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Boundary value problems of integrodifferential equations under boundary conditions taking into account physical nonlinearity

When solving integrodifferential equations under boundary conditions, taking into account physical nonlinearity, a broad class of boundary-value problems of oscillations arises associated with various boundary conditions at the edges of a flat element. When taking into account non-stationary external influences, the main parameters is the frequency of natural vibrations of a flat component, taking into account temperature, prestressing, and other factors. The study of such problems, taking into account complicating factors, reduces to solving rather complex problems. The difficulty of solving these problems is due to both the type of equations and the variety. We analyze the results of previous works on the boundary problems of vibrations of plane elements. Possible boundary conditions at the edges of a flat element and the necessary initial conditions for solving particular problems of self-oscillation and forced vibrations, and other problems are considered. The set of equations, boundaries, and initial conditions make it possible to formulate and solve various boundary value problems of vibrations for a flat element. The oscillation equations for a flat element in the form of a plate given in this paper contain viscoelastic operators that describe the viscous behavior of the materials of a flat component. In studying oscillations and wave processes, it is advisable to take the kernels of viscoelastic operators regularly, since only such operators describe instantaneous elasticity and then viscous flow.

Keywords: physical nonlinearity, plates, oscillations, boundary value problems, wave process, isotropic plates, integrodifferential equation, approximate equation, nonlinear operators.

Introduction

When solving integrodifferential equations under boundary conditions, taking into account physical nonlinearity, a broad class of boundary-value problems of oscillations arises associated with various boundary conditions at the edges of a flat element. When taking into account non-stationary external influences, the main parameters is the frequency of natural vibrations of a flat element, taking into account temperature, prestressing, and other factors. The study of such problems, taking into account complicating factors, reduces to solving rather complex problems. The difficulty of solving these problems is due to both the type of equations and the variety. Let us systematize the results of previous works on boundary value problems of oscillations of flat elements. Possible boundary conditions at the edges of a flat element and initial conditions necessary for solving particular problems of natural and forced vibrations, and other problems are considered. The set of equations, boundary, and initial conditions make it possible to formulate and solve various boundary value problems of vibrations for a flat element. Integrodifferential equations with regular kernels are known to be equivalent to partial differential equations. For other approximate equations of oscillations of a plane element, these equations for regular nuclei can also be reduced to partial differential equations, which will be shown below.

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The assumed mathematical approach allows us to consider problems in a nonlinear setting when the nonlinearity is physical. The necessary theoretical information on the substantiation of the nonlinear dependence law $\sigma_{ij} \sim \varepsilon_{ij}$ for a viscoelastic isotropic body was presented in other papers.

1 *General staging*

For simplicity, we will consider a flat structure in the form of a plate and a base in the plane (x, z) or when external forces do not depend on the y coordinate. In this case, displacements u_l, w_l are non-zero, and displacement $v_l = 0$, i.e. absent. We assume that vibrations of a plate lying on a deformable base can be caused both by external forces on the surface of the plate and by disturbances propagating from the side of the base. In addition, we will assume that along the boundaries of the contact of the plate with the base, these contacts are ideal, i.e. there is no friction. Let us consider the case when the base material is isotropic and the dependence of stresses on strains is linear, i.e. Boltzmann-type relations hold [1–3]:

$$\sigma_{jj}^{(2)} = L_2(\varepsilon_{jj}^{(2)}) + 2M_2(\varepsilon_{jj}^{(2)}),$$

$$\sigma_{ij}^{(2)} = M_2(\varepsilon_{ij}^{(2)}), \quad (i, j = x, z; i \neq j).$$

Let us assume that the dependences of stresses on deformations for a plate are cubic.

$$\sigma_{jj}^{(1)} = 3K_1R_0^{(1)}\{\varepsilon_0^{(1)}[1 + \alpha\chi_0^{(1)}K_2^{(1)}(\varepsilon_0^{(1)2})]\} + 2G_1R^{(1)}\{(\varepsilon_{jj}^{(1)} - \varepsilon_0^{(1)}) \cdot [1 + \alpha\gamma_0^{(1)}G_1^{(1)}(\psi_0^{(1)2})]\}, \tag{1}$$

$$\sigma_{ij}^{(1)} = G_1R^{(1)}\varepsilon_{ij}^{(1)}[1 + \alpha\gamma_0^{(1)}G_1^{(1)}(\psi_0^{(1)2})], (i \neq j; i = x, y; l = 1, 2),$$

where $\varepsilon^{(1)}$ is the average volumetric strain. $(\psi_0^{(1)2})$ is the square strain intensity, i.e.

