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Bending of the rigidly fixed elastic-plastic plate

The number of plate structures include coatings, as well as box systems, working as a complete spatial system. The plate structures include systems elements of which are both rods and plates, for example, frame-panel structures with the bearing capacity of frame-panel buildings when considering the theory of bending. In the theory of elastic-plastic plates, the algorithm of calculation according to this theory, its basic formulas, the basic relations of the theory of elasticity are considered. This article discusses the bending of the rigidly fixed elastic-plastic plate. Calculations were made using the method of separation of variables. This method was implemented in the program MathCAD. Values of the laws of stress and modulus of materials, bending functions, shear components, deformation components, bending moment values, torsion and internal forces, stresses are obtained and plots of distribution functions are plotted. The values of the distribution functions of the rigidly fixed elastic-plastic plate are determined.

Keywords: plate, elastic, plastic, malleability, bending, deformation, stress.

Introduction

Plates of flagstones are called elastic cylindrical or prismatic bodies thickness of which is small comparing with two others in the size. Plate is widespread in building: panels, ferro-concrete or slab, the bases of the big buildings, flagstones, etc. Depending on foundation plates differ: round, pillow-like, triangular, quadrangular and polygonal. At present, plates are used in many technical branches: in building, in aircraft, in ship building, at assemblage of cars. Plates are the important designs in different branches of technics and in building. Therefore plate predesign is very important. To make or use any construction predesign is necessary. As the plate is the main element of many constructions predesign of bending and bearing ability is the important problem. To determine regularity of change of internal forces of elastic-plastic plates. In the calculation of the elastic, plastic, elastic-plastic plates with different bearing on the bending deflections and compare distributed functions [1, 2].

Main part

Consider a rigid-reinforced elastic-plastic plate of rectangular shape in a Cartesian coordinate system (l_1, l_2, h – plate dimensions in the direction of the coordinate axes) (Fig. 1) [3–11].

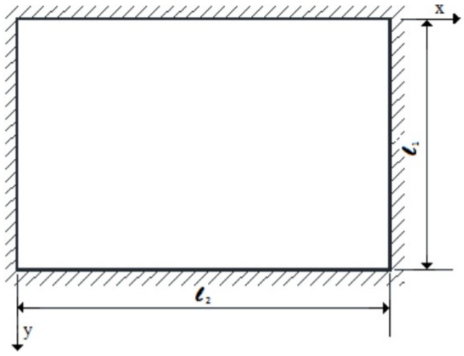


Figure 1. Rigidly fixed plate

Let it be acted q_0 on by a uniformly distributed load. The calculation is made using the method of separation of variables. This method will be implemented in the program MathCAD.

Given:

$$l_1 = 1,\ l_2 = 1,\ a_1 = 1,\ b_1 = 1,\ a_2 = 2,\ b_2 = 2,\ h = \frac{1}{2},\ E_0 = 2 \cdot 10^5,\ v = \frac{1}{4},\ q_0 = 1.$$

The boundary values of the parameters when rigidly fixed form will take the following form:

$$\alpha_0 = 0,\ \beta_0 = 0.$$

We calculate the dimensionless function of the plate bend:

$$f(x,\ y) = X(x) \cdot Y(y) = \frac{1}{24^2}\left(x^4 - 2x^3 + x^2\right) \cdot \left(y^4 - 2y^3 + y^2\right).$$

Bending parameters are defined as follows:

$$m = \frac{l_1}{l_2} = 1,\ I_1 = \frac{1}{720},\ I_2 = 0,\ J_1 = \frac{1}{720},\ J_2 = 0,$$

$$\alpha = \frac{1}{\frac{1}{m^2} \cdot J_1 + 2 \cdot I_2 \cdot J_2 + m^2 \cdot I_1} = 360.$$

Cylindrical plate stiffness:

$$D_0 = \frac{E_0 \cdot h^3}{12 \cdot (1 - v^2)} = \frac{20000}{9}.$$

The laws of stress and module functions of materials (Fig. 2):

$$\varphi(\xi) = \sin\left(\frac{\pi}{2} \cdot \xi\right),\ \ \psi(\xi) = \frac{\sin\left(\frac{\pi}{2} \cdot \xi\right)}{\xi},$$

where ξ - deformation parameter.

