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On construction of a field of forces along given trajectories in the presence of random perturbations

In this paper, a force field is constructed along a given integral manifold in the presence of random perturbing forces. In this case, two types of integral manifolds are considered separately: 1) trajectories that depend on generalized coordinates and do not depend on generalized velocities, and 2) trajectories that depend on both generalized coordinates and generalized velocities. The construction of the force field is carried out in the class of second-order stochastic Ito differential equations. It is assumed that the functions in the right-hand sides of the equation must be continuous in time and satisfy the Lipschitz condition in generalized coordinates and generalized velocities. Also this functions satisfy the condition for linear growth in generalized coordinates and generalized velocities.These assumptions ensure the existence and uniqueness up to stochastic equivalence of the solution to the Cauchy problem of the constructed equations in the phase space, which is a strictly Markov process continuous with probability 1. To solve the two posed problems, stochastic differential equations of perturbed motion with respect to the integral manifold are constructed. Moreover, in the case when the trajectories depend on generalized coordinates and do not depend on generalized velocities, the second order equations of perturbed motion are constructed, and in the case when the trajectories depend on both generalized coordinates and generalized velocities, the first order equations of perturbed motion are constructed. And further, in both cases by Erugin's method necessary and sufficient conditions for solving the posed problems are derived.

Keywords: stochastic differential equations, inverse problems, stability, integral manifold.

Introduction

The theory of inverse problems of differential systems in the class of ordinary differential equations is quite fully developed in [1–6, etc.]. And set of ordinary differential equations is constructed along a given integral curve in [1]. This work later turned out to be fundamental in the formation and development of the theory of inverse problems of the dynamics of systems described by ordinary differential equations. Formulations, classification of inverse problems of differential systems are stated and general methods of their solving in the class of ordinary differential equations are developed in [2–6]. We also note [7–9], in which inverse problems of dynamics of automatic control systems are considered in the class of ordinary differential equations. Methods for solving inverse problems in the class of ordinary differential equations are generalized to the class of Ito stochastic differential equations in [10–14]. In this paper, the results of [15, 16], obtained in the class of ordinary differential equations, are extended to the class of Ito stochastic differential equations.

1 The problem of construction of a force field along given trajectories (independent of velocities) in the presence of random perturbations

Let the trajectory

$$\Lambda : \lambda(x, y, t) = 0, \text{ где } \lambda = \lambda(x, y, t) \in C^{222}_{xyt},\ \lambda \in R^1 \tag{1.1}$$

be given. It is required to construct a force field in the presence of random perturbing forces so that the constructed force field has a given trajectory as an integral manifold

$$\begin{cases} \ddot{x} = X_1(x, y, t) + \widehat{\sigma}_1(x, y, t)\dot{\xi}, \\ \ddot{y} = Y_1(x, y, t) + \widetilde{\sigma}_1(x, y, t)\dot{\xi}, \end{cases} \tag{1.2}$$

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here $\xi = \xi(t, \omega)$ is random process with independent increments, which, following [17], can be represented as a sum $\xi = \xi_0 + \int c(\mu) P^0(t, d\mu)$, where ξ_0 is a Wiener process and P^0 is a Poisson process. $P^0(t, d\mu)$ is the number of jumps of P^0 on the interval $[0, t]$, that fall on the set $d\mu$. $c(\mu)$ is a scalar function mapping the space R^2 into the space R^1 of values of the process $\xi(t)$ for any t.

Definition 1. A function $f(z, t)$ belongs to the class K, $f \in K$, if z is continuous in t, $t \in [0, \infty]$, and is Lipschitz continuous in x and y $\|f(z,t) - f(\widetilde{z}, t\| \leq B\|z - \widetilde{z}\|$ in the entire space $z = (x, y)^T \in R^2$ and satisfies the condition $\|f(z,t)\| \leq B(1 + \|z\|)$ of linear growth with respect to z with some constant B.

It is assumed that, $X_1(x,y,t)$, $Y_1(x,y,t)$, $\widehat{\sigma}_1(x,y,t)$ and $\widetilde{\sigma}_1(x,y,t)$, belong to the class K, which ensures the existence and uniqueness in the space R^4 up to stochastic equivalence of solution $(x(t), y(t), \dot{x}(t), \dot{y}(t))^T$ of the system of equations (1.1) with the initial condition $(x(t_0), y(t_0), \dot{x}(t_0), \dot{y}(t_0))^T = (x_0, y_0, \dot{x}_0, \dot{y}_0)^T$ which is continuous with probability 1 strictly Markov process [17].

