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Spectral problem for the sixth order nonclassical differential equations

In this article we investigate the correctness of boundary value problems for a sixth order quasi-hyperbolic equation in the Sobolev space

$$Lu = -D_t^6 u + \Delta u - \lambda u$$

($D_t = \frac{\partial}{\partial t}$, $\Delta = \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$ – Laplace operator, λ – real parameter). For the given operator L two spectral problems are introduced and uniqueness of these problems is established. The eigenvalues and eigenfunctions of the first spectral problem are calculated for the sixth order quasi-hyperbolic equation. In this work we show that the equation $Lu = 0$ for $\lambda < 0$ under uniform conditions has a countable set of nontrivial solutions. Usually, this does not happen when the operator L is an ordinary hyperbolic operator.

Keywords: a sixth order quasi-hyperbolic equation, eigenvalues, eigenfunctions, nontrivial solutions.

Formulation of the problem

Let Ω – be the limited area of space $\mathbb{R}^n$ variables $x_1, x_2, ..., x_n$ with smooth compact boundary $\Gamma = \partial\Omega$. Let's consider the following differential operator in the cylindrical area $Q = \Omega \times (0,T),\ S = \Gamma \times (0,T),\ \ 0 < T < +\infty$

$$Lu \equiv -\frac{\partial^6 u}{\partial t^6} + \Delta u - \lambda u = f(x,t), \quad x \in \Omega, \quad t \in (0,T), \tag{1}$$

where $f(x,t)$ is a given function.

Boundary value problem $I_{3,\lambda}$: It is required to find a function $u(x,t)$ which is a solution to equation (1) in the cylinder Q that satisfies following conditions

$$u(x,t)|_S = 0, \tag{2}$$

$$u(x,0) = \frac{\partial u}{\partial t}(x,0) = \frac{\partial^2 u}{\partial t^2}(x,0) = \frac{\partial^3 u}{\partial t^3}(x,0) = 0,\ x \in \Omega, \tag{3}$$

$$\frac{\partial u}{\partial t}(x,T) = \frac{\partial^2 u}{\partial t^2}(x,T) = 0,\ x \in \Omega. \tag{4}$$

Boundary value problem $II_{3,\lambda}$: It is required to find a function $u(x,t)$ which is a solution to equation (1) in the cylinder Q that satisfies conditions (2), (3) and

$$D_t^4 u(x,t)|_{t=T} = D_t^5 u(x,t)|_{t=T} = 0,\ x \in \Omega. \tag{5}$$

The study of the solvability of boundary value problems for quasi-hyperbolic equations began, apparently, with the works of V.N. Vragov [1, 2]. Studies in [3–7] are related to further investigations of operators similar to L. One of the main conditions for correctness in these studies was the condition that the parameter λ is non-negative. Investigations of nonlocal problems with integral conditions for linear parabolic equations, for differential equations of odd order, and for some classes of non-stationary equations have been actively carried out recently in the works of A.I. Kozhanov [4, 6, 7]. In [5], the solvability of problem (2), (3), (5) for fourth order quasi- hyperbolic equations with $p = 2$ is investigated. In the work [8] boundary value problems with normal derivatives were studied for elliptic equations of the $2l-$st order with constant real coefficients. For these problems, sufficient conditions for the Fredholm solvability of the problem are obtained and formulas for the index of this problem are given. An explicit form of the Green function of the Dirichlet problem for the model-polyharmonic equation $\Delta^l u = f$ in a multidimensional sphere was constructed in [9]. [10, 11] are devoted to an investigations of the solvability of various boundary value problems of order $0 \le k_1 < k_2 < ... < k_l \le 2l-1$ for the polyharmonic equation in a multidimensional ball.

In this paper, we describe calculation of eigenvalues $\lambda_m^{(1)}(\lambda_m^{(2)})$ of spectral problems $I_{3,\lambda}(II_{3,\lambda})$ for a sixth order quasi-hyperbolic equation and study solvability of boundary value problems $I_{3,\lambda}(II_{3,\lambda})$ for cases when λ coincides or does not coincide with $\lambda_m^{(1)}(\lambda_m^{(2)})$.

