МАТЕМАТИКА

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On non-degenerate singular points of normalized Ricci flows on some generalized Wallach spaces

The present paper devoted to problems of Riemannian geometry and planar dynamical systems. In particular we study non-degenerate singular points of normalized Ricci flows on special type of generalized Wallach spaces. Our main goal is to prove the absence of such points. The main idea is based on a special set Ω introduced in [1, 2] for studying general properties of degenerate singular points of Ricci flows. More concretely, for solving the mentioned problem we use the facts that the set $\left(0,1/2\right)^3\cap\Omega$ is connected and the set $\left(0,1/2\right)^3\setminus\Omega$ consists of three connected components as it was proved in [3].

Key words: generalized Wallach space, normalized Ricci flow, dynamical system, singular point of dynamical system, real algebraic surface.

Introduction

In the present work we consider the autonomous system of nonlinear ODEs obtained in [1]:

$$\frac{dx_1}{dt}=f(x_1,x_2,x_3),\quad \frac{dx_2}{dt}=g(x_1,x_2,x_3),\quad \frac{dx_3}{dt}=h(x_1,x_2,x_3),$$

where $x_i=x_i(t)>0$;

$$f(x_1,x_2,x_3)=-1-a_1x_1\left(\frac{x_1}{x_2x_3}-\frac{x_2}{x_1x_3}-\frac{x_3}{x_1x_2}\right)+x_1B;$$

$$g(x_1,x_2,x_3)=-1-a_2x_2\left(\frac{x_2}{x_1x_3}-\frac{x_3}{x_1x_2}-\frac{x_1}{x_2x_3}\right)+x_2B;$$

$$h(x_1,x_2,x_3)=-1-a_3x_3\left(\frac{x_3}{x_1x_2}-\frac{x_1}{x_2x_3}-\frac{x_2}{x_1x_3}\right)+x_3B;$$

$$B:=\left(\frac{1}{a_1x_1}+\frac{1}{a_2x_2}+\frac{1}{a_3x_3}-\frac{x_1}{x_2x_3}-\frac{x_2}{x_1x_3}-\frac{x_3}{x_1x_2}\right)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)^{-1},\quad a_i\in(0,1/2],\quad i=1,2,3.$$

Recall that the system above arises at investigations of Ricci flows on generalized Wallach spaces (see details in [1, 2]) and could be equivalently reduced to the system of two differential equations of the type as it was proved in [1]:

$$\frac{dx_1}{dt}=\tilde{f}(x_1,x_2),\quad \frac{dx_2}{dt}=\tilde{g}(x_1,x_2), \tag{1}$$

where

$$\tilde{f}(x_1,x_2)=f(x_1,x_2,\varphi(x_1,x_2)),\ \tilde{g}(x_1,x_2)=g(x_1,x_2,\varphi(x_1,x_2)),\ \varphi(x_1,x_2)=x_1^{-\frac{a_3}{a_1}}x_2^{-\frac{a_3}{a_2}}.$$

Let

$$a_1^0 = \frac{n}{2(l+m+n-2)},\ a_2^0 = \frac{m}{2(l+m+n-2)},\ a_3^0 = \frac{k}{2(l+m+n-2)}.$$

where $n, m, k \in N$.

The case $(a_1, a_2, a_3) = (a_1^0, a_2^0, a_3^0)$ corresponds to the special family of generalized Wallach spaces $SO(k+m+n)/(SO(k)\times SO(m)\times SO(n))$ as it was shown in [4] (a more detailed information concerning geometric aspects of this problem could be found in [4–8]).

The main result of this work is contained in the following theorem.

Theorem 1. Let $m = k$. Then the system (1) has no non-degenerate stable nodes at $(a_1, a_2, a_3) = (a_1^0, a_2^0, a_3^0)$.