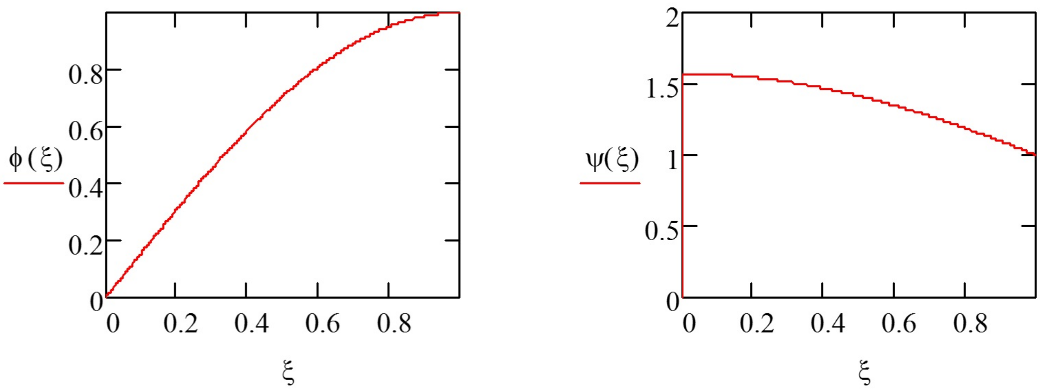


Figure 2. The laws of stress and module functions of materials

The value of the stress change laws and material functions of the module are shown in Table 1.

Table 1

The value of thestress change laws and functions of materials module

ξ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\varphi(\xi)$	0	0.156	0.309	0.454	0.588	0.707	0.809	0.891	0.951	0.988	1
$\psi(\xi)$	0	1.564	1.545	1.513	1.469	1.414	1.348	1.273	1.189	1.097	1

Using dimensionless function of bending plates, as follows: define the great bend feature $W(x, y, \xi)$ (Fig. 3):

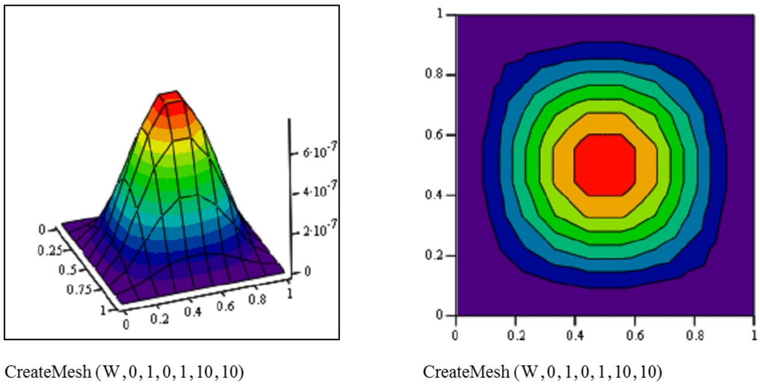


Figure 3. Bending function

Using the components of the shear, determine the strain components and their diagrams at $z = 0.5$, $\xi = 0.5$ (Fig. 4) [12]:

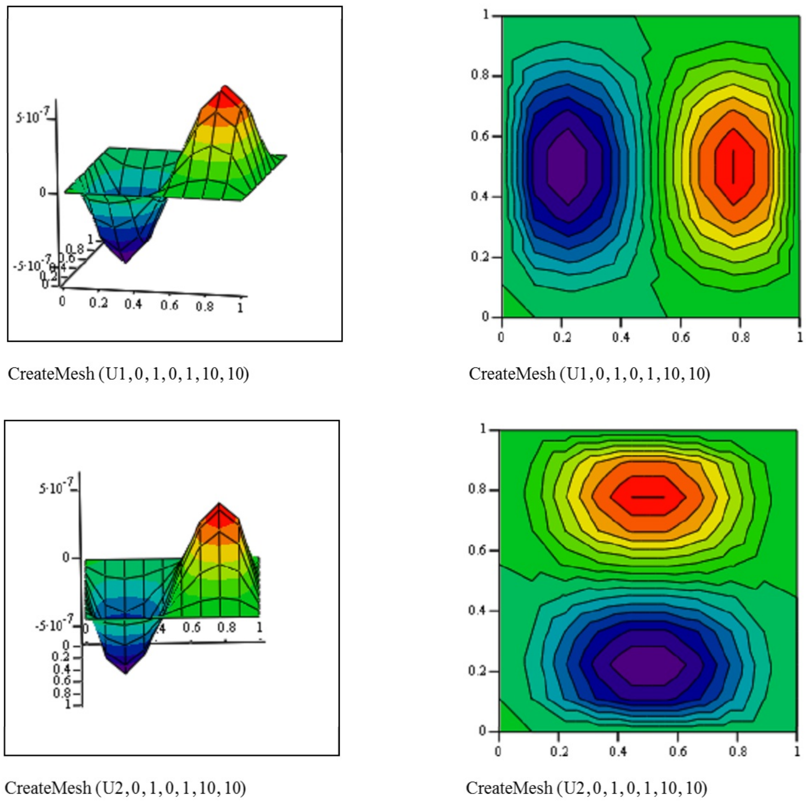
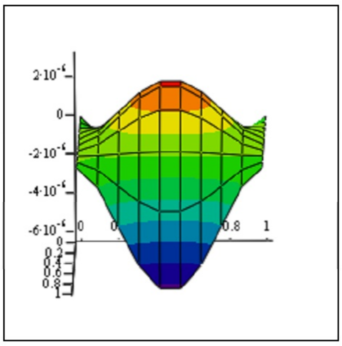
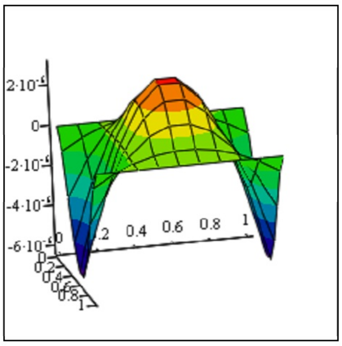


Figure 4. Shear components

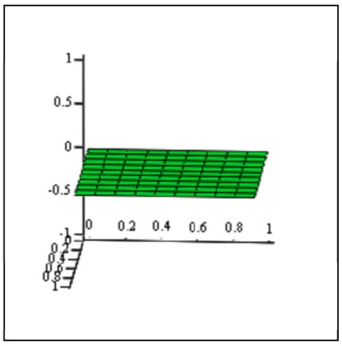
Using shear components, we define the deformation components and their diagrams at $z = 0.5$ (Fig. 5) [13]:



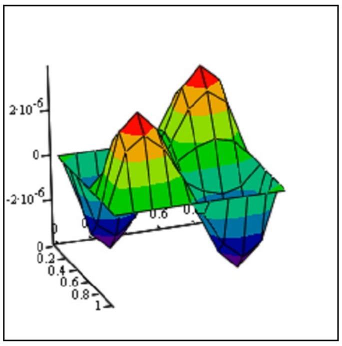
CreateMesh (ε1 , 0, 1, 0, 1, 10, 10)



CreateMesh (ε2 , 0, 1, 0, 1, 10, 10)



CreateMesh (ε3 , 0, 1, 0, 1, 10, 10)



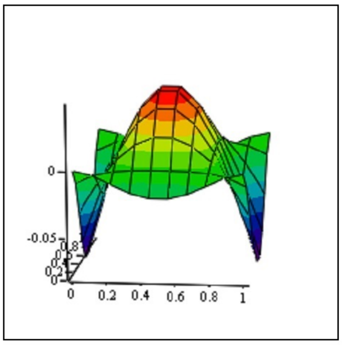
CreateMesh (γ12 , 0, 1, 0, 1, 10, 10)

Figure 5. Deformation components

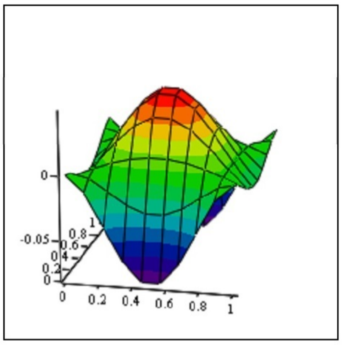
We calculate the internal forces (Fig. 6):

$$\overline{E}_0 = \frac{640000}{3};\ \overline{E}(\xi) = \frac{640000 \cdot \sin\left(\frac{\pi \cdot \xi}{2}\right)}{3 \cdot \xi};\ D(\xi) = \frac{20000 \cdot \sin\left(\frac{\pi \cdot \xi}{2}\right)}{9 \cdot \xi};$$