Supporting statement

We denote by V_3 – the linear set of functions $v(x,t)$, belonging to the space $L_2(Q)$ and having generalized derivatives with respect to spatial variable up to the second order inclusively belonging to the same space and with respect to the variable t up to the order 6 inclusively, with the norm

$$\|v\|_{V_3} = \left(\int_Q \Big[v^2 + \sum_{i,j=1}^n \Big(\frac{\partial^2 v}{\partial x_i \partial x_j}\Big)^2 + \Big(\frac{\partial^6 v}{\partial t^6}\Big)^2\Big] dxdt\right)^{\frac{1}{2}}.$$

Obviously, the space V_3 with this norm is a Banach space.

Let $v(x)$ be function from the space $\overset{\circ}{W}{}_2^1(\Omega)$. The following inequality is true

$$\int_\Omega v^2(x)dx \le c_0 \int_\Omega \sum_{i=1}^n v_{x_i}^2(x)dx, \tag{6}$$

where constant c_0 defined only by area Ω (see, example [12]).

For the function from the space V_3 satisfying condition (3), the following inequality holds:

$$\int_\Omega v^2(x,t_0)dx \le T^3 \int_0^T \int_\Omega v_{ttt}^2(x,t)dxdt, \quad t_0 \in [0,T], \tag{7}$$

$$\int_0^T \int_\Omega v^2(x,t)dxdt \le \frac{T^6}{8} \int_0^T \int_\Omega v_{ttt}^2(x,t)dxdt. \tag{8}$$

Let $\omega_j(x)$ be the eigenfunction of the Dirichlet problem for the Laplace operator corresponding to the eigenvalue μ_j:

$$\Delta\omega_j(x) = \mu_j \omega_j(x), \quad \omega_j(x)|_\Gamma = 0.$$

3 Main results

Theorem 1. Let $\lambda > c_1, c_1 = \min\{-\frac{1}{c_0}, -\frac{40}{T^6}\}$, c_0 from (6). Then the homogeneous boundary value problem $I_{3,\lambda}$ has only zero solution in the space V_3. On the interval $(-\infty, c_1)$ there exists a countable set of numbers $\lambda_m^{(1)}$ such that for $\lambda = \lambda_m^{(1)}$ the homogeneous boundary value problem $I_{3,\lambda}$ has a non-trivial solution.

Proof. First, we prove the uniqueness of the solution to the problem $I_{3,\lambda}$. Let $A > T$. We consider the equality

$$\int_0^T \int_\Omega (A-t)Lu \cdot u_t dxdt = 0.$$

Integrating by parts and using conditions (2), (3) we get

$$\frac{A-T}{2} \int_\Omega [u_{ttt}^2(x,T) + \sum_{i=1}^n u_{x_i}^2(x,T)]dx + \frac{5}{2}\int_0^T \int_\Omega u_{ttt}^2 dxdt+$$

$$+\frac{1}{2}\sum_{i=1}^n \int_0^T \int_\Omega u_{x_i}^2 dxdt = -\frac{\lambda(A-T)}{2}\int_\Omega u^2(x,T)dx - \frac{\lambda}{2}\int_0^T \int_\Omega u^2 dxdt = I. \tag{9}$$

When $\lambda \geq 0$ it follows from this equality that $u(x,t) \equiv 0$.

We now consider the case of negative values of λ. On the one hand due to expressions (6) and (7), there is an inequality

$$|I| = | - \frac{\lambda(A-T)}{2} \int_{\Omega} u^2(x,T)dx - \frac{\lambda}{2} \int_0^T \int_{\Omega} u^2 dxdt| \leq$$

$$\leq \frac{|\lambda|(A-T)}{2} T^3 \int_0^T \int_{\Omega} u_{ttt}^2 dxdt + \frac{|\lambda|}{2} c_0 \sum_{i=1}^{n} \int_0^T \int_{\Omega} u_{x_i}^2 dxdt. \tag{10}$$

On the other hand, due to inequalities (7) and (8) we get

$$|I| \leq \frac{|\lambda|(A-T)}{2} T^3 \int_0^T \int_{\Omega} u_{ttt}^2 dxdt + \frac{|\lambda| T^6}{2 \cdot 2^3} \int_0^T \int_{\Omega} u_{ttt}^2 dxdt.$$