Recall some well-known definitions of the qualitative theory of ODEs: (x_1^0, x_2^0) is called a non-degenerate stable node of (1) if

1) $\tilde{f} = \tilde{g} = 0$ at (x_1^0, x_2^0);

2) $\lambda_1 < 0,\ \lambda_2 < 0,$ where λ_1, λ_2 are the eigenvalues of the Jacobian matrix $J = J(x_1^0, x_2^0) = \left.\begin{pmatrix} \tilde{f}_{x_1} & \tilde{f}_{x_2} \\ \tilde{g}_{x_1} & \tilde{g}_{x_2} \end{pmatrix}\right|_{(x_1,x_2)=(x_1^0,x_2^0)}$ calculating by the formula $\lambda_{1,2} = \frac{\rho \pm \sqrt{\sigma}}{2}$, $\delta := \det(J)$; $\rho := trace(J)$, $\sigma := \rho^2 - 4\delta$.

The paper is organized as follows. In Section 1 we reformulate some well-known facts. In Section 2 we prove Lemmas 2, 3 and 4. In Section 3 we prove Theorem 1.

1. Preliminaries

In [1] the special set

$\Omega' = \{(a_1, a_2, a_3) \in R^3 \mid$ the system (1) has at least one degenerate singular point $\}$ was introduced and the following lemma was proved.

Lemma 1 (Lemma 4 in [1]). If a point (a_1, a_2, a_3) with $a_1a_2 + a_1a_3 + a_2a_3 \neq 0$ and $a_1, a_2, a_3 \neq 0$ lies in the set Ω', then $Q(a_1, a_2, a) = 0$, where

$$\begin{aligned} Q(a_1,a_2,a) = {} & (2s_1+4s_3-1)(64s_1^5+64s_1^4+8s_1^3+12s_1^2-6s_1+1+ \\ & +240s_3s_1^2-240s_3s_1-153s_3^2s_1-4096s_3^3+60s_3+758s_3^2)- \\ & -8s_1(2s_1+4s_3-1)(2s_1-32s_3-1)(10s_1+32s_3-5)s_2- \\ & -16s_1^2(13-52s_1+640s_3s_1+1024s_3^2-320s_3+52s_1^2)s_2^2+ \\ & +64(2s_1-1)(2s_1-32s_3-1)s_3^2+2048s_1(2s_1-1)s_2^4; \\ & s_1 = a_1+a_2+a_3,\ s_2 = a_1a_2+a_1a_3+a_2a_3,\ s_3 = a_1a_2a_3. \end{aligned} \tag{2}$$

$Q(a_1, a_2, a_3)$ is a symmetric polynomial in a_1, a_2, a_3 of degree 12. Therefore, as it was remarked in [1] the equation $Q(a_1, a_2, a) = 0$ (without the restrictions $a_1a_2 + a_1a_3 + a_2a_3 \neq 0$ and $a_1a_2a_3 \neq 0$) defines an real algebraic surface in R^3 that we will denote by Ω according to [1]. From Lemma 1 we see that $\Omega' \subset \Omega$.

By [3] in the set $(0,1/2)^3 \cap \Omega$ is connected and there are exactly three connected components in the set $(0,1/2)^3 \setminus \Omega$. Preserving the original notations of [1] denote by O_1, O_2 and O_3 the components in $(0,1/2)^3 \setminus \Omega$ containing the points $(1/6, 1/6, 1/6)$, $(7/15, 7/15, 7/15)$ and $(1/6, 1/4, 1/3)$ respectively.

In [2] the following theorem was proved.

Theorem 2 (Theorem 7 in [2]). For $(a_1, a_2, a_3) \in O_j$ the following possibilities for singular points of the system (1) can occur:

i) If $j=1$ then there is one singular point with $\delta>0$ (an unstable node) and three singular points with $\delta<0$ (saddles);

ii) If $j=2$ then there is one singular point with $\delta>0$ (a stable node) and three singular points with $\delta<0$ (saddles);

iii) If $j=3$ then there are two singular points with $\delta<0$ (saddles).