$$(\psi_0^{(1)2}) = \frac{2}{\sqrt{3}}\left[\frac{2}{3}(\varepsilon_{xx}^{(1)2} + \varepsilon_{zz}^{(1)2} - \varepsilon_{xx}^{(1)2}\varepsilon_{zz}^{(1)2} + \frac{1}{2}\varepsilon_{xz}^{(1)2}\right].$$

$\chi_0^{(1)}, \gamma_0^{(1)}$ are the elongation and shear functions, respectively, which are expressed by the formulas:

$$\chi_0^{(1)} = 1 + F_0^{(1)}(\varepsilon_0^1); \gamma_0^1(\psi_0^{(1)2}) = 1 + F_1^1(\psi_0^{(1)2}); \; F_j^{(1)}(0) = 0.$$

In this case, the functions $F_0^{(1)}$ and $F_1^{(1)}$ are expanded in a power series

$$F_0^{(1)}(\varepsilon_0^{(1)}) = \sum_{n=0}^{\infty} \alpha_n \cdot (\varepsilon_0^{(1)})^{2(n+1)},$$

$$F_1^{(1)}(\psi_0^{(1)2}) = \sum_{n=0}^{\infty} \gamma_n \cdot (\psi_0^{(1)})^{2(n+1)}.$$

$R_0^{(1)}$ and $R^{(1)}$ are linear integral operators of Voltaire type

$$R_0^{(1)}(\zeta) = \zeta(t) - \int_0^t F_{10}(t - \xi)\zeta(\xi)d\xi,$$

$$R^{(1)}(\zeta) = \zeta(t) - \int_0^t F_{20}(t - \xi)\zeta(\xi)d\xi.$$

$K_2^{(1)}$ and $G_1^{(1)}$ are non-linear viscoelastic operators.

$$K_2^{(1)}(\varepsilon_0^{(1)2}) = \varepsilon_0^{(1)2} - \int\limits_0^t \int\limits_0^t F_0^{(1)}[(t-\xi_1)(t-\xi_2)]\varepsilon_0^{(1)}(\xi_2)d\xi_1 d\xi_2,$$

$$G_1^{(1)}(\psi_0^{(1)2}) = \psi_0^{(1)2} - \int\limits_0^t F_1^{(1)}(t-\xi)\psi_0^{(1)2}(\xi)d\xi.$$

Constants K_1 and G_1 are equal

$$K_1 = \lambda_1 + \frac{2}{3}\mu_1; G_1 = \mu_1.$$

The vibration equations for a plate as a viscoelastic layer have the form: [4].

$$\left(K_2^{(1)}R_0^{(1)} + \frac{4}{3}G_1R^{(1)}\right)\frac{\partial^2 u_1}{\partial x^2} + G_1R^{(1)}\frac{\partial^2 u_1}{\partial z^2} + \left(K_1R_0^{(1)} + \frac{1}{3}G_1R^{(1)}\right) \times$$
$$\times \frac{\partial^2 w_1}{\partial x \partial z} + \alpha F_1^{(1)}(u_1, w_1) = \rho_1 \frac{\partial^2 u_1}{\partial t^2}, \quad (2)$$
$$\left(K_1R_0^{(1)} + \frac{1}{3}G_1R^{(1)}\right)\frac{\partial^2 u_1}{\partial x \partial z} + G_1R^{(1)}\frac{\partial^2 w_1}{\partial z^2} + \left(K_1R_0^{(1)} + \frac{4}{3}G_1R^{(1)}\right) \times$$
$$\times \frac{\partial^2 w_1}{\partial z^2} + \alpha F_2^{(1)}(u_1, w_1) = \rho_1 \frac{\partial^2 w_1}{\partial t^2},$$

where $F_1^{(1)}, F_2^{(1)}$ are non-linear operators.

