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Zh.A. Balkizov

Institute of Applied Mathematics and Automation of Kabardin-Balkar Scientific Centre of RAS, Nalchik, Russia (E-mail: Giraslan@yandex.ru)

The first boundary value problem with deviation from the characteristics for a second-order parabolic-hyperbolic equation

We pose and investigate the first boundary value problem using a model second order equation of parabolic-hyperbolic type with A.M. Nakhushev's conditions violated relative to coefficients. Despite these conditions are violated an a priori estimate similar to the a priori estimate obtained by A.M. Nakhushev takes place for solving the first boundary-value problem under study.

Keywords: equation of mixed parabolic-hyperbolic type, the first boundary value problem, a priori estimate of the solution.

Introduction

Consider the equation

$$f = Lu \equiv \begin{cases} u_{yy} - k(y)u_{xx}, & y < 0, \\ u_{yy} + u_x, & y > 0, \end{cases} \tag{1}$$

where $k(y) \geq k_1 > 0$, $f = f(x,y)$ are given functions, and $u = u(x,y)$ is an unknown one. For $y < 0$, equation (1) coincides with the Chaplygin equation [1; 21], and for $y > 0$, it is a parabolic equation backward in time (with x standing for a time variable). Thus, equation (1) is a second-order parabolic-hyperbolic equation with non-characteristic line of degeneracy [2].

A great number of scientific researches are devoted to the study of boundary-value problems for second-order parabolic-hyperbolic equations with non-characteristic line of degeneracy. For example, in [3], by the spectral method, a priori estimates in the L_p- and C-classes for solution of the Tricomi problem for an equation of the form (1), are obtained. In [4–8], boundary value problems with deviation from the characteristics for parabolic-hyperbolic equations are studied.

In [9], a method enabling one to formulate well-posed boundary value problems for a class of linear parabolic-hyperbolic equations of the form

$$Lu \equiv u_{yy} - k(x,y)u_{xx} + a(x,y)u_x + b(x,y)u_y + c(x,y)u = f(x,y), \tag{2}$$

in a bounded domain Ω with piecewise smooth boundary Σ, is given. It is assumed that the coefficients of (2) are continuous and satisfy the Nakhushev conditions, namely:

$$k_x(x,y),\ a_x(x,y),\ c_x(x,y) \in C\left(\overline{\Omega}\right), \quad f(x,y) \in L_2\left(\Omega\right), \tag{3}$$

$$k(x,y) \geq 0, \qquad \forall (x,y) \in \Omega, \tag{4}$$

$$a(x,y) > 0, \quad 2a(x,y) + k_x(x,y) \geq 0, \qquad \forall (x,y) \in \Omega_0, \tag{5}$$

where Ω_0 is the domain of parabolicity of (2). In particular, all parts of Σ which should be free from boundary data, are specified. Moreover, an a priori estimate for such a problem for (2) is obtained. The latter implies the uniqueness of a regular solution and the existance of a weak one. In [10], this problem is called the first boundary value problem for parabolic-hyperbolic equations.

In what follows, by Ω we denote the union of Ω_1, Ω_2 and J_r, i.e. $\Omega = \Omega_1 \cup \Omega_2 \cup J_r$, where

$$\Omega_1 = \{(x,y) : 0 < x < r,\ 0 < y < \varphi(x)\};$$

$$\Omega_2 = \{(x,y) : 0 < x \leq l,\ \gamma_1(x) < y < 0\} \cup \{(x,y) : l \leq x < r,\ \gamma_2(x) < y < 0\}$$

and

$$J_r = \{(x,y) : 0 < x < r,\ y = 0\}.$$

It is assumed that

$$\varphi(x) \in C^1[0,r], \quad \gamma_1(x) \in C^1[0,l], \quad \gamma_2(x) \in C^1[l,r]$$

and

$$\varphi(x) > 0, \quad \gamma_1'(x) < 0, \quad \gamma_2'(x) > 0, \qquad \gamma_1(0) = \gamma_2(r) = 0, \quad \gamma_1(l) = \gamma_2(l).$$