The projections of the velocity of a material point $\dot{x}, \dot{y}$ onto the coordinate axes x, y are determined from the equation

$$\dot{\lambda} = \lambda_t + \lambda_x \dot{x} + \lambda_y \dot{y}.$$

Differentiating the last expression with respect to time, we obtain

$$\ddot{\lambda} = \lambda_{tt} + (\lambda_{tx} + \lambda_{xx}\dot{x} + \lambda_{xy}\dot{y})\dot{x} + \lambda_x \ddot{x} + (\lambda_{ty} + \lambda_{xy}\dot{x} + \lambda_{yy}\dot{y})\dot{y} + \lambda_y \ddot{y} =$$

$$= \lambda_{tt} + \lambda_{tx}\dot{x} + \lambda_{ty}\dot{y} + \lambda_{xx}\dot{x}^2 + 2\lambda_{xy}\dot{x}\dot{y} + \lambda_x(X + \widehat{\sigma}_1\dot{\xi}) + \lambda_{yy}\dot{y}^2 + \lambda_y(Y + \widetilde{\sigma}_1\dot{\xi}). \tag{1.3}$$

To ensure the integrality of set (1.1) for the system of differential equations (1.2), following Erugin's method, we introduce the vector function A and the matrix B

$$A_1 = A_1(\lambda, \dot{\lambda}; x, y, \dot{x}, \dot{y}, t), \;\; B_1 = B_1(\lambda, \dot{\lambda}; x, y, \dot{x}, \dot{y}, t)$$

with properties $A_1(0, 0; x, y, \dot{x}, \dot{y}, t) \equiv B_1(0, 0; x, y, \dot{x}, \dot{y}, t) \equiv 0$, such that

$$\ddot{\lambda} = A_1 + B_1\dot{\xi}. \tag{1.4}$$

In view of (1.3) and (1.4), we have

$$A_1 = \lambda_{tt} + \lambda_{tx}\dot{x} + \lambda_{ty}\dot{y} + \lambda_{xx}\dot{x}^2 + 2\lambda_{xy}\dot{x}\dot{y} + \lambda_{yy}\dot{y}^2 + \lambda_x X + \lambda_y Y,$$

$$B_1 = \lambda_x\widehat{\sigma}_1 + \lambda_y\widetilde{\sigma}_1.$$

Let $\lambda_x^2 + \lambda_y^2 \neq 0$ takes place, then
a) if $\lambda_x \neq 0$ for any x, y, then

$$\begin{cases} X_1 = \lambda_x^{-1}(A_1 - \lambda_{xx}\dot{x}^2 - 2\lambda_{xy}\dot{x}\dot{y} - \lambda_{yy}\dot{y}^2 - \lambda_y Y) \\ \widehat{\sigma}_1 = \lambda_x^{-1}(B_1 - \lambda_y\widetilde{\sigma}_1) \end{cases} \tag{1.5}$$

for any $Y_1, \widetilde{\sigma}_1$ from the class K;
б) if $\lambda_y \neq 0$ for any x, y, then

$$\begin{cases} Y_1 = \lambda_y^{-1}(A_1 - \lambda_{xx}\dot{x}^2 - 2\lambda_{xy}\dot{x}\dot{y} - \lambda_{yy}\dot{y}^2 - \lambda_x X) \\ \widetilde{\sigma}_1 = \lambda_y^{-1}(B_1 - \lambda_x\widehat{\sigma}_1) \end{cases} \tag{1.6}$$

for any $X_1, \widehat{\sigma}_1$ from the class K.

Theorem 1. A necessary and sufficient condition that the set of force fields (1.2) has a given trajectory (1.1) in the presence of random perturbations from the class of processes with independent increments is that one of conditions (1.5) or (1.6) be satisfied.

2 The problem of construction of a force field along given trajectories (depending on velocities) in the presence of random perturbations

Let us consider the case when the given trajectory λ depends on both generalized coordinates and generalized velocities

$$\Lambda : \lambda(x, \dot{x}, y, \dot{y}, t) = 0, \text{ where } \lambda = \lambda(x, \dot{x}, y, \dot{y}, t) \in C^{12121}_{x\dot{x}y\dot{y}t}, \; \lambda \in R^1. \tag{2.1}$$

It is required to construct a force field in the presence of random perturbing forces so that the constructed force field has a given trajectory as an integral manifold

$$\begin{cases} \ddot{x} = X_2(x,y) + \widehat{\sigma}_2(x,y)\dot{\eta}, \\ \ddot{y} = Y_2(x,y,t) + \widetilde{\sigma}_2(x,y,t)\dot{\eta}, \end{cases} \tag{2.2}$$

here $\eta = \eta(t,\omega)$ is scalar Wiener process [17].