$$F_1^{(1)}(u_1, w_1) = 3K_1\chi_0^{(1)}R_0^{(1)}\left\{\frac{\partial}{\partial x}\left[\varepsilon_0^{(1)}K_2^{(1)}(\varepsilon_0^{(1)2})\right]\right\} +$$
$$+\gamma_0\left\{G_1R^{(1)}\frac{\partial}{\partial x}[(\varepsilon_{xx}^{(1)} - \varepsilon_0^{(1)})G_1^{(1)}(\psi_0^{(1)2})]\right\} + \gamma_0 G_1R^{(1)}\frac{\partial}{\partial z}[\varepsilon_{xz}^{(1)}G_1^{(1)}(\psi_0^{(1)2})] \quad (3)$$
$$F_2^{(1)}(u_1, w_1) = 3K_1\chi_0^{(1)}R_0^{(1)}\left\{\frac{\partial}{\partial z}\left[\varepsilon_0^{(1)}K_2^{(1)}(\varepsilon_0^{(1)2})\right]\right\} +$$
$$+\gamma_0\left\{G_1R_1\frac{\partial}{\partial z}\left[(\varepsilon_{xx}^{(1)} - \varepsilon_0^{(1)})G_1^{(1)}(\psi_0^{(1)2})\right]\right\} + \gamma_0 G_1R^{(1)}\frac{\partial}{\partial x}\left[\varepsilon_{xz}^{(1)}G_1^{(1)}(\psi_0^{(1)2})\right]$$

Boundary conditions: at $z = h$.

$$\sigma_{zz}^{(1)} = f_z^{(1)}(x,t), \sigma_{xz}^{(1)} = 0 \quad (4)$$

at $z = -h$.

$$\sigma_{zz}^{(1)} = \sigma_{zz}^{(2)}; \sigma_{xz}^{(1)} = 0; \sigma_{xz}^{(2)} = 0; w_1 = w_2. \quad (5)$$

The initial conditions are zero, i.e. $u_l = \frac{\partial u_l}{\partial t} = w_l = \frac{\partial w_l}{\partial t} = 0$, at $t = 0$.

Thus, the boundary-value problem of the vibration of isotropic plates lying on a deformable foundation, taking into account the physical nonlinearity of stresses from deformation, is reduced to solving integrodifferential equations (2) under boundary and initial conditions (4)–(5).

Let us consider the oscillation equations taking into account the physical nonlinearity of stresses due to deformations [5].

Relations (1) hold for the plate material.

We will look for the displacements of the u and v plates in the form of a series with respect to parameter α.

$$u(x,z,t)=\sum_{n=0}^{\infty} a^n u_n(x,z,t),$$

$$\omega(x,z,t)=\sum_{n=0}^{\infty} a^n w_n(x,z,t). \tag{6}$$

In this case, the parameter α will be considered small, i.e. the nonlinearity is considered weak. We restrict ourselves to the first two terms in the series (6). Then for u_0, w_0 and u_1, w_1 we have the equations:

$$L_1\left(\frac{\partial^2 u_0}{\partial x^2}\right)+M_1\left(\frac{\partial^2 u_0}{\partial z^2}\right)+(L_1+M_1)\left(\frac{\partial^2 w_0}{\partial x\partial z}\right)=\rho_1\frac{\partial^2 u_0}{\partial t_2}, \tag{7}$$

$$(L_1+M_1)\left(\frac{\partial^2 u_0}{\partial x\partial z}\right)+M_1\left(\frac{\partial^2 w_0}{\partial x^2}\right)+L_1\left(\frac{\partial^2 w_0}{\partial x^2}\right)=\rho_1\frac{\partial^2 w_0}{\partial t_2},$$

$$L_1\left(\frac{\partial^2 u_1}{\partial x^2}\right)+M_1\left(\frac{\partial^2 u_1}{\partial z^2}\right)+(L_1+M_1)\left(\frac{\partial^2 w_1}{\partial x\partial z}\right)+F_1(u_0,w_0)=\rho_1\frac{\partial^2 u_1}{\partial t_2},$$

$$(L_1+M_1)\left(\frac{\partial^2 u_1}{\partial x\partial z}\right)+M_1\left(\frac{\partial^2 w_1}{\partial x^2}\right)+L_1\left(\frac{\partial^2 w_1}{\partial z^2}\right)+F_2(u_0,w_0)=\rho_1\frac{\partial^2 w_1}{\partial t_2}, \tag{8}$$

where $L_1=K_1R_0^{(1)}=\frac{4}{3}G_1R^{(1)}; M_1=G_1R^{(1)}$.