We will use the following notations:

$$A = A(0,0); \quad B = B(r,0); \quad C = C(l,\gamma_1(l)); \quad A_0 = A_0(0,\varphi(0)); \quad B_0 = B_0(r,\varphi(r));$$

$$\sigma_0 = \{(x,y) : 0 < x < r,\ y = \varphi(x)\}; \quad \sigma_1 = \{(x,y) : 0 < x < l,\ y = \gamma_1(x)\}$$

and

$$\sigma_2 = \{(x,y) : l < x < r,\ y = \gamma_2(x)\}.$$

We also require $f(x,y)$ and $k(y)$ to be continuous, i.e.

$$f \in C\left(\bar{\Omega}_i\right), \quad i = 1,2; \qquad k(y) \in C\left[\gamma_1(l), 0\right].$$

As it is shown in [9], the well-posedness of the first boundary-value problem for equation (1) strongly depends on the mutual location of curves σ_1 and σ_2, and the characteristics

$$AC_1 : x + \int\limits_0^y \sqrt{k(t)}dt = 0 \qquad \text{and} \qquad C_1B : x - \int\limits_0^y \sqrt{k(t)}dt = r.$$

Here, C_1 is the point of intersection of the characteristics passing through the points A and B.

In this paper, we consider equation (1) with coefficients not satisfying the Nakhushev conditions. We state and solve the first boundary value problem in the case when the curves σ_1 and σ_2 lie entirely in the characteristic triangle ABC_1. We also prove an analogue of the a priori estimate obtained in [9].

Formulation of the problem

By substitution [9]:

$$u(x,y) = \upsilon(x,y)\,\exp\left(\mu\, x\right), \tag{6}$$

the given operator Lu associated with the operator $L_\mu \upsilon$ according to the formula

$$L_\mu \upsilon = \begin{cases} \upsilon_{yy} - k(y)\,\upsilon_{xx} - 2\mu\, k(y)\,\upsilon_x - \mu^2\, k(y)\,\upsilon, & y < 0; \\ \upsilon_{yy} + \upsilon_x + \mu\upsilon, & y > 0, \end{cases}$$

where μ is some negative number. The operators Lu and $L_\mu \upsilon$ when replacing (6) will be related by

$$Lu = \exp\left(\mu x\right) L_\mu \upsilon.$$

The Regular solution of the equation

$$L_\mu \upsilon = f_\mu,\ f_\mu = \exp\left(-\mu x\right) f \tag{7}$$

in the domain Ω we call any function $\upsilon = \upsilon(x,y)$ for the class $C\left(\bar{\Omega}\right) \cap C^1\left(\Omega\right) \cap C^2\left(\Omega_1\right) \cap C^2_y\left(\Omega_2\right)$, which turns the equation into an identity upon substitution.

Problem 1. Find in the domain Ω the regular solution $\upsilon = \upsilon(x,y)$ to the equation (7) of $\upsilon(x,y) \in C^1\left(\bar{\Omega}\right) \cap C^2\left(\Omega_i\right)$, $i = \overline{1,2}$ satisfying the boundary conditions

$$\upsilon = 0 \qquad \forall\ (x,y) \in BB_0 \cup \sigma_0 \cup \sigma_2; \tag{8}$$

$$\upsilon_y = 0 \qquad \forall\ (x,y) \in \sigma_1. \tag{9}$$

Theorem to get a priori estimation of the solution to problem 1

Further, we use the following notation:

$$(u,\, \upsilon)_0 = \int\limits_{\Omega} u\, \upsilon d\Omega; \quad \|u\|_0^2 = \int\limits_{\Omega} u^2 d\Omega;$$

$$\|u\|_1^2 = \int\limits_{\Omega} \left[u^2 + u_x^2 + u_y^2\right] d\Omega.$$

The following Theorem holds true.