Let us compose the equation of the perturbed motion relative to a given trajectory (2.1). To do this, we differentiate (2.1) with respect to time and obtain

$$\dot{\lambda} = \lambda_t + \lambda_x \dot{x} + \lambda_{\dot{x}} \ddot{x} + \lambda_y \dot{y} + \lambda_{\dot{y}} \ddot{y} =$$

$$= \lambda_x \dot{x} + \lambda_{\dot{x}}(X_2 + \widehat{\sigma}_2 \dot{\eta}) + \lambda_y \dot{y} + \lambda_{\dot{y}}(Y_2 + \widetilde{\sigma}_2 \dot{\eta}) + \frac{1}{2}(\lambda_{\dot{x}\dot{x}}\widehat{\sigma}_2^2 + \lambda_{\dot{y}\dot{y}}\widetilde{\sigma}_2^2). \tag{2.3}$$

Further, following Erugin's method [1], we introduce the vector function A_2 the matrix B_2

$$A_2 = A_2(\lambda; x, y, \dot{x}, \dot{y}), \ \ B_2 = B_2(\lambda; x, y, \dot{x}, \dot{y})$$

with properties $A_2(0; x, y, \dot{x}, \dot{y}) = 0, \ B_2(0; x, y, \dot{x}, \dot{y}) = 0$ such that

$$\dot{\lambda} = A_2 + B_2 \dot{\eta}. \tag{2.4}$$

In view of (2.3) and (2.4), we arrive at the relations

$$A_2 = \lambda_x \dot{x} + \lambda_{\dot{x}} X + \lambda_y \dot{y} + \lambda_{\dot{y}} Y + \frac{1}{2}(\lambda_{\dot{x}\dot{x}}\widehat{\sigma}_2^2 + \lambda_{\dot{y}\dot{y}}\widetilde{\sigma}_2^2),$$

$$B_2 = \lambda_{\dot{x}}\widehat{\sigma}_2 + \lambda_{\dot{y}}\widetilde{\sigma}_2.$$

Let $\lambda_{\dot{x}}^2 + \lambda_{\dot{y}}^2 \neq 0$ takes place. Then
a) if $\lambda_{\dot{x}} \neq 0$ for any x, y, then

$$\begin{cases} X_2 = \lambda_{\dot{x}}^{-1}(A_2 - \lambda_x \dot{x} - \lambda_y \dot{y} + \lambda_{\dot{y}} Y_2 - \frac{1}{2}(\lambda_{\dot{x}\dot{x}}\widehat{\sigma}_2^2 + \lambda_{\dot{y}\dot{y}}\widetilde{\sigma}_2^2)) \\ \widehat{\sigma}_2 = \lambda_{\dot{x}}^{-1}(B_2 - \lambda_{\dot{y}}\widetilde{\sigma}_2) \end{cases} \tag{2.5}$$

for any $Y_2, \widetilde{\sigma}_2$ from K;
b) if $\lambda_{\dot{y}} \neq 0$ for any x, y, then

$$\begin{cases} Y_2 = \lambda_{\dot{y}}^{-1}(A_2 - \lambda_x \dot{x} - \lambda_{\dot{x}} X_2 + \lambda_y \dot{y} - \frac{1}{2}(\lambda_{\dot{x}\dot{x}}\widehat{\sigma}_2^2 + \lambda_{\dot{y}\dot{y}}\widetilde{\sigma}_2^2)) \\ \widetilde{\sigma}_2 = \lambda_{\dot{y}}^{-1}(B_2 - \lambda_{\dot{x}}\widehat{\sigma}_1) \end{cases} \tag{2.6}$$

for any $X_2, \widehat{\sigma}_2$ from the class K.

The following theorem holds.

Theorem 2. A necessary and sufficient condition that the set of force fields (2.2) has a given trajectory (2.1) in the presence of random perturbations from the class of Wiener processes is that one of conditions (2.5) or (2.6) be satisfied.