That problem was reduced to systems of two linear problems.

Problem (7) under boundary conditions (4) and (5) was solved in the second chapter in a three-dimensional formulation, so we will consider it solved. For example, the exact equations for the longitudinal-transverse oscillation of a plate lying on a deformable base in the first or linear approximations in a flat setting have the form:

$$M_{1(n)}(U^{(1)})+M_{2(n)}(W_1^{(1)})+M_{3(n)}(U_1^{(1)})+M_{4(n)}(W^{(1)})=M_1^{-1}f_z,$$

$$K_{1(n)}(U^{(1)})+K_{2(n)}(W_1^{(1)})+K_{3(n)}(U_1^{(1)})+K_{4(n)}(W^{(1)})=0,$$

$$D_{1(n)}(U^{(1)})+D_{2(n)}(W_1^{(1)})+D_{3(n)}(U_1^{(1)})+D_{4(n)}(W^{(1)})=0,$$

$$-K_{1(n)}(U^{(1)})-K_{2(n)}(W_1^{(1)})+K_{3(n)}(U_1^{(1)})+K_{4(n)}(W^{(1)})=0,$$

where the operators $M_{j(n)}, K_{j(n)}, D_{j(n)}$ are expressed from a system of general equations describing the longitudinal-transverse oscillation of a plate of constant thickness located in a deformable medium under the surface obtained in [5–7].

In particular, for the main part of the transverse displacement $W^{(1)}$ in the classical approximation we have the equation

$$\rho_1\frac{\partial^2 W_0^{(1)}}{\partial t^2}+\frac{h^2}{6}\left[\rho_1^2(N_1^{-1}+3M_1^{-1})-4\rho_1(3-2ML^{-1})\frac{\partial^4 W_0^{(1)}}{\partial t^2\partial x^2}+8(1-M_1L_1^{-1})\frac{\partial^4 W_0^{(1)}}{\partial x^4}\right]+$$

$$+P(W_0^{(1)}=\Phi_1(x,t),$$

where the operator P is equal to

$$P=\frac{s}{2h}\rho_1\left\{\frac{\partial}{\partial t}+\frac{h^2}{2}\left[\rho_1(M_1^{-1}+3L_1^{-1})\frac{\partial^3}{\partial t^3}-4\frac{\partial^3}{\partial t\partial x^2}\right]\right\}.$$

For equations (8), the boundary conditions have the form:

$$\text{at } z = h, \ \sigma_{zz}^{(1)} = 0; \ \sigma_{xz}^{(1)} = 0, \tag{9}$$

$$\text{at } z = -h, \ \sigma_{zz}^{(1)} = R(w_1); \ \sigma_{xz}^{(1)} = 0, \tag{10}$$

where the operator R is found after the invocation of the operator

$$R_0 = \frac{(\beta^2 + k^2 + q^2)^2 - 4\alpha_2\beta_2(k^2 + q^2)}{\alpha^2(\beta_2^2 - k^2 - q^2)}$$

for k, q, ρ (k and q are the parameters of the Fourier transform, ρ is the parameter of the Laplace transform) [8].

It should be noted that from the boundary conditions (9), namely $\sigma_{xz}^{(1)} = 0 \ w_1 = w_2$ at $z = -h$ we have eliminated the base parameters, and R is the base reaction.

Thus, we have the problem (5) of vibrations of an isotropic plate under boundary conditions (9) and (10) taking into account the physical nonlinearity of stress versus strain [9, 10].

With this formulation of the problem, we have a linear problem (8) under boundary conditions (9) and (10), and in the left parts of equation (8), there are nonlinear terms $F_1(u_0, w_0)$ and $F_2(u_0, w_0)$ depending on displacements u_0, w_0 and having the form (3). Representing displacements u_1, w_1 as

$$u_1 = \int_0^\infty \left. \begin{matrix} sinkx \\ -coskx \end{matrix} \right\} dk \int_l u_{10}\, exp\,(pt) dt$$

$$w_1 = \int_0^\infty \left. \begin{matrix} coskx \\ sinkx \end{matrix} \right\} dk \int_l w_{10}\, exp\,(pt) dt$$

for quantities u_{10}, w_{10} from equations (8) we obtain ordinary differential equations