Theorem 1. Assume the curves σ_0, σ_1 and σ_2, restricting the domain Ω are such that they possess the following properties:

$$\sigma_0 \colon y = \varphi(x) \in C^1\,[0,\, r]\,, \text{ and besides } \varphi'(x) \le 0 \quad \forall\, x \in [0, r]; \tag{10}$$

$$\sigma_1 \,:\, y = \gamma_1(x) \in C^1\,[0,\, l]\,;\; \sigma_2 = CB \,:\, y = \gamma_2(x) \in C^1\,[l,\, r]\,; \tag{11}$$

$$-1 < \sqrt{k(y)}\,\gamma_1'(x) < 0 \qquad \forall\; (x, y) \in AC; \tag{12}$$

$$0 < \sqrt{k(y)}\,\gamma_2'(x) \le 1 \qquad \forall\; (x, y) \in CB. \tag{13}$$

Then to solve $\upsilon = \upsilon(x, y)$ of problem 1 we have the energy inequality

$$\|\upsilon\|_1 \le M_1\, \|f_\mu\|_0\,, \tag{14}$$

where the function $\upsilon = \upsilon(x, y)$ is associated with the solution $u = u(x, y)$ of original equation (1) according to the formula (6), and M_1 is the positive constant that does not depend on the sought function $\upsilon(x, y)$.

Proof. For the operator $L_\mu \upsilon$ as $y < 0$ the equality

$$2\,\delta(x)\,\upsilon_x\,L_\mu \upsilon = 2\,\delta(x)\,\upsilon_x\,\left[\upsilon_{yy} - k(y)\upsilon_{xx} - 2\mu\,k(y)\,\upsilon_x - \mu^2 k(y)\upsilon\right] =$$

$$= \frac{\partial}{\partial y}\left[2\,\delta(x)\,\upsilon_x\,\upsilon_y\right] - \frac{\partial}{\partial x}\left[\delta(x)\,\left(k(y)\upsilon_x^2 + \upsilon_y^2 + \mu^2 k(y)\upsilon^2\right)\right] +$$

$$+\delta(x)\left[\alpha\left(k(y)\upsilon_x^2 + \upsilon_y^2\right) - 4\mu\,k(y)\upsilon_x^2 + \alpha\mu^2 k(y)\upsilon^2\right],$$

holds true while as $y > 0$ we have the equality

$$2\delta(x)\,L_\mu \upsilon = 2\,\delta(x)\,\left[\upsilon_{yy} +\, \upsilon_x + \mu\upsilon\right] = \frac{\partial}{\partial x}\left[\delta(x)\,\left(\mu\upsilon^2 - \upsilon_y^2\right)\right] + \frac{\partial}{\partial y}\left[2\delta(x)\upsilon_x\upsilon_y\right] +$$

$$+\delta(x)\left[2\upsilon_x^2 + \alpha\,\upsilon_y^2 - \mu\alpha\upsilon^2\right],$$

where $\delta(x) = \exp{(\alpha x)}$, $\alpha > 0$. With the above equalities, it is easy to verify that for any function $\upsilon(x, y) \in C^1\left(\bar{\Omega}\right) \cap C^2\left(\Omega_i\right)$, $i = \overline{1, 2}$ holds

$$2\left(\delta(x)\upsilon_x,\, L_\mu \upsilon\right)_0 = \int\limits_{\Omega} 2\delta(x)\,\upsilon_x\,L_\mu \upsilon d\Omega = \int\limits_{\Omega_1} 2\delta(x)\,\upsilon_x\,L_\mu \upsilon d\Omega_1 + \int\limits_{\Omega_2} 2\delta(x)\,\upsilon_x\,L_\mu \upsilon d\Omega_2 =$$

$$= \int\limits_{\Omega_1} 2\,\delta(x)\,\upsilon_x\,\left[\upsilon_{yy} - k(y)\upsilon_{xx} - 2\mu\,k(y)\,\upsilon_x - \mu^2 k(y)\upsilon\right] d\Omega_1 +$$

$$+ \int\limits_{\Omega_2} 2\delta(x)\,\upsilon_x\,\left[\upsilon_{yy} + \upsilon_x + \mu\upsilon\right] d\Omega_2 =$$