In [2] the following question was formulated: let $\left(a_1^0, a_2^0, a_3^0\right)$ be any triple in $(0,1/2)^3 \setminus \Omega$. Is there a way to decide on which connected component O_1, O_2, or O_3 does this triple belong to? The answer was affirmative according to the following remark.

Remark 1 (Remark 8 in [2]). Consider first the simplest case where $a_1^0 = a_2^0 = a_3^0 =: a^0$. Then obviously $\left(a_1^0, a_2^0, a_3^0\right) \in O_1$ for $a^0 < 1/4$ and $\left(a_1^0, a_2^0, a_3^0\right) \in O_2$ for $a^0 > 1/4$ (recall that $(1/4, 1/4, 1/4)$ is a very special of Ω).

Assume now that $a_1^0 : a_2^0 : a_3^0 \neq 1:1:1$. Then it is easy to find (solving an algebraic equation of degree at most 12 with respect to t) the intersection of Ω with the interval I containing points of the form $\left(a_1, a_2, a_3\right) = \left(a_1^0 t, a_2^0 t, a_3^0 t\right)$, where $0<t<1$. This means that we need to give numerical values to $\left(a_1^0, a_2^0, a_3^0\right)$ and then solve the corresponding equation with respect to t (it could be done approximately e.g. by Maple or by Matematica).

From simple geometric arguments we have the following: If the number of intersection points is 0, 1, or 2, then $\left(a_1^0, a_2^0, a_3^0\right)$ belongs to O_1, O_2, or O_3 respectively. For instance, if all solutions of the corresponding equation are complex, then the number of intersection points is 0 and $\left(a_1^0, a_2^0, a_3^0\right) \in O_1$.

For our aims we need also the well-known Sturm's theorem.

Theorem 3 (Sturm, [3]). If the real numbers a and b, $a<b$, are not the roots of a polynomial $f(x)$ which does not have any multiple roots, then $W(a) \geq W(b)$ and the difference $W(a)-W(b)$ is equal to the number of real roots of $f(x)$ in the interval between a and b, where $W(c)$ denotes the number of variations in sign in the sequence

$$f_0(x),\ f_1(x),\ \ldots,\ f_s(x),\ x=c.$$

Remark 2. Recall that the Sturm sequence $f_0(x),\ f_1(x),\ \ldots,\ f_s(x)$ may be constructed as following:

$$f_0 := f,\ f_1 := f_0',\ f_2 := -\operatorname{rem}(f_0, f_1),\ \ldots, 0 = -\operatorname{rem}(f_{s-1}, f_s),$$

where $\operatorname{rem}(f_{i-1}, f_i)$ means a remainder of the polynomial division of f_{i-1} by f_i.

2. Auxiliary results

Return to the family of generalized Wallach spaces $SO(k+m+n)/(SO(k)\times SO(m)\times SO(n))$.

The case $m=k=n,\ k\in N$.

Lemma 2. If $m=k=n$ then $\left(a_1, a_2, a_3\right)$ can belong only to O_1.

Proof. Substituting $\left(a_1, a_2, a_3\right) = \left(a_1^0 t, a_2^0 t, a_3^0 t\right)$ into $Q\left(a_1, a_2, a_3\right)$ according to Remark 1 we get the following polynomial of degree 12

$$p(t) := Q\left(a_1^0 t, a_2^0 t, a_3^0 t\right) = -\frac{(kt+3k-2)^4 (2kt-3k+2)^8}{(3k-2)^{12}}. \qquad (3)$$

As the calculations show the equation $p(t)=0$ has the unique real solution $t=\dfrac{(3k-2)}{2k}$ of algebraic multiplicity 4. It is clear that the condition $0<t<1$ could be satisfied only at $k=1$, however $a_1 = a_2 = a_3 = 1/2$ in this case. The lemma is proved.

The case $m=k,\ n\neq k,\ k\in N$, Consider the case where two of a_i^0's are equal in (3).