$$M_{10}\frac{d^2u_{10}}{dz^2} - [\rho_1 p^2 + k^2 L_{10}]u_{10} - k[L_{10} + M_{10}]\frac{dw_{10}}{dz} = F_{10}(u_0, w_0) \tag{11}$$

$$L_{10}\frac{d^2w_{10}}{dz^2} - [\rho_1 p^2 + k^2 M_{10}]w_{10} - k[L_{10} + M_{10}]\frac{du_{10}}{dz} = F_{20}(u_0, w_0),$$

where F_{10} and F_{20} are Fourier and Laplace-transformed nonlinear functions $F_1(u_0, w_0)$, $F_2(u_0, w_0)$.

$$F_{10} = \int_0^\infty \left. \begin{matrix} sinkx \\ -coskx \end{matrix} \right\} dk \int_l F_1\, exp\,(pt) dp$$

$$F_{20} = \int_0^\infty \begin{matrix} coskx \\ sinkx \end{matrix} \ dk \int_l F_2\, exp\,(pt) dp.$$

General solutions of equations (11) are sought in the form

$$u_{10} = k\left[A_1 ch(\alpha z) + B_1 sh(\alpha z)\right] + \beta\left[A_2 ch(\beta z) + B_2 sh(\beta z)\right] - \frac{1}{\alpha(\beta^2 - \alpha^2)}$$

$$\int_0^z F(\xi) sh\left[\alpha(z - \xi)\right] d\xi + \frac{1}{\beta(\beta^2 - \alpha^2)} \int_0^z F(\xi) sh\left[(\beta(z - \xi)\right] d\xi$$

$$w_{10} = -\alpha\left[A_1 sh(\alpha z) + B_1 ch(\alpha z)\right] - k\left[A_2 sh(\beta z) + B_2 ch(\beta z)\right] + \frac{1}{k(\beta^2-\alpha^2)}$$

$$\int\limits_0^z F(\xi) ch\left[\alpha(z-\xi)\right] d\xi - \frac{k}{\beta^2(\beta^2-\alpha^2)}\int\limits_0^z F(\xi) ch\left[(\beta(z-\xi)\right] d\xi$$

where $F(z) = \frac{k(L_{10}+M_{10})}{L_{10}\cdot M_{10}}\frac{dF_{20}}{dz} + \frac{1}{L_{10}}\frac{d^2F_{10}}{dz^2} - \frac{\beta^2}{L_{10}}F_{10}$.

In this case, function $F(z)$ is considered to be given, and the integrals $\int\limits_0^z ch\left[\gamma(z-\xi)\right] d\xi$ and $\int\limits_0^z sh\left[\gamma(z-\xi)\right] d\xi$ can be expanded into power series.

Expanding the expressions for u_{10} and w_{10} into power series in coordinate z and introducing the main parts of the displacement according to the formulas [11]:

$$U_{10} = kA_1 + \beta A_2;\ U_{10}^{(1)} = kB_1\alpha + \beta^2 B_2$$

$$W_{10}^{(1)} = -\alpha^2 A_1 - k\beta A_2; W_0^{(1)} = -\alpha B_1 - kB_2$$

and reversing k and ρ we get:

$$u_1 = \sum_{n=0}^{\infty}\left\{\left[\lambda_1^{(n)} - \lambda_1^{(1)} c_t Q_n\right] U_1 + c_t Q_n \frac{\partial W_1^{(1)}}{\partial x} + F_{2n}^{(1)}\right\}\frac{Z^{2n}}{(2n)!} +$$

$$+\sum_{n=0}^{\infty}\left\{\left[\lambda_2^{(n)} - \frac{\partial^2}{\partial x^2} D_1 Q_n\right] U_1^{(1)} + D_1 Q_n \frac{\partial}{\partial x}\lambda_2^{(1)} W^{(1)} + F_{2n+1}^{(2)}\right\}\frac{Z^{2n+1}}{(2n+1)!},$$

$$w_1 = \sum_{n=0}^{\infty}\left\{\left[c_t \lambda_1^{(1)} Q_n\right]\frac{\partial U_1}{\partial x} + \left[\lambda^{(n)} - c_t \frac{\partial^2}{\partial x^2} Q_n\right] W_1^{(1)} + F_{2n}^{(3)}\right\}\frac{Z^{2n+1}}{(2n+1)!}$$