$$= \int\limits_{\Omega_1} \frac{\partial}{\partial y}\left[2\,\delta(x)\,\upsilon_x\,\upsilon_y\right] - \frac{\partial}{\partial x}\left[\delta(x)\left(k(y)\upsilon_x^2 + \upsilon_y^2 + \mu^2 k(y)\upsilon^2\right)\right] d\Omega_1 +$$

$$+\int\limits_{\Omega_2}\frac{\partial}{\partial x}\left[\delta(x)\,\left(\mu\upsilon^2-\upsilon_y^2\right)\right]+\frac{\partial}{\partial y}\left[2\delta(x)\upsilon_x\upsilon_y\right]d\Omega_2+$$

$$+\int\limits_{\Omega_1}\delta(x)\left[\alpha\left(k(y)\upsilon_x^2+\upsilon_y^2\right)-4\mu\,k(y)\upsilon_x^2+\alpha\mu^2k(y)\upsilon^2\right]d\Omega_1+$$

$$+\int\limits_{\Omega_2}\delta(x)\left[2\upsilon_x^2+\alpha\,\upsilon_y^2-\mu\alpha\upsilon^2\right]d\Omega_2.$$

Applying the Green formula to the last equation, we obtain

$$2\left(\delta(x)\upsilon_x,\,L_\mu\upsilon\right)_0=-\int\limits_{\Gamma_1}\left[2\,\delta(x)\,\upsilon_x\,\upsilon_y\right]dx+\left[\delta(x)\left(k(y)\upsilon_x^2+\upsilon_y^2+\mu^2k(y)\upsilon^2\right)\right]dy+$$

$$+\int\limits_{\Gamma_2}\left[\delta(x)\left(\mu\upsilon^2-\upsilon_y^2\right)\right]dy-\left[2\delta(x)\upsilon_x\upsilon_y\right]dx+$$

$$+\int\limits_{\Omega_1}\delta(x)\left[(\alpha-4\mu)\,k(y)\upsilon_x^2\;+\alpha\,\upsilon_y^2+\alpha\mu^2k(y)\upsilon^2\right]d\Omega_1+$$

$$+\int\limits_{\Omega_2}\delta(x)\left[2\upsilon_x^2+\alpha\,\upsilon_y^2-\mu\alpha\upsilon^2\right]d\Omega_2=$$

$$=\int\limits_{A}^{A_0}\delta(x)\left[\upsilon_y^2\,(0,\,y)-\mu\upsilon^2\,(0,\,y)\right]dy+\int\limits_{B}^{B_0}\delta(x)\left[\mu\upsilon^2\,(r,y)-\upsilon_y^2\,(r,y)\right]dy+$$

$$+\int\limits_{A_0}^{B_0}\delta(x)\left[2\upsilon_x\upsilon_ydx+\left(\upsilon_y^2-\mu\,\upsilon^2\right)dy\right]-$$

$$-\int\limits_{A}^{C}\delta(x)\left[2\upsilon_x\,\upsilon_ydx+\left(k(y)\upsilon_x^2+\upsilon_y^2+\mu^2k(y)\upsilon^2\right)dy\right]-$$

$$-\int\limits_{C}^{B}\delta(x)\left[2\upsilon_x\,\upsilon_ydx+\left(k(y)\upsilon_x^2+\upsilon_y^2+\mu^2k(y)\upsilon^2\right)dy\right]+$$

$$+\int\limits_{\Omega_1}\delta(x)\left[(\alpha-4\mu)\,k(y)\upsilon_x^2\;+\alpha\,\upsilon_y^2+\alpha\mu^2k(y)\upsilon^2\right]d\Omega_1+$$

$$+\int\limits_{\Omega_2}\delta(x)\left[2\upsilon_x^2+\alpha\,\upsilon_y^2-\mu\alpha\upsilon^2\right]d\Omega_2=$$

$$=I_1+I_2+I_3+I_4+I_5+I_6+I_7, \tag{15}$$

where $\Gamma_1=AA_0\cup A_0B_0\cup BB_0\cup AB;\;\Gamma_2=AC\cup CB\cup AB$ — are boundaries of the domains Ω_1 and Ω_2, respectively.