If $c_1 = -\frac{1}{c_0}$, then by evaluating the right side of (9) by (10), we get

$$\frac{A-T}{2} \int_{\Omega} [u_{ttt}^2(x,T) + \sum_{i=1}^{n} u_{x_i}^2(x,T)]dx +$$

$$+ \frac{5 - |\lambda|(A-T)T^3}{2} \int_0^T \int_{\Omega} u_{ttt}^2 dxdt + \frac{1 - |\lambda| c_0}{2} \sum_{i=1}^{n} \int_0^T \int_{\Omega} u_{x_i}^2 dxdt \leq 0. \tag{11}$$

Since inequality $|\lambda| c_0 < 1$ holds and we can choose number A close to number T, the inequality

$$5 - |\lambda|(A-T)T^3 > 0$$

holds for fixed values of λ. Then, from (11) it follows that $u(x,t) \equiv 0$.

In the case of $c_1 = -\frac{40}{T^6}$, we have

$$\frac{A-T}{2} \int_{\Omega} [u_{ttt}^2(x,T) + \sum_{i=1}^{n} u_{x_i}^2(x,T)]dx +$$

$$+ \frac{40 - 8|\lambda|(A-T)T^3 - |\lambda| T^6}{2 \cdot 2^3} \int_0^T \int_{\Omega} u_{ttt}^2 dxdt + \frac{1}{2} \sum_{i=1}^{n} \int_0^T \int_{\Omega} u_{x_i}^2 dxdt \leq 0. \tag{12}$$

Since $40 - |\lambda| T^6 > 0$, then choosing again A close to the T,

$$40 - 8|\lambda|(A-T)T^3 - |\lambda| T^6 > 0$$

inequality can be achieved. Then, from (12) we also get $u(x,t) \equiv 0$.

The solution to equation (1) is sought in the form $u(x,t) = \varphi(t)\omega_j(x)$. Then function $\varphi(t)$ must be the solution to equation

$$-D_t^6 \varphi(t) + [\mu_j - \lambda]\varphi(t) = 0, \tag{13}$$

satisfying condition

$$\varphi(0) = \varphi'(0) = \varphi''(0) = \varphi'''(0) = \varphi'(T) = \varphi''(T) = 0. \tag{14}$$

a) If $\mu_j - \lambda > 0$, then general solution (13) has the form

$$\varphi(t) = C_1 e^{\gamma_j t} + C_2 e^{\frac{\gamma_j t}{2}} cos \frac{\sqrt{3}}{2} \gamma_j t + C_3 e^{\frac{\gamma_j t}{2}} sin \frac{\sqrt{3}}{2} \gamma_j t +$$

$$+ C_4 e^{-\gamma_j t} + C_5 e^{-\frac{\gamma_j t}{2}} cos \frac{\sqrt{3}}{2} \gamma_j t + C_6 e^{-\frac{\gamma_j t}{2}} sin \frac{\sqrt{3}}{2} \gamma_j t, \tag{15}$$

where $\gamma_j = (\mu_j - \lambda)^{\frac{1}{6}}$. Taking in account (14), the numbers $C_j, j = \overline{1,6}$, should be a solution to an algebraic system

$$\begin{cases} C_1+C_2+C_4+C_5=0, \\ C_1+\frac{1}{2}C_2+\frac{\sqrt{3}}{2}C_3-C_4-\frac{1}{2}C_5+\frac{\sqrt{3}}{2}C_6=0, \\ C_1-\frac{1}{2}C_2+\frac{\sqrt{3}}{2}C_3+C_4-\frac{1}{2}C_5-\frac{\sqrt{3}}{2}C_6=0, \\ C_1-C_2-C_4+C_5=0, \\ E^2C_1+E(\frac{1}{2}C-\frac{\sqrt{3}}{2}S)C_2+E(\frac{\sqrt{3}}{2}C+\frac{1}{2}S)C_3- \\ E^{-2}C_4-E^{-1}(\frac{1}{2}C+\frac{\sqrt{3}}{2}S)C_5+E^{-1}(\frac{\sqrt{3}}{2}C-\frac{1}{2}S)C_6=0, \\ E^2C_1-E(\frac{1}{2}C+\frac{\sqrt{3}}{2}S)C_2+E(\frac{\sqrt{3}}{2}C-\frac{1}{2}S)C_3+ \\ E^{-2}C_4+E^{-1}(-\frac{1}{2}C+\frac{\sqrt{3}}{2}S)C_5-E^{-1}(\frac{\sqrt{3}}{2}C+\frac{1}{2}S)C_6=0, \end{cases}$$

where

$$E=e^{\frac{\gamma_j T}{2}}, C=cos\frac{\sqrt{3}}{2}\gamma_j T, S=sin\frac{\sqrt{3}}{2}\gamma_j T.$$