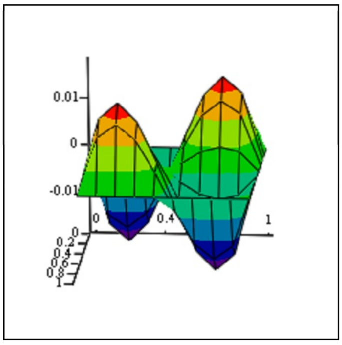
$$PU(x,\ y) = 1;\ q(x,\ y) = q_0 \cdot PU(x,\ y) = 1.$$



CreateMesh (M1, 0, 1, 0, 1, 10, 10)



CreateMesh (M2, 0, 1, 0, 1, 10, 10)



CreateMesh (M12, 0, 1, 0, 1, 10, 10)

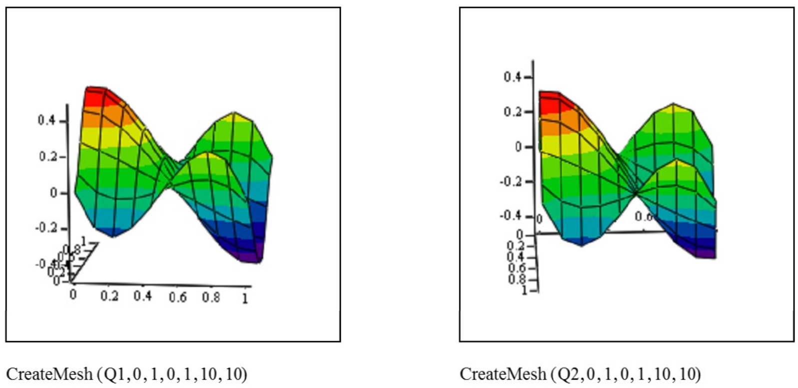


Figure 6. Moments of bending, torsion and internal forces

We find the components of stress, using the found internal forces [14–16]. At $x = 0.5$, $y = 0.5$ and $x = -0.5$, $y = -0.5$ and their diagrams are as follows (Fig. 7):