By the same way as in proof of Lemma 2 we get the polynomial of degree 12

$$p(t) := Q\left(a_1^0 t, a_2^0 t, a_3^0 t\right) = p_1(t) p_2(t) p_3^3(t), \quad (4)$$

where

$$p_1(t) := -nt - (n+2k-2);$$
$$p_2(t) := 2n(k+n)t^2 - 2(k+n)(n+2k-2)t + (n+2k-2)^2;$$
$$p_3(t) := 2k^2(k+n)t^3 - (n+2k)(n+2k-2)^2 t + (n+2k-2)^3.$$

Obviously that $p_1(t)$ has only negative roots: $-(n+2k-2)n^{-1} < 0.$

Hence our purpose is to find conditions on n, k ensuring the polynomial $p_2(t)p_3(t)$ distinct real roots from the interval $(0,1).$ We will consider the subcases $n<k$ and $n>k$ separately.

The subcase $n<k.$

Lemma 3. Let $m=k,\ n<k.$ Then the following assertions hold for the polynomial $p(t)$ given by (4):

(1) If $n < 2\sqrt{k-1},$ then $p(t)$ has a unique zero $t^* \in (0,1);$

(2) If $2\sqrt{k-1} \le n < k$ then $p(t)$ has no zeroes between 0 and $1.$

Proof. Roots of $p_2(t).$ It is easy to show that the quadratic equation $p_2(t)=0$ has the following two real roots

$$t^* := \frac{k+n-\sqrt{k^2-n^2}}{2n(k+n)}(n+2k-2), \quad t^{**} := \frac{k+n+\sqrt{k^2-n^2}}{2n(k+n)}(n+2k-2).$$

Note that the condition $0 < t^{**} < 1$ is equivalent to the inequality

$$\sqrt{k^2-n^2} < \frac{2n(k+n)}{n+2k-2} - (k+n) = \frac{(k+n)(n+2-2k)}{n+2k-2}$$

which has no solution since $n+2-2k = (n-k)-(k-2) < 0$ at $k \ge 2.$ Therefore there is no values of $n,k(n<k)$ such that $t^{**} \in (0,1).$

Consider the root $t^*.$ Since $t^* > 0$ for $n<k$ then the condition $t^* < 1$ leads to the inequality

$$\sqrt{k^2-n^2} > \frac{(k+n)(n+2-2k)}{n+2k-2} > 0$$

which admits the solution $n < 2\sqrt{k-1}.$ Since $2\sqrt{k-1} \le k$ (with equality only at $k=2$) then the condition $n<k$ is preserved too.

Roots of $p_3(t)$. *Further we will prove that* $p_3(t)$ *has no zeros in the interval* $(0,1)$ *if* $n<k.$

As the calculations show the explicit formulas for roots of the cubic equation $p_3(t)=0$ does not allow further in-depth study of real roots. Moreover, since we are interested only in detecting of boundaries of such roots, then the Sturm's theorem can provide some opportunities.

To use Theorem 3, at first, we will show that $p_3(t)$ has no multiple zeroes. Reduce $p_3(t)=0$ to the form $t^3+pt+q=0,$ where

$$p := -(n+2k)(n+2k-2)^2(2k^2(k+n))^{-1} < 0;$$
$$q := (n+2k-2)^3(2k^2(k+n))^{-1} > 0.$$

The discriminant of the last qubic equation is

$$D := \frac{p^3}{27} + \frac{q^2}{4} = -\frac{(n+2k-2)^6(2n^2+14nk+11k^2)(n-k)}{432k^6(k+n)^3}.$$

As we know from algebra multiple roots are possible only at $D=0$ that never can occur because of $n \ne k.$ In particular, $D>0$ for $n<k$ and $p_3(t)$ has an unique real root (and two complex roots).

