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Well-posedness of a periodic boundary value problem for the system of hyperbolic equations with delayed argument

The periodic boundary value problem for the system of hyperbolic equations with delayed argument is considered. By method of introduction a new functions the investigated problem reduce to an equivalent problem, consisting the family of periodic boundary value problem for a system of differential equations with delayed argument and integral relations. Relationship of periodic boundary value problem for the system of hyperbolic equations with delayed argument with the family of periodic boundary value problems for the system of ordinary differential equations with delayed argument is established. Algorithms for finding solutions of the equivalent problem are constructed and their convergence is proved. Sufficient and necessary conditions of well-posedness of periodic boundary value problem for the system of hyperbolic equations with delayed argument are obtained.

Keywords: periodic boundary value problem, system of hyperbolic equations, delayed argument, family of periodic boundary value problems, system of differential equations with delayed argument, algorithm, unique solvability, well-posedness.

Introduction

Numerous problems of application such as problems of population dynamics, management of technical systems, the problem of physics, mathematical economics, ecology and etc., variational problems related to the regulatory process, the optimal control problem with delay systems leads to boundary value problems for a differential equations with deviating argument [1]. One of the rapidly growing field of the theory of differential equations with deviating argument is the theory of boundary value problems for a various classes of the differential equations with delayed argument [2, 3]. Mathematical modeling of the various processes in physics, chemistry, mechanics, biology leads to the hyperbolic differential equations with delayed argument [1], which in combination with periodic boundary conditions allow us to describe important classes of models. To investigate the questions of solvability of these classes of problems there have been applied the methods of the qualitative theory of differential equations, Riemann's method, the method of monotone iteration, asymptotic methods, the method of upper and lower solutions, numerical-analytical method and others. On their base, there have been obtained the solvability conditions for the considered problems and suggested the ways of finding solutions. Study of qualitative properties of nonlocal boundary value problems for the differential equations of hyperbolic type with delayed argument, as well as the conditions of solvability and finding solutions us associated with many problems, such as: the complexity of considered objects, the impossibility of constructing of analytical solution, the lack of universal methods of solving, difficulties with adaptation of known methods, etc. Note that the periodic boundary value problems for hyperbolic equations with delayed argument are widely used in various applications. Nevertheless, the problem of finding effective features of unique solvability of periodic boundary value problems for hyperbolic equations with delayed argument still holds actual today.

In this paper we investigate of the questions existence and uniqueness of solution to the periodic boundary value problem for the system of hyperbolic equations with delayed argument. Periodic boundary value problems for the system of hyperbolic equations with delayed argument will be reduced to the family of periodic boundary value problems for the system of ordinary differential equations with delayed argument and the integral relations. We establish a connection between conditions of the solvability to the periodic boundary value problem for the system of hyperbolic equations with delayed argument and the solvability of the family of periodic boundary value problems for the system of ordinary differential equations with delay argument.

We consider the periodic boundary value problem for the system of the hyperbolic equations second order with delay argument on the domain $\Omega_\tau = [-\tau, T] \times [0, \omega]$

$$\frac{\partial^2 u(t,x)}{\partial t \partial x} = A(t,x)\frac{\partial u(t,x)}{\partial x} + A_0(t,x)\frac{\partial u(t-\tau,x)}{\partial x} + B(t,x)\frac{\partial u(t,x)}{\partial t} + C(t,x)u(t,x) + f(t,x);$$

$$(t,x) \in [0,T] \times [0,\omega], \tag{1}$$

$$\frac{\partial u(z,x)}{\partial x} = diag\Big[\frac{\partial u(0,x)}{\partial x}\Big] \cdot \varphi(z), \quad z \in [-\tau, 0], \qquad x \in [0,\omega]; \tag{2}$$

$$u(0,x) = u(T,x), \qquad x \in [0,\omega]; \tag{3}$$

$$u(t,0) = \psi(t), \qquad t \in [-\tau, T], \tag{4}$$

where $u(t,x) = col(u_1(t,x), u_2(t,x), ..., u_n(t,x))$ is unknown function, the $(n \times n)$ matrices $A(t,x)$; $A_0(t,x)$, $B(t,x)$, $C(t,x)$ and n vector-function $f(t,x)$ are continuous on $\Omega = [0,T] \times [0,\omega]$; the n vector-function $\varphi(t)$ is continuously differentiable and given on the initial set $[-\tau, 0]$ such that $\varphi_i(0) = 1$, $i = \overline{1,n}$, $\tau > 0$ is constant delay, the n vector-function $\psi(t)$ is continuously differentiable on $[-\tau, T]$, and the compatibility condition is valid: $\psi(0) = \psi(T)$.