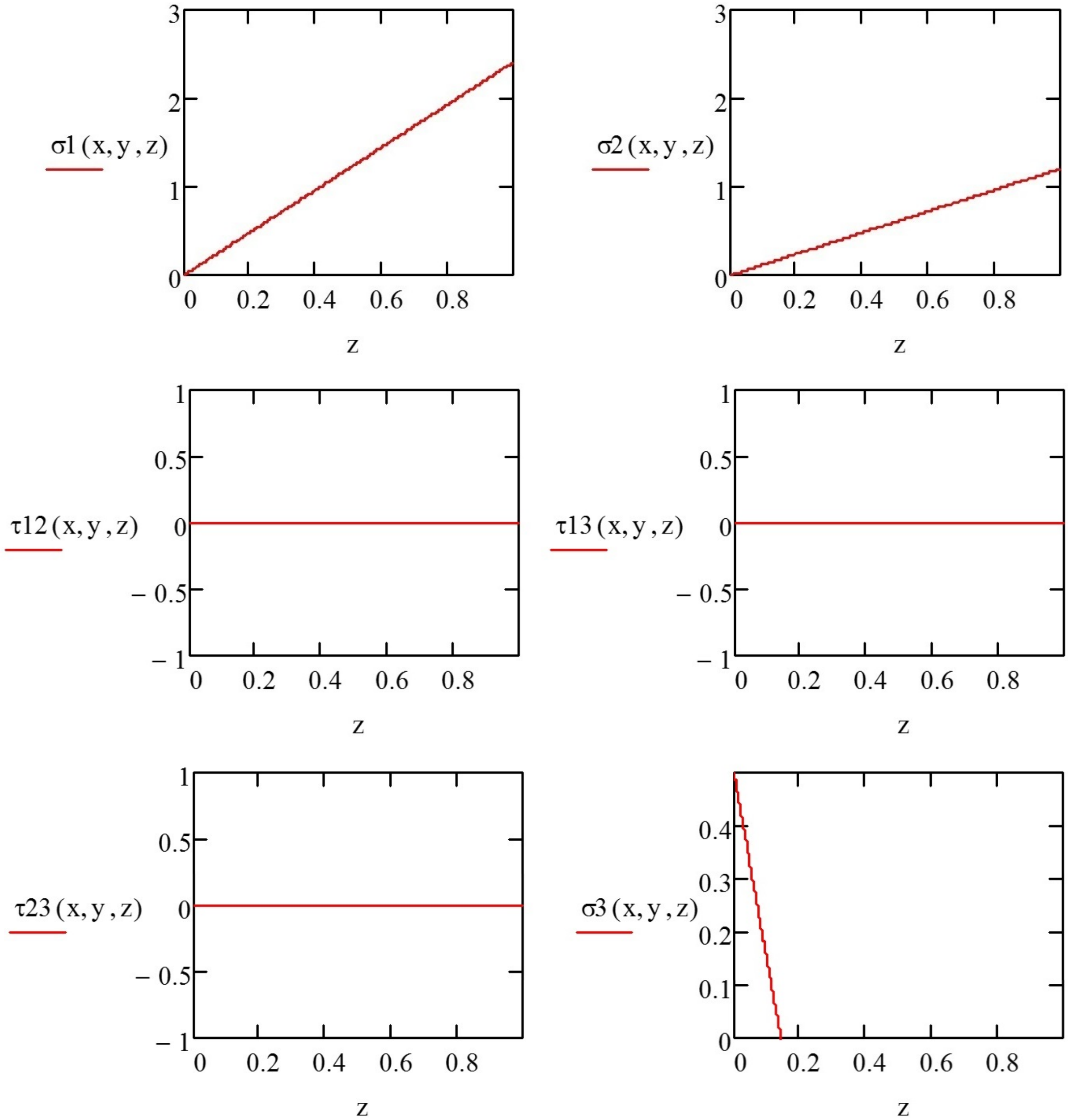


Figure 7. Stresses

Diagrams of the distribution functions (Fig. 8):

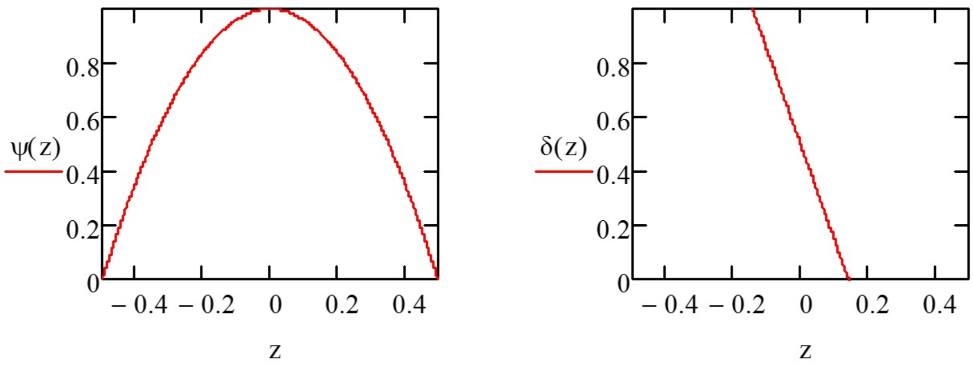


Figure 8. Distribution functions

The values of the distribution functions of the rigidly fixed elastic-plastic plate are given in Table 2 [17, 18].