$$+\sum_{n=0}^{\infty}\left\{-D_1 Q_n \frac{\partial U_1^{(1)}}{\partial x} + \left[\lambda_2^{(n)} - \lambda_2^{(1)} D_1 Q_n\right] W^{(1)} + F_{2n+1}^{(4)}\right\}\frac{Z^{2n}}{(2n)!},$$

where

$$F_{2n}^{(1)} = F\left[\frac{\beta^2-k^2}{k(\beta^2-\alpha^2)} + \cdots + \frac{\alpha^{2n}(\beta^2+k^2) - 2k^2\beta^{2n}}{k(\beta^2-\alpha^2)}\right]$$

$$F_{2n+1}^{(2)} = F\left[\frac{\beta^2-k^2}{\beta^2(\beta^2-\alpha^2)} + \cdots + \frac{2\beta^{2(n+1)} - (\beta^2+k^2)\beta^{2n}}{\beta^2(\beta^2-\alpha^2)}\right]$$

$$F_{2n}^{(3)} = \frac{\partial}{\partial z}F_{2n}^{(1)}/_{Z=0};\ \ F_{2n+1}^{(4)} = \frac{\partial}{\partial z}F_{2n+1}^{(2)}/_{Z=0}.$$

Then from the boundary conditions (9) and (10) we obtain a system of four equations for U_1, $U_1^{(1)}$, $W_1^{(1)}$ and $W^{(1)}$.

$$M'_{1(n)}(U_1) + M'_{2(n)}(W_1^{(1)}) + M'_{3(n)}(U_1^{(1)}) + M_{4(n)}(W^{(1)}) = M_{5(n)}(F_{2n}^{(1,3)})$$

$$K'_{1(n)}(U_1) + K'_{2(n)}(W_1^{(1)}) + K'_{3(n)}(U_1^{(1)}) + K'_{4(n)}(W^{(1)}) = K'_{5(n)}(F_{2n}^{(1,3)})$$

$$D_{1(n)}^{(R)'}(U_1) + D_{2(n)}^{(R)'}(W_1^{(1)}) + D_{3(n)}^{(R)'}(U_1^{(1)}) + D_{4(n)}^{(R)'}(W^{(1)}) = D_{5(n)}(F_{2n}^{(i)}, F_{2n+1}^{(j)})$$

$$-K'_{1(n)}(U_1) - K'_{2(n)}(W_1^{(1)}) + K'_{3(n)}(U_1^{(1)}) + K'_{4(n)}(W^{(1)}) = -K'_{5(n)}(F_{2n+1}^{(2,4)}), \qquad (12)$$

where the operators $M'_{j(n)},\ K'_{j(n)}\ D^{(R)'}_{j(n)},\ j = \overline{1,5}$ have the form:

$$M'_{1(n)} = \sum_{n=0}^{\infty} \left\{ - \left[c_t \left(\lambda_2^{(1)} - \frac{\partial^2}{\partial x^2} \right) Q_n - (1 + c_t)\lambda_2^{(n)} \right] \right\} \frac{h^{2n}}{(2n)!}$$

$$M'_{2(n)} = \sum_{n=0}^{\infty} \left\{ \left[c_t \left(\lambda_2^{(1)} - \frac{\partial^2}{\partial x^2} \right) Q_n + (1 - c_t)\lambda_2^{(1)} \right] \right\} \frac{h^{2n}}{(2n)!}$$

$$M'_{3(n)} = \sum_{n=0}^{\infty} \left\{ \left[2\lambda_2^{(1)} D_1 Q_n \psi_n + \lambda_1^{(n)} \right] \right\} \frac{h^{2n+1}}{(2n+1)!}$$

$$M'_{4(n)} = \sum_{n=0}^{\infty} \left\{ \lambda_2^{(1)} \left(2\frac{\partial^2}{\partial x^2} F_1 Q_n + \lambda_1^{(n)} \right) \right\} \frac{h^{2n+1}}{(2n+1)!}$$

$$M'_{5(n)} = \sum_{n=0}^{\infty} \left\{ -F_{2n}^{(1)} \frac{h^{(2n)}}{(2n)!} + F_{2n}^{(3)} \frac{h^{2n+1}}{(2n+1)!} \right\}$$