Since $\mu<0$ then

$$I_1=\int\limits_{A}^{A_0}\delta(x)\;\left[\upsilon_y^2\,(0,\,y)-\mu\upsilon^2\,(0,\,y)\right]dy\geq 0$$

and due to boundary condition (8)

$$I_2 = \int\limits_{B}^{B_0} \delta(x)\ \left[\mu\, \upsilon^2\,(r,\,y) - \upsilon_y^2\,(r,\,y)\right] dy = 0.$$

Next, in view of boundary condition (8) $\upsilon(x,y)\,|_{A_0B_0} = \upsilon(x,y)\,\big|_{y=\varphi(x)} = 0$, and therefore in the line A_0B_0 the equality: $\upsilon_x + \upsilon_y\varphi'(x) = 0$ holds. Thus,

$$I_3 = \int\limits_{A_0}^{B_0} \delta(x)\left[2\upsilon_x\upsilon_y dx + \left(\upsilon_y^2 - \mu\,\upsilon^2\right) dy\right] = \int\limits_{0}^{r} \delta(x)\left[2\upsilon_x\upsilon_y + \upsilon_y^2\,\varphi'(x)\right] dx =$$

$$= \int\limits_{0}^{r} \delta(x)\left[-2\varphi'(x)\upsilon_y^2 + \varphi'(x)\upsilon_y^2\right] dx = -\int\limits_{0}^{r} \delta(x)\,\varphi'(x)\,\upsilon_y^2 dx$$

and with condition (10) for the function $y = \varphi(x)$ the integral $I_3 \geq 0$.

On the curve $\sigma_1:\ y = \gamma_1(x)$ taking into account the boundary condition (9) we have

$$I_4 = -\int\limits_{A}^{C} \delta(x)\left[2\upsilon_x\,\upsilon_y dx + \left(k(y)\upsilon_x^2 + \upsilon_y^2 + \mu^2 k(y)\upsilon^2\right) dy\right] =$$

$$= -\int\limits_{0}^{l} \delta(x)\left[2\upsilon_x\,\upsilon_y + \left(k(y)\upsilon_x^2 + \upsilon_y^2 + \mu^2 k(y)\upsilon^2\right)\ \gamma_1'(x)\right] dx =$$

$$= -k(y)\int\limits_{0}^{l} \gamma_1'(x)\,\delta(x)\left[\upsilon_x^2 + \mu^2\upsilon^2\right] dx \geq 0.$$

Similarly on the curve $\sigma_2:\ y = \gamma_2(x)$ get

$$I_5 = -\int\limits_{C}^{B} \delta(x)\left[2\upsilon_x\,\upsilon_y dx + \left(k(y)\upsilon_x^2 + \upsilon_y^2 + \mu^2 k(y)\upsilon^2\right) dy\right] =$$

$$= -\int\limits_{l}^{r} \delta(x)\left[2\upsilon_x\,\upsilon_y + \left(k(y)\upsilon_x^2 + \upsilon_y^2 + \mu^2 k(y)\upsilon^2\right)\gamma_2'(x)\right] dx =$$

$$= \int\limits_{l}^{r} \delta(x)\left[-k(y)\,\gamma_2'(x)\,\upsilon_x^2 - 2\upsilon_x\,\upsilon_y - \gamma_2'(x)\,\upsilon_y^2 - \mu^2\,k(y)\,\gamma_2'(x)\,\upsilon^2\right] dx.$$

Due to condition (8) of problem 1: $\upsilon(x,\ y)\,|_{\sigma_2} = \upsilon(x,\ y)\,\big|_{y\ =\ \gamma_2(x)} = 0$. Hence $[\upsilon_x(x,y) + \upsilon_y(x,y)\,\gamma_2'(x)]\,|_{\sigma_2=CB} = 0$. Considering this, for the integral I_5 we have

$$I_5 = \int\limits_{l}^{r} \delta(x)\left[\gamma_2'(x)\,\upsilon_y^2 - k(y)\,\gamma_2'^3(x)\,\upsilon_y^2\right] dx = \int\limits_{l}^{r} \delta(x)\,\gamma_2'(x)\ \left[1 - k(y)\,\gamma_2'^2(x)\right] \upsilon_y^2 dx.$$