At second, we will check the inequalities $p_3(0) \ne 0,\ p_3(1) \ne 0.$ It is clear that $p_3(0) = (n+2k-2)^3 \ne 0.$

Easy calculations show that

$$p_3(1) = -2n^2 + 2(k-2)^2 n + 2(k^3-4k^2+8k-4) = -2(n-n_1)(n-n_2),$$

where

$$n_1 := \frac{(k-2)^2 - k\sqrt{(k-2)^2+4}}{2} < 0;$$

$$n_2 := \frac{(k-2)^2 + k\sqrt{(k-2)^2+4}}{2} \ge k. \qquad (5)$$

Since $n<k$ then $n<n_2$, and consequently we have $p_3(1)\neq 0$.

Hence it is reasonable to construct the Sturm sequence f_0, f_1, f_2, f_3 for $p_3(t)$ using Remark 2:

$$f_0(t) := p_3(t),$$

$$f_1(t) := f_0'(t) = 6k^2(k+n)t^2 - (n+2k)(n+2k-2)^2;$$

$$f_2(t) := \mathrm{rem}\,(f_0, f_1) = \frac{2(n+2k)(n+2k-2)^2}{3}t - (n+2k-2)^3;$$

$$f_3(t) := \mathrm{rem}\,(f_1, f_2) = \frac{(n+2k-2)^2(2n^2+14nk+11k^2)(n-k)}{2(n+2k)^3}. \qquad (6)$$

Now we are ready to calculate the values $W(0)$ and $W(1)$. It is obvious that $W(0)=1$ (see Table 1).

Table 1

Values of $W(0)$ in the case $n<k$

$t=0$	$\mathrm{sgn}\,(f_0(0))$	$\mathrm{sgn}\,(f_1(0))$	$\mathrm{sgn}\,(f_2(0))$	$\mathrm{sgn}\,(f_3(0))$	$W(0)$
$n<k$	+	–	–	–	1

At $t=1$ we have

$$f_0(1) := p_3(1) = -2(n-n_1)(n-n_2); \qquad (7)$$

$$f_1(1) := -2k^3 - 2(3n-8)k^2 - 2(3n-2)(n-2)k - n(n-2)^2; \qquad (8)$$

$$f_2(1) := -\frac{(n+2(k-3))(n+2k-2)^2}{3}. \qquad (9)$$

Obviously, $f_3(1)<0$ and $f_0(1)>0$ since $n<k<n_2$. Signs of $f_1(1)$, $f_2(1)$ will be detected from the following cases.

Case 1. Let $k=2$, $n=1$. Then $f_1(1)=27$, $f_2(1)=3$.

Case 2. Let $k\ge 2$, $n<k$. Then $f_2(1)<0$. In this case we do not need to detect the sign of $f_1(1)$ since for any possible event ($+$, $-$, or 0) only one sign change expected in the second row of Table 2.

Thus from Tables 1,2 we get $W(0)-W(1)=0$. Using Theorem 3 we conclude that there is no roots of $p_3(t)$ in the interval $(0,1)$ whenever $n<k$.

Table 2

Values of $W(1)$ in the case $n<k$

$t=1$	$\mathrm{sgn}\,(f_0(1))$	$\mathrm{sgn}\,(f_1(1))$	$\mathrm{sgn}\,(f_2(1))$	$\mathrm{sgn}\,(f_3(1))$	$W(1)$
$k=2$, $n=1$	+	–	–	–	1
$k\ge 3$, $n<k$	+	$\pm$ or 0	–	–	1

The lemma is proved.

The subcase $n>k$.

Lemma 4. Let $m=k$, $n>k$. Then the following assertions hold for the polynomial $p(t)$ given by (4):

(1) If $k\le 3$ or $k\ge 4$, $k<n_2<n$ then $p(t)$ has an unique zero $t\in(0,1)$;

(2) If $k\ge 4$, $k<n_2<n$ then $p(t)$ has zeroes between 0 and 1, where n_2 is given by (5).