$$K'_{1(n)} = \sum_{n=0}^{\infty} = \left\{ - \left[Q_n \left(\lambda_1^{(1)} + \lambda_2^{(1)} \right) + \lambda_1^{(n)} \right] \right\} \frac{h^{2n+1}}{(2n+1)!}$$

$$K'_{2(n)} = \sum_{n=0}^{\infty} \left\{ \left[2\lambda_1^{(1)} Q_1 c_t + (1 + c_t)\lambda_2^{(n)} \right] \right\} \frac{h^{2n+1}}{(2n+1)!}$$

$$K'_{3(n)} = \sum_{n=0}^{\infty} \left\{ \left[\left(\lambda_2^{(1)} + \frac{\partial^2}{\partial x^2} \right) D_1 Q_n + \lambda_1^{(n)} \right] \right\} \frac{h^{2n}}{(2n)!}$$

$$K'_{4(n)} = \sum_{n=0}^{\infty} \left\{ \left(\lambda_2^{(1)} - \frac{\partial^2}{\partial x^2} \right) D_1 Q_n - \lambda_1^{(n)} \right\} \frac{h^{2n}}{(2n)!}$$

$$K'_{5(n)} = \sum_{n=0}^{\infty} \left\{ -F_{2n+1}^{(2)} \frac{h^{(2n+1)}}{(2n+1)!} + F_{2n+1}^{(4)} \frac{h^{2n}}{(2n)!} \right\}$$

$$D^{(R)'}_{1(n)} = \sum_{n=0}^{\infty} \left\{ \left[(1 + c_t)\lambda_2^{(n)} - c_t \left(\lambda_2^{(1)} - \frac{\partial^2}{\partial x^2} \right) Q_n \right] \frac{h^{2n}}{(2n)!} + R \left[c_t \lambda_1^{(1)} Q_n \frac{\partial}{\partial x} \right] \frac{h^{(2n+1)}}{(2n+1)!} \right\}$$

$$D^{(R)'}_{2(n)} = \sum_{n=0}^{\infty} \left\{ \left[c_t \left(\lambda_2^{(1)} - \frac{\partial^2}{\partial x^2} \right) Q_n + (1 - c_t)\lambda_2^{(n)} \right] \frac{h^{2n}}{(2n)!} + R \left[\lambda_1^{(n)} - c_t \frac{\partial^2}{\partial x^2} Q_n \right] \frac{h^{(2n+1)}}{(2n+1)!} \right\}$$

$$D^{(R)'}_{3(n)} = -\sum_{n=0}^{\infty} \left\{ \left[2\lambda_2^{(1)} D_1 Q_n + \lambda_1^{(n)} \right] \frac{h^{2n+1}}{(2n+1)!} + R \left[-D_1 Q_n \frac{\partial}{\partial x} \right] \frac{h^{(2n)}}{(2n)!} \right\}$$

$$D^{(R)'}_{4(n)} = -\sum_{n=0}^{\infty} \left\{ \left[\lambda_2^{(1)} \left(2\frac{\partial}{\partial x^2} D_1 Q_n + \lambda_1^{(n)} \right) \right] \frac{h^{(2n+1)}}{(2n+1)!} + R \left[\lambda_2^{(n)} - \lambda_2^{(1)} D_1 Q_n \right] \frac{h^{2n}}{(2n)!} \right\}$$

$$D^{(R)'}_{5(n)} = \sum_{n=0}^{\infty} \left\{ [R F_{2n+1}^{(4)} - F_{2n}^{(1)}] \frac{h^{2n}}{(2n)!} - (1 + R) F_{2n}^{(3)} \frac{h^{2n+1}}{(2n+1)!} \right\}$$

The system of equations (12) are the equations of the longitudinal-transverse oscillation of a plate in a non-linear formulation, lying on a deformable foundation in the first approximation.

2 Conclusions

Thus, the boundary-value problem of plate oscillations, taking into account the physical nonlinearity of stresses, is reduced to solving integrodifferential equations, under given boundary and initial conditions.