Table 2

Values of distribution functions of the rigidly fixed elastic-plastic plate

$W(x,\ y,\ \xi)$	$U_1(x,\ y,z,\ \xi)$	$\varepsilon_1(x,\ y,z,\ \xi)$	$M_1(x,\ y)$	$M_{12}(x,\ y)$	$Q_1(x,\ y,\ \xi)$	$\sigma_1(x,\ y,z)$
0	0	0	-0.02	0	0	-0.469
$1.0068\cdot10^{-6}$	0	$-1.611\cdot10^{-5}$	$-3.922\cdot10^{-3}$	0	0.09	-0.094
$3.1820\cdot10^{-6}$	0	$-5.091\cdot10^{-5}$	0.015	0	0.12	0.365
$5.4815\cdot10^{-6}$	0	$-8.77\cdot10^{-5}$	0.033	0	0.105	0.783
$7.1595\cdot10^{-6}$	0	$-1.146\cdot10^{-4}$	0.045	0	0.06	1.07
$7.7685\cdot10^{-6}$	0	$-1.243\cdot10^{-4}$	0.049	0	0	1.172
$7.1595\cdot10^{-6}$	0	$-1.146\cdot10^{-4}$	0.045	0	-0.06	1.07
$5.4815\cdot10^{-6}$	0	$-8.77\cdot10^{-5}$	0.033	0	-0.105	0.783
$3.1820\cdot10^{-6}$	0	$-5.091\cdot10^{-5}$	0.015	0	-0.12	0.365
$1.0068\cdot10^{-6}$	0	$-1.611\cdot10^{-5}$	$-3.922\cdot10^{-3}$	0	-0.09	-0.094
0	0	0	-0.02	0	0	-0.469

Conclusions

Thus, this calculation was performed using elastic-plastic theory. The influence of the plate bending function in the form of polynomials in the elastic-plastic state is taken as a parameter. It is obtained analytically by applying the bending function of the beam taking into account the deformation. The law of change, which corresponds to the elastic-plastic material of stress, is used. In General, the calculation of the bending of a rigidly fixed elastic-plastic plate is shown.

It was found that many problems of the technique can be solved by the method of separating variables. Using this method simultaneously with the calculation of the plate bending, it is possible to solve the problem of free oscillations and other problems.

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Л.С. Әсетова, Б.М. Нурланова, И.А. Самойлова

Жақтары қатты бекітілген серпімді-иілімді пластинаның иілуі

Пластиналық конструкциялар қатарына жабулар, сонымен қатар тұтас кеңістіктік жүйе ретінде жұмыс істейтін қорап тәріздес жүйелер жатады. Пластиналық конструкциялар қатарына өзектер де, пластиналар да элементтер болып табылатын жүйелер кіреді, мысалы, каркасты-панельдік ғимараттардың көтеруші қабілеті бар белдік конструкциялар. Серпімді-иілімді пластинаның иілу теориясын қарастырғанда осы теория бойынша есептеу алгоритмі, оның неізгі формулалары, серпімділік теорияның негізгі қатыстары қарастырылды. Мақалада жақтары қатты бекітілген серпімді-иілімді пластинаның иілуі зерттелді. Есептеу айнымалыларды бөлу әдісінің көмегімен жүргізілген. Бұл әдіс MathCAD бағдарламасы бойынша жүзеге асырылды. Кернеу мен материал модулінің функциясының өзгеру заңдарының мәндері, майысу функциясы, жылжулар компоненттері, деформация компоненттері, иілу, бұралу моменттері және ішкі күштер, кернеулердің мәндері алынды және үлестірім функцияларының эпюралары салынды. Жақтары қатты бекітілген серпімді-иілімді пластинаның таралу функцияларының мәндері анықталды.

Кілт сөздер: пластина, серпімді, икемді, иілімді, иілу, деформация, кернеу.

Л.С. Асетова, Б.М. Нурланова, И.А. Самойлова

Изгиб жестко закрепленной упруго-пластической пластины

К числу пластинных конструкций относятся покрытия, а также коробчатые системы, работающие как целостная пространственная система. В состав пластинных конструкций входят системы, элементами которых являются как стержни, так и пластины, например, при рассмотрении теории изгиба это каркасно-панельные конструкции с несущей способностью каркасно-панельных зданий. В теории упруго-пластического изгиба пластин рассмотрены алгоритм расчета, его основные формулы, основные соотношения теории упругости. В статье рассчитан изгиб жестко закрепленной упруго-пластической пластины. Расчеты производились с помощью метода разделения переменных, реализованного с использованием программы MathCAD. Авторами получены значения законов изменения напряжений и функции модуля материалов, функции изгиба, компонентов сдвига и деформации, значения моментов изгиба, кручения и внутренних сил, напряжений. Построены эпюры функций распределения. Определены значения функций распределения жестко закрепленной упруго-пластической пластины.

Ключевые слова: пластина, упругий, пластический, податливость, изгиб, деформация, напряжение.

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