \sup_{\overline{Q}} \left|U_\alpha\left(t,x\right)\right|. \qquad (23)$$

The estimate (23) in the mathematical theory of the Navier-Stokes equations is fundamentally-key. As in [2–4], et al. on the basis of (23) in selected spaces to prove the uniqueness of the weak and the existence of strong solutions of the NSE for the whole time.

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Ә.Ш.Ақыш

Навье-Стокс теңдеулері үшін максимум қағидасының жаңа түрі туралы

Навье-Стокс теңдеулеріндегі жылдамдық векторының, кинетикалық энергия тығыздығы мен қысым функцияларының облыс нүктелеріндегі экстремалдық мәндерінің жаңа байланысы айқындалған, осының негізінде бейсызықты Навье-Стокс теңдеулері үшін математикалық тұрғыдан іргелі де тиекті мәселе максимум қағидасының орындалатындығы көрсетілген.

А.Ш.Акыш

О новом варианте принципа максимума для уравнений Навье-Стокса

Показаны новые связи экстремальных значений вектора скорости, плотности кинетической энергии (в частности, локального максимума) и давления уравнений Навье-Стокса. С помощью последнего показана справедливость принципа максимума для нелинейной системы уравнений Навье-Стокса, что с математической точки зрения является принципиально-ключевым.