Conclusion

Thus, the article deals with stochastic problems of constructing a force field along given trajectories. In the first section trajectories depend on generalized coordinates and do not depend on generalized velocities. And in the second section trajectories depend on both generalized coordinates and generalized velocities. The obtained results extend Galiullin's some statements [2,3] on the construction of a force field from a given family of trajectories in the class of ordinary differential equations to the class of second-order stochastic Ito differential equations.

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М.Ы. Тілеубергенов, Г.Қ. Василина, Г.А. Түзелбаева

Кездейсоқ түрткі болғанда берілген траекториялар бойынша күштер өрісін тұрғызу туралы

Мақалада кездейсоқ түрткілеуші күштер болғанда берілген интегралдық көпбейне бойынша күштік өріс тұрғызылған. Бұл арада интегралдық көпбейненің екі түрі жеке қарастырылды: 1) жалпыланған координаттарға тәуелді және жалпыланған жылдамдықтардан тәуелсіз емес траекториялар және 2) жалпыланған координаттарға да жалпыланған жылдамдыққа да тәуелді траекториялар. Күштік өрісті тұрғызу екінші ретті Ито стохастикалық дифференциалдық теңдеулер класында жүргізіледі. Бұл арада, теңдеудің оң жағына кіретін функциялар, уақыт бойынша үзіліссіз және де жалпыланған координаттар және жалпыланған жылдамдықтар бойынша Липшиц шартын қанағаттандыратын, сонымен бірге жалпыланған координаттар және жалпыланған жылдамдықтар бойынша сызықтық өсуді қанағаттандыратын болуы болжамдалды. Бұл болжамдар фазалық кеңістікте, тұрғызылған теңдеулердің Коши есебінің шешімінің, үзіліссіз 1 ықтималдықты қатаң марковтік үдеріс болып табылатын, стохастикалық эквиваленттікке дейінгі бар болуын және жалқылығын қамтамасыз етеді. Қойылған екі есепті шешу үшін түрткіленген қозғалыстың стохастикалық дифференциалдық теңдеулері интегралдық көпбейне бойынша тұрғызылды. Жалпыланған координаттарға тәуелді және жалпыланған жылдамдықтардан тәуелсіз траекториялар жағдайында, екінші ретті түрткіленген қозғалыс теңдеулері; ал жалпыланған координаттарға да жалпыланған жылдамдықтарға да тәуелді траекториялар жағдайында, бірінші ретті түрткіленген қозғалыс теңдеулері тұрғызылады. Әрі қарай, Еругин әдісі бойынша екі жағдайда да қойылған есептердің шешілуінің қажетті және жеткілікті шарттары қорытылып шығарылды.

Кілт сөздер: стохастикалық дифференциалдық теңдеулер, кері есептер, орнықтылық, интегралдық көпбейне.

М.И. Тлеубергенов, Г.К. Василина, Г.А. Тузелбаева

О построении поля сил по заданным траекториям при наличии случайных возмущений

В статье построено силовое поле по заданному интегральному многообразию при наличии случайных возмущающих сил. При этом отдельно рассмотрены два вида интегральных многообразий: 1) траектории, зависящие от обобщенных координат и не зависящие от обобщенных скоростей, и 2) траектории, зависящие как от обобщенных координат, так и от обобщенных скоростей. Построение силового поля проводится в классе стохастических дифференциальных уравнений Ито второго порядка. При этом предполагается, что функции, входящие в правые части уравнения, должны быть непрерывными по времени и удовлетворять условию Липшица по обобщенным координатам и обобщенным скоростям, а также условию линейного роста по обобщенным координатам и обобщенным скоростям. Эти предположения обеспечивают в фазовом пространстве существование и единственность до стохастической эквивалентности решения задачи Коши построенных уравнений, являющегося непрерывным с вероятностью 1 строго марковским процессом. Для решения поставленных двух задач строятся стохастические дифференциальные уравнения возмущенного движения относительно интегрального многообразия. Причем, в случае, когда траектории зависят от обобщенных координат и не зависят от обобщенных скоростей, строятся уравнения возмущенного движения второго порядка, а в случае, когда траектории зависят как от обобщенных координат, так и от обобщенных скоростей, строятся уравнения возмущенного движения первого порядка. И далее, методом Еругина в обоих случаях выводятся необходимые и достаточные условия решения поставленных задач.

Ключевые слова: стохастические дифференциальные уравнения, обратные задачи, устойчивость, интегральное многообразие.

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