The determinant of this system will be equal to

$$D(\gamma_j)=\frac{3}{2}\left[2E^3C-3E^2-6EC+10+4C^2-6E^{-1}C-3E^{-2}+2E^{-3}C\right],$$

and it can not be zero, therefore, in this case, problem (13), (14) have not non-trivial solutions.

b) If $\mu_j-\lambda<0$, then general solution (13) has a form

$$\varphi(t)=C_1e^{\frac{\sqrt{3}}{2}\gamma_j t}cos\frac{\gamma_j t}{2}+C_2e^{\frac{\sqrt{3}}{2}\gamma_j t}sin\frac{\gamma_j t}{2}+C_3e^{-\frac{\sqrt{3}}{2}\gamma_j t}cos\frac{\gamma_j t}{2}+$$
$$+C_4e^{-\frac{\sqrt{3}}{2}\gamma_j t}sin\frac{\gamma_j t}{2}+C_5cos\gamma_j t+C_6sin\gamma_j t, \quad (16)$$

where $\gamma_j=(\lambda-\mu_j)^{\frac{1}{6}}$. Considering (14), the number $C_j, j=\overline{1,6}$, should be a solution to an algebraic system

$$\begin{cases} C_1+C_3+C_5=0, \\ \frac{\sqrt{3}}{2}C_1+\frac{1}{2}C_2-\frac{\sqrt{3}}{2}C_3+\frac{1}{2}C_4+C_6=0, \\ \frac{1}{2}C_1+\frac{\sqrt{3}}{2}C_2+\frac{1}{2}C_3-\frac{\sqrt{3}}{2}C_4-C_5=0, \\ C_2+C_4-C_6=0, \\ E(\frac{\sqrt{3}}{2}C-\frac{1}{2}S)C_1+E(\frac{1}{2}C+\frac{\sqrt{3}}{2}S)C_2-E^{-1}(\frac{\sqrt{3}}{2}C+\frac{1}{2}S)C_3+ \\ +E^{-1}(\frac{1}{2}C-\frac{\sqrt{3}}{2}S)C_4-2CSC_5+(C^2-S^2)C_6=0, \\ E(\frac{1}{2}C-\frac{\sqrt{3}}{2}S)C_1+E(\frac{\sqrt{3}}{2}C+\frac{1}{2}S)C_2+E^{-1}(\frac{1}{2}C+\frac{\sqrt{3}}{2}S)C_3+ \\ +E^{-1}(-\frac{\sqrt{3}}{2}C+\frac{1}{2}S)C_4+(-C^2+S^2)C_5-2CSC_6=0, \end{cases}$$

where $E=e^{\frac{\sqrt{3}}{2}\gamma_j T}, C=cos\frac{\gamma_j T}{2}, S=sin\frac{\gamma_j T}{2}$.

This system has a nontrivial solution if the determinant

$$D(\gamma_j)=-C^2S^2=-\frac{1}{4}sin^2\gamma_j T=0, \quad (17)$$

is equal to zero. From (17) we get desired set of eigenvalues

$$\lambda_{jk}^{(1)}=\mu_{jk}+\left(\frac{k\pi}{T}\right)^6, \quad k=1,2,... \quad (14)$$

The theorem 1 is proved.

Consequence 1. The problem $I_{3,\lambda}$ does not have real eigenvalues other than the numbers $\lambda_{jk}^{(1)}$ from (18) and the family $\{\lambda_{jk}^{(1)}\}_{j,k=1}^{\infty}$ does not have finite limit points. All eigenvalues of $\{\lambda_{jk}^{(1)}\}_{j,k=1}^{\infty}$ are finite multiplicity.

Proof. The fact that the problem $I_{3,\lambda}$ does not have real eigenvalues other than the numbers $\lambda_{jk}^{(1)}$, follows from the basis of the system of functions

$$\{\omega_j(x)\}_{j=1}^{\infty}$$

in space $W_2^2(\Omega)$.