By the last formula it is clear under conditions (11), (13) on the curve $\sigma_2 = CB:\ y = \gamma_2(x)$ for integral I_5 we get the inequality

$$I_5 = \int\limits_{l}^{r} \delta(x)\gamma_2'(x)\left[1 - k(y)\,\gamma_2'^2(x)\right] \upsilon_y^2 dx \geq 0.$$

Thus, under conditions (10)–(13) of theorem 1 the integrals $I_n \geq 0,\ n = \overline{1,5}$. Discarding the nonnegative integrals $I_n,\ \ n = \overline{1,5}$ by (15) we arrive at the inequality

$$2\left(\delta(x)\upsilon_x,\ L_\mu\upsilon\right)_0 \geq$$
$$\geq \int\limits_{\Omega_1} \delta(x)\left[(\alpha-4\mu)\,k(y)\upsilon_x^2 + \alpha\upsilon_y^2 + \alpha\mu^2 k(y)\upsilon^2\right] d\Omega_1 +$$
$$+ \int\limits_{\Omega_2} \delta(x)\left[2\upsilon_x^2 + \alpha\,\upsilon_y^2 - \mu\alpha\upsilon^2\right] d\Omega_2 =$$
$$= \int\limits_{\Omega} \delta(x)\left[\left(2H(y) + (\alpha-4\mu)\,k(y)H(-y)\right)\upsilon_x^2 + \alpha\upsilon_y^2\right] d\Omega +$$
$$+\alpha \int\limits_{\Omega} \delta(x)\left[\mu^2 H(-y) - \mu H(y)\right]\upsilon^2 d\Omega, \tag{16}$$

where $H(y)$ is the Heaviside function.

On the other hand, with the Cauchy-Bunyakovskii inequality, for all $\varepsilon > 0$ find

$$2\left(\delta(x)\upsilon_x, L_\mu\upsilon\right)_0 = 2\left(\sqrt{\varepsilon\delta(x)}\,\upsilon_x,\ \frac{\sqrt{\delta(x)}\,L_\mu\upsilon}{\sqrt{\varepsilon}}\right)_0 \leq$$
$$\leq \varepsilon\left\|\sqrt{\delta(x)}\upsilon_x\right\|_0^2 + C(\varepsilon)\ \left\|\sqrt{\delta(x)}\,L_\mu\upsilon\right\|_0^2. \tag{17}$$

By (16) and (17) the inequality

$$\int\limits_{\Omega} \delta(x)\left[\left(2H(y) + (\alpha-4\mu)\ k(y)\,H(-y)\right)\ \upsilon_x^2 + \right.$$
$$\left. + \alpha\upsilon_y^2 + \alpha\left(\mu^2 H(-y) - \mu H(y)\right)\ \upsilon^2\right] d\Omega \leq$$
$$\leq \varepsilon\ \left\|\sqrt{\delta(x)}\,\upsilon_x\right\|_0^2 + C(\varepsilon)\ \left\|\sqrt{\delta(x)}\,L_\mu\upsilon\right\|_0^2,$$

is implied, whence

$$\int\limits_{\Omega} \delta(x)\ \left[\left(2H(y) + (\alpha-4\mu)\ k(y)\,H(-y) - \varepsilon\right)\ \upsilon_x^2 + \alpha\upsilon_y^2 + \right.$$
$$\left. + \alpha\left(\mu^2 H(-y) - \mu H(y)\right)\ \upsilon^2\right] d\Omega \leq C_1(\varepsilon)\ \|\,L_\mu\upsilon\|_0^2, \tag{18}$$