Proof. Roots of $p_2(t)$. *It is clear that* $p_2(t)$ *has no real zeros at* $n > k$. Roots of $p_3(t)$. *It is clear that* $W(0) = 0$ *(see Table 3).*

Table 3

Values of $W(0)$ in the case $n > k$

$t = 0$	$\operatorname{sgn}(f_0(0))$	$\operatorname{sgn}(f_1(0))$	$\operatorname{sgn}(f_2(0))$	$\operatorname{sgn}(f_3(0))$	$W(0)$
$n > k$	+	–	–	–	2

Detail analysis in (6)–(9) show that $W(1)$ can take signs depicted in Table 4. Note that $n_2 < n$ for $k \le 3$ (see (5)).

Table 4

Values of $W(1)$ in the case $n > k$

$t = 1$	$\operatorname{sgn}(f_0(1))$	$\operatorname{sgn}(f_1(1))$	$\operatorname{sgn}(f_2(1))$	$\operatorname{sgn}(f_3(1))$	$W(1)$
$k = 1,\ n = 2$	–	+	+	+	1
$k = 1,\ n = 3$	–	–	+	+	1
$k = 1,\ n = 4$	–	–	0	+	1
$k = 1,\ n \ge 5$	–	–	–	+	1
$k = 1,\ n \ge 3$	–	–	–	+	1
$k = 1,\ n \ge 4$	–	–	–	+	1
$k \ge 4,\ k < n_2 < n$	–	–	–	+	1
$k \ge 4,\ k < n < n_2$	+	–	–	+	2

Therefore $W(0) - W(1) = 0$ can take only the values 0 or 1 in the case $n > k$. The lemma is proved.

3. Proof of the main result

Proof of Theorem 1. From Lemmas 2, 3 and 4 we get the following: if $m = k$ then any triple (a_1^0, a_2^0, a_3^0) determined by (3) can belong only to either O_1 or O_2. Therefore, according to Theorem 2 the normalized Ricci flow on generalized Wallach spaces of the type $SO(k+m+n)/(SO(k)\times SO(m)\times SO(n))$, $m = k$, cannot have non-degenerate stable nodes. Theorem 1 is proved.

Remark 3. When this paper has been written the first author was informed about the recent preprint [9] where the classification of generalized Wallach spaces was obtained. The mentioned classification and Theorem 2 provide a general result concerning the classification of non-degenerate singular points of the system (1) for all generalized Wallach spaces.

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Н.А.Әбиев, З.О.Тұртқұлбаева

Кейбір жалпыланған Уоллах кеңістіктеріндегі нормалдастырылған Риччи ағымдарының өзгешеленбеген ерекше нүктелері туралы

Мақала римандық геометрия мен жазық динамикалық жүйелер есептеріне арналған. Дербес жағдайда жалпыланған Уоллах кеңістіктерінің арнайы типтеріндегі нормалдастырылған Риччи ағымдарының өзгешеленбеген ерекше нүктелері зерттелді. Басты мақсат — осындай нүктелердің болмауын дәлелдеу. Негізгі идея Риччи ағымдарының өзгешеленген ерекше нүктелерін зерттеу үшін [1, 2] жұмыстарында енгізілген Ω жиыны қасиеттерін пайдалануға негізделген. Анығырақ айтқанда, қойылған есепті шешу үшін, [3] жұмысында дәлелденгендей, $(0,1/2)^3 \cap \Omega$ жиыны тұтас, ал $(0,1/2)^3 \setminus \Omega$ жиыны үш тұтастық компонентасынан тұратыны пайдаланылады.

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О невырожденных особых точках нормализованных потоков Риччи на некоторых обобщенных пространствах Уоллаха

Статья посвящена проблемам римановой геометрии и плоских динамических систем. В частности, изучены невырожденные особые точки нормализованных потоков Риччи на специальных типах обобщенных пространств Уоллаха. Главная цель — доказать отсутствие таких точек. Основная идея зиждется на использовании свойств специального множества Ω, введенного в [1, 2], для изучения общих свойств вырожденных особых точек потоков Риччи. Отмечено, что для решения поставленной задачи используется то, что множество $(0,1/2)^3 \cap \Omega$ связно, а множество $(0,1/2)^3 \setminus \Omega$ состоит из трех связных компонент, как доказано в [3].

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