Suppose that the family $\{\lambda_{jk}^{(1)}\}_{j,k=1}^{\infty}$ has a finite limit point. Then there is a family (j_i,k_i) of pairs of natural numbers such that $j_i+k_i\to\infty$ such $i\to\infty$ and the sequence $\lambda_{jk}^{(1)}$ will be fundamental. Note that the indices

j_i, cannot be limited together, since in this case $\lambda_{jk}=\mu_{jk}+\left(\frac{k\pi}{T}\right)^6, \quad k=1,2,...,$ which cannot be true for a fundamental sequence.

Further, the indices k_i also cannot be limited together, since in this case the sequence $\{\mu_{j_i}-\mu_{j_{i+m}}\}$, will be limited, which is not the case. Therefore, for the indices j_i and k_i, $j_i\to\infty$, $k_i\to\infty$ hold for $i\to\infty$. But then $\lambda_{j_k k_i}\to-\infty$, which again does not hold for a fundamental sequence. From the above, the validity of second part of consequence follows. The finite multiplicity of each eigenvalue $\lambda^{(1)}_{jk}$ follows from the fact that for fixed numbers j and k the equality $\lambda^{(1)}_{jk}=\lambda^{(1)}_{j_1k_1}$ is only possible for a finite set of indices j_1 and k_1. Consequence is proved.

Note that for the case $n=1$ the eigenvalues μ_j could be in exact form, and then it is easy to give constructive conditions for the simplicity of each eigenvalue $\lambda^{(1)}_{jk}$ or to provide examples in which the eigenvalues will have a multiplicity greater than one. In the general case, it is also easy to give simplicity conditions, but it seems that they will not be constructive.

Consequence 2. The eigenvalues $\lambda^{(1)}_{jk}$ of the problem $I_{3,\lambda}$ correspond to the eigenfunctions

$$u^{(1)}_{jk}(x,t)=\omega_j(x)\varphi^{(1)}_k(t),$$

where function $\varphi^{(1)}_k(t)$ represented as

$$\varphi^{(1)}_k(t)=\frac{C}{12S_k(E_k-E_k^{-1})}\Big[-(3C_k(E_k-E_k^{-1})+5\sqrt{3}S_k(E_k+E_k^{-1})+6)e^{\frac{\sqrt{3}}{2}\gamma_k t}cos\frac{\gamma_k t}{2}-$$
$$(3\sqrt{3}C_k(E_k+E_k^{-1})-15S_k(E_k-E_k^{-1})+3\sqrt{3})e^{\frac{\sqrt{3}}{2}\gamma_k t}sin\frac{\gamma_k t}{2}+$$
$$(-3C_k(E_k+E_k^{-1})+(4+5\sqrt{3})S_k(E_k-E_k^{-1})-6)e^{-\frac{\sqrt{3}}{2}\gamma_k t}cos\frac{\gamma_k t}{2}+$$
$$(3\sqrt{3}C_k(E_k+E_k^{-1})+15S_k(E_k-E_k^{-1})-6\sqrt{3})e^{-\frac{\sqrt{3}}{2}\gamma_k t}sin\frac{\gamma_k t}{2}+$$
$$(6C_k(E_k+E_k^{-1})-6\sqrt{3}S_k(E_k-E_k^{-1})+12)cos\gamma_k t+12S_k(E_k-E_k^{-1})sin\gamma_k t\Big],$$

$$E_k=e^{\frac{\sqrt{3}\pi k}{2}},\ C_k=cos\frac{\pi k}{2},\ S_k=sin\frac{\pi k}{2},\ C=Const,\quad k=1,2,...$$

Now consider the problem II_3. The study of problem II_3 is similar to I_3. The following theorem holds.

Theorem 2. For $\lambda>c_1, c_1=\min\{-\frac{1}{c_0},-\frac{40}{T^6}\}$, the homogeneous boundary problem $II_{3,\lambda}$ has only zero solution in the space V_3. On the interval $(-\infty,c_1)$ there doesn't exist a countable set of the numbers $\lambda^{(2)}_m$ such that for $\lambda=\lambda^{(2)}_m$ homogeneous boundary problem $II_{3,\lambda}$ has only trivial solution.