where $C_1(\varepsilon) = \exp(\alpha r)\,C(\varepsilon)$. Select in the latter inequality the numbers $\varepsilon > 0,\ \alpha > 0,\ \mu < 0$ so that $\varepsilon < \min\left\{(\alpha-4\mu)\,k_1,\ 2\right\}$. Then for the left-hand side the following estimate holds:

$$\int\limits_{\Omega} \delta(x)\ \left[\left(2H(y) + (\alpha-4\mu)\ k(y)\,H(-y) - \varepsilon\right)\ \upsilon_x^2 + \alpha\upsilon_y^2 + \right.$$
$$\left. + \alpha\left(\mu^2 H(-y) - \mu H(y)\right)\ \upsilon^2\right] d\Omega \geq M\int\limits_{\Omega}\left(\upsilon^2 + \upsilon_x^2 + \upsilon_y^2\right) d\Omega = M\ \|\upsilon\|_1^2, \tag{19}$$

where $M = \min\left\{2-\varepsilon;\ (\alpha-4\mu)k_1 - \varepsilon;\ \alpha;\ \mu^2\alpha,\ |\mu|\alpha\right\}$.

By inequalities (18)–(19) we arrive at the a priori estimate (14). Theorem 1 is proved.

By the Theorem 1 we conclude that if $u = u(x,y)$ in the domain Ω is the solution of original equation (1) for the class $C^1\left(\bar{\Omega}\right)$ with the right-hand side $f(x,y) \in L_2(\Omega)$ satisfying the boundary conditions

$$u = 0\quad \forall\ (x,y) \in BB_0 \cup \sigma_0 \cup \sigma_2,$$

then we have the estimate

$$\int\limits_{\Omega}\left[\left(u_x - \mu u\right)^2 + u_y^2 + u^2\right] d\Omega \leq M_1^2\ \|f\|_0^2. \tag{20}$$

By the a priori estimate (20) implies the uniqueness of the regular solution of Problem 1 and the existence of a weak solution for the dual of Problem 1 for any right-hand side $f(x,y) \in L_2(\Omega)$.

We note that boundary condition (9) in the statement of Problem 1 can be replaced either by the condition $\upsilon_x = 0 \quad \forall\,(x,y) \in \sigma_1$ or the condition $\upsilon_n = 0 \quad \forall\,(x,y) \in \sigma_1$, where υ_n is the derivative of the function $\upsilon = \upsilon(x,y)$ in the direction of the outward-pointing normal to the curve $\sigma_1 = AC$.

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Ж.А. Балкизов

Гиперболану облысында сипаттаушылардан алыстаған екінші ретті парабола-гиперболалық типті моделді теңдеу үшін бірінші шеттік есеп

Екінші ретті моделді парабола-гиперболалық типті теңдеу мысалында коэффициенттеріне қатысты А.М. Нахушевтің шарттары орындалмаған жағдайы үшін бірінші шеттік есеп қойылып зерттелген. Коэффициенттеріне қатысты А.М. Нахушевтің шарттары орындалмаған жағдайға қарамастан, жұмыста зерттеліп отырған бірінші шеттік есеп үшін А.М. Нахушевтің жұмыстарында алынған априорлық бағалауға ұқсас априорлық бағалаудың орын алатыны көрсетілген.

Кілт сөздер: параболикалық-гиперболалық теңдеу, бірінші шекаралық есеп, проблеманы шешуге априорлы бағалау.

Ж.А. Балкизов

Первая краевая задача для модельного уравнения параболо-гиперболического типа второго порядка с отходом от характеристик в области гиперболичности

На примере модельного уравнения параболо-гиперболического типа второго порядка, относительно коэффициентов которого нарушены условия А.М. Нахушева, сформулирована и исследована первая краевая задача. Показано, что несмотря на то, что относительно коэффициентов рассматриваемого уравнения нарушены условия А.М. Нахушева, для решения исследуемой в работе первой краевой задачи будет иметь место априорная оценка, аналогичная априорной оценке, полученной А.М. Нахушевым.

Ключевые слова: уравнение смешанного параболо-гиперболического типа, первая краевая задача, априорная оценка решения.

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