A general formulation of the boundary value problem of vibrations of isotropic plates in a nonlinear formulation, lying on a deformable foundation, is given. To solve specific problems, instead of exact equations, it is advisable to use approximate ones, which include one or another finite order in derivatives: such approximate equations can be easily obtained from exact equations, limited to a finite number of first terms. If the nonlinear dependence on the stress intensity does not depend, i.e. parameter $\gamma_0 = 0$, the obtained results are greatly simplified. Of theoretical and applied interest is the problem of the effects of a normal load on the surface of an elastic plate lying on an absolutely rigid half-space with ideal contact between them. As above, it is assumed that the dependencies of stresses on strains are non-linear (physical non-linearity).

Due to the ideality of the contact, the desired displacements of the points of the plate are symmetrical with respect to displacement u and antisymmetric with respect to displacement v.

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Физикалық бейсызықты негізіндегі шекаралық шарттардағы интегралдық-дифференциалдық теңдеулердің шеттік есептері

Физикалық бейсызықты негізінде, шекаралық шарттарда интегралдық-дифференциалдық теңдеулерді шешу кезінде жазық элементтің шеттеріндегі әртүрлі шекаралық шарттармен байланысты тербелістердің шеттік есептерінің кең класы туындайды. Стационарлы емес сыртқы әсерлерді есепке алғанда, температураны, алдын ала кернеуді және басқа факторларды ескере отырып, жазық элементтің табиғи тербелістерінің жиілігі негізгі параметрлердің негізгісі болып табылады. Күрделі факторларды ескере отырып, мұндай проблемаларды зерттеу өте күрделі мәселелерді шешу жолына әкеледі. Бұл есептерді шешудің қиындығы теңдеулердің түріне және әртүрлілігіне байланысты. Жазық элементтер тербелістерінің шекаралық есептері бойынша алдыңғы жасалған жұмыстардың нәтижелері талданған. Жазық элементтің шеттеріндегі мүмкін болатын шекаралық шарттар мен меншікті және мәжбүрлі тербелістердің дербес есептерін шешуге қажетті бастапқы шарттар және басқа да есептер қарастырылады. Бұл теңдеулер жиыны, шекаралық және бастапқы шарттар жазық элемент үшін тербелістердің әртүрлі шекаралық есептерін құрастыруға және шешуге мүмкіндік береді. Осы жұмыста берілген пластина түріндегі жазық элементтің тербелістерінің теңдеулері жалпақ элемент материалдарының тұтқырлық әрекетін сипаттайтын тұтқыр серпімді операторларды қамтиды. Тербелістер мен толқындық процестерді зерттеуде тұтқыр серпімді операторлардың ядроларын жүйелі түрде қабылдаған жөн, өйткені тек осындай операторлар лездік серпімділікті, содан кейін тұтқыр ағынды сипаттайды.

Кілт сөздер: физикалық бейсызық, пластиналар, тербеліс, шекаралық есептер, толқындық процесс, изотропты қалақшалар, интегралдық-дифференциалдық теңдеу, жуықтық теңдеу, сызықтық емес операторлар.

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Краевые задачи интегро–дифференциальных уравнений при граничных условиях с учетом физической нелинейности

При решении интегро-дифференциальных уравнений при граничных условиях с учетом физической нелинейности возникает широкий класс краевых задач колебаний, связанных с различными граничными условиями на краях плоского элемента. При учете нестационарных внешних воздействий основным из главных параметров является частота собственных колебаний плоского элемента с учетом температуры, предварительной напряженности и других факторов. Исследование таких задач, с учетом усложняющих факторов, сводится к решению достаточно сложных задач. Трудность решения данных задач обусловлена как типом уравнений, так и разнообразием. Проанализированы результаты предыдущих работ по краевым задачам колебания плоских элементов. Рассмотрены возможные граничные условия на краях плоского элемента и необходимые начальные условия для решения частных задач собственных и вынужденных колебаний и другие задачи. Совокупность уравнений, граничных и начальных условий позволяют формулировать и решать различные краевые задачи колебания для плоского элемента. Приведенные в данной работе уравнения колебания плоского элемента в виде пластинки содержат вязкоупругие операторы, описывающие вязкое поведение материалов плоского элемента. При исследовании колебания и волновых процессов ядра вязкоупругих операторов целесообразно брать регулярными, так как только такие операторы описывают мгновенную упругость, а затем вязкое течение.

Ключевые слова: физическая нелинейность, пластинки, колебания, краевые задачи, волновой процесс, изотропные пластинки, интегро-дифференциальное уравнение, приближенные уравнения, нелинейные операторы.

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