The solution to equation (1) is sought in the form $u(x,t)=\varphi(t)\omega_j(x)$. Then, function $\varphi(t)$ must be solution to equation (13) that satisfy conditions

$$\varphi(0)=\varphi'(0)=\varphi''(0)=\varphi'''(0)=\varphi''''(T)=\varphi'''''(T)=0. \tag{19}$$

a) If $\mu_j-\lambda>0$, then general solution $\varphi(t)$ has a form

$$\varphi(t)=C_1e^{\gamma_j t}+C_2e^{\frac{\gamma_j t}{2}}cos\frac{\sqrt{3}}{2}\gamma_j t+C_3e^{\frac{\gamma_j t}{2}}sin\frac{\sqrt{3}}{2}\gamma_j t+$$
$$+C_4e^{-\gamma_j t}+C_5e^{-\frac{\gamma_j t}{2}}cos\frac{\sqrt{3}}{2}\gamma_j t+C_6e^{-\frac{\gamma_j t}{2}}sin\frac{\sqrt{3}}{2}\gamma_j t,$$

where $\gamma_j=(\mu_j-\lambda)^{\frac{1}{6}}$. Considering (15), $C_j, j=\overline{1,6}$, should be a solution to an algebraic system

$$\begin{cases} C_1+C_2+C_4+C_5=0,\\ C_1+\frac{1}{2}C_2+\frac{\sqrt{3}}{2}C_3-C_4-\frac{1}{2}C_5+\frac{\sqrt{3}}{2}C_6=0,\\ C_1-\frac{1}{2}C_2+\frac{\sqrt{3}}{2}C_3+C_4-\frac{1}{2}C_5-\frac{\sqrt{3}}{2}C_6=0,\\ C_1-C_2-C_4+C_5=0,\\ E^2C_1+E(-\frac{1}{2}C+\frac{\sqrt{3}}{2}S)C_2-E(\frac{\sqrt{3}}{2}C+\frac{1}{2}S)C_3+\\ E^{-2}C_4-E^{-1}(\frac{1}{2}C+\frac{\sqrt{3}}{2}S)C_5+E^{-1}(\frac{\sqrt{3}}{2}C-\frac{1}{2}S)C_6=0,\\ E^2C_1+E(\frac{1}{2}C+\frac{\sqrt{3}}{2}S)C_2+E(-\frac{\sqrt{3}}{2}C+\frac{1}{2}S)C_3-\\ E^{-2}C_4+E^{-1}(-\frac{1}{2}C+\frac{\sqrt{3}}{2}S)C_5-E^{-1}(\frac{\sqrt{3}}{2}C+\frac{1}{2}S)C_6=0, \end{cases}$$

where $E = e^{\frac{\gamma_j T}{2}}, C = cos\frac{\sqrt{3}}{2}\gamma_j T, S = sin\frac{\sqrt{3}}{2}\gamma_j T$. The determinant of this system will be equal to

$$D(\gamma_j) = -\frac{3}{2}\left[2E^3C + 3E^2 + 6EC + 10 + 4C^2 + 6E^{-1} + 3E^{-2} + 2E^{-3}C\right],$$

and it can not be zero, therefore, in this case, there are no non-trivial solutions.

b) If $\mu_j - \lambda < 0$, then function $\varphi(t)$ has a form

$$\varphi(t) = C_1 e^{\frac{\sqrt{3}}{2}\gamma_j t} cos\frac{\gamma_j t}{2} + C_2 e^{\frac{\sqrt{3}}{2}\gamma_j t} sin\frac{\gamma_j t}{2} + C_3 e^{-\frac{\sqrt{3}}{2}\gamma_j t} cos\frac{\gamma_j t}{2} +$$

$$C_4 e^{-\frac{\sqrt{3}}{2}\gamma_j t} sin\frac{\gamma_j t}{2} + C_5 cos\gamma_j t + C_6 sin\gamma_j t,$$

where $\gamma_j = (\lambda - \mu_j)^{\frac{1}{6}}$. In this case, $C_j, j = \overline{1,6}$, should be a solution to an algebraic system

$$\begin{cases} C_1 + C_3 + C_5 = 0, \\ \frac{\sqrt{3}}{2}C_1 + \frac{1}{2}C_2 - \frac{\sqrt{3}}{2}C_3 + \frac{1}{2}C_4 + C_6 = 0, \\ \frac{1}{2}C_1 + \frac{\sqrt{3}}{2}C_2 + \frac{1}{2}C_3 - \frac{\sqrt{3}}{2}C_4 - C_5 = 0, \\ C_2 + C_4 - C_6 = 0, \\ -E(\frac{1}{2}C + \frac{\sqrt{3}}{2}S)C_1 + E(\frac{\sqrt{3}}{2}C - \frac{1}{2}S)C_2 + E^{-1}(-\frac{1}{2}C + \frac{\sqrt{3}}{2}S)C_3 - \\ E^{-1}(\frac{\sqrt{3}}{2}C + \frac{1}{2}S)C_4 + (C^2 - S^2)C_5 + 2CSC_6 = 0, \\ -E(\frac{\sqrt{3}}{2}C + \frac{1}{2}S)C_1 + E(\frac{1}{2}C - \frac{\sqrt{3}}{2}S)C_2 + E^{-1}(\frac{\sqrt{3}}{2}C - \frac{1}{2}S)C_3 + \\ +E^{-1}(\frac{1}{2}C + \frac{\sqrt{3}}{2}S)C_4 - 2CSC_5 + (C^2 - S^2)C_6 = 0, \end{cases}$$

where $E = e^{\frac{\sqrt{3}}{2}\gamma_j T}, C = cos\frac{\gamma_j T}{2}, S = sin\frac{\gamma_j T}{2}$. The determinant of this system will be equal to

$$D(\gamma_j) = \frac{3}{4}\left[E^2 + 8EC^3 + 6 + 12C^2 + 8E^{-1}C^3 + E^{-2}\right],$$

also can not be zero.

In conclusion, the problem $II_{3,\lambda}$ does not have real eigenvalues $\lambda_{jk}^{(2)}$. The theorem 2 is proved.

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Алтыншы ретті классикалық емес дифференциалдық теңдеуге арналған спектрлік есеп

Мақалада С.Л. Соболев кеңістігінде алтыншы ретті квазигиперболалық теңдеу үшін шеттік есептердің

$$Lu = -D_t^6 u + \Delta u - \lambda u$$

($D_t = \frac{\partial}{\partial t}$, $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ – Лаплас операторы, λ – нақты параметр) тиянақты шешімділігі зерттелген. Берілген L операторы үшін классикалық емес дифференциалдық теңдеуге екі спектрлік есеп қарастырылған. Қойылған есептің шешімінің жалғыздығы, бірінші есептің меншікті мәндері мен функцияларының бар екендігі дәлелденген, яғни бұл есептің нөлдік емес шешімдері табылған. Авторлар $\lambda < 0$ үшін $Lu = 0$ және теңдеудің біртектілік шарты орындалғанда спектрлік есептің меншікті функцияларының нөлден өзгеше шешімдер жүйесінің бар екендігін көрсетеді. Әдетте, L операторы қарапайым гиперболалық оператор болғанда, мұндай қасиет орындалмайды.

Кілт сөздер: алтыншы ретті квазигиперболалық теңдеу, меншікті мәндер, меншікті функциялар, нөлдік емес шешімдер.

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Спектральная задача для неклассических дифференциальных уравнений шестого порядка

В статье исследована корректная разрешимость краевых задач для квазигиперболического уравнения шестого порядка в пространстве С.Л. Соболева

$$Lu = -D_t^6 u + \Delta u - \lambda u,$$

$D_t = \frac{\partial}{\partial t}$, $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ – оператор Лапласа; λ – вещественный параметр. Рассмотрены две неклассические спектральные задачи для данного оператора L и устанавливается единственность поставленных задач. Вычислены собственные значения и собственные функции поставленной первой задачи для квазигиперболического уравнения шестого порядка. Авторами показано, что уравнение $Lu = 0$ при $\lambda < 0$ и при выполнении однородных условий обладает счетным множеством нетривиальных решений. Обычно такой факт не имеет места, когда оператор L есть обычный гиперболический оператор.

Ключевые слова: квазигиперболические уравнения шестого порядка, собственные значения, собственные функции, нетривиальные решения.

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