Let $C(\Omega_\tau, R^n)$ be the space of continuous on Ω_τ vector functions $u(t,x)$ with the norm

$$||u||_0 = \max_{(t,x)\in\Omega_\tau} ||u(t,x)||, \quad ||u(t,x)|| = \max_{i=\overline{1,n}} |u_i(t,x)|;$$

$C([0,\omega], R^n)$ be a space of continuous on $[0,\omega]$ vector functions $\varphi(x)$ with the norm

$$||\varphi||_{0,1} = \max_{x\in[0,\omega]} ||\varphi(x)||;$$

$C^1([-\tau,T], R^n)$ be a space of continuously differentiable on $[-\tau,T]$ vector functions $\psi(t)$ with the norm $||\psi||_{1,0} = \max\Big(\max\limits_{t\in[-\tau,T]} ||\psi(t)||, \max\limits_{t\in[-\tau,T]} ||\dot{\psi}(t)||\Big)$;

$$\Omega_0 = \{(t,x) : t = 0, 0 \le x \le \omega\}.$$

The function $u(t,x) \in C(\Omega_\tau, R^n)$, that has partial derivatives $\frac{\partial u(t,x)}{\partial x} \in C(\Omega_\tau, R^n)$, $\frac{\partial u(t,x)}{\partial t} \in C(\Omega_\tau \backslash \Omega_0, R^n)$, $\frac{\partial^2 u(t,x)}{\partial t \partial x} \in C(\Omega_\tau \backslash \Omega_0, R^n)$ is called a *classical solution* to periodic boundary value problem (1)–(4) if it satisfies system (1) for all $(t,x) \in \Omega$ and the condition (2) in the initial set $[-\tau, 0]$, the boundary conditions (3), (4).

1 Problem (1)–(4) and its relationship with the family of periodic boundary value problems for ordinary differential equations with delayed argument

In [4] a nonlocal boundary value problem with an integral condition in time was studied for the system of hyperbolic equations. By introducing new functions the problem was reduced to the equivalent problem consisting of a family of boundary value problems with an integral condition for ordinary differential equations and integral relations. Family of boundary value problems with integral conditions for ordinary differential equations was solved by the parametrization method. Necessary and sufficient conditions for the well-posedness of nonlocal boundary value problem with an integral condition for a system of hyperbolic equations were set. The results of [4] will be developed to the periodic boundary value problems for differential equations of hyperbolic type with delayed argument. There will be established the conditions of unique solvability of the considered problem.

We introduce a new unknown functions $v(t,x) = \frac{\partial u(t,x)}{\partial x}$ and $w(t,x) = \frac{\partial u(t,x)}{\partial t}$ and reduce problem (1)–(4) to the equivalent problem

$$\frac{\partial v(t,x)}{\partial t} = A(t,x)v(t,x) + A_0(t,x)v(t-\tau,x) + F(t,x,w(t,x),u(t,x)), \qquad (t,x) \in \Omega; \tag{1.1}$$

$$v(z,x) = diag[v(0,x)] \cdot \varphi(z), \quad z \in [-\tau,0], \qquad x \in [0,\omega]; \tag{1.2}$$

$$v(0,x) = v(T,x), \qquad x \in [0,\omega]; \tag{1.3}$$

$$u(t,x) = \psi(t) + \int_0^x v(t,\xi)d\xi, \qquad w(t,x) = \dot{\psi}(t) + \int_0^x \frac{\partial v(t,\xi)}{\partial t} d\xi, \tag{1.4}$$

where $F(t,x,w(t,x),u(t,x)) = B(t,x)w(t,x) + C(t,x)u(t,x) + f(t,x)$.
In the problem (1.1)–(1.4) the condition $u(t,0) = \psi(t)$ is taken into account in relation (1.4).

A triple $\{v(t,x), u(t,x), w(t,x)\}$ of functions is called a *solution* to problem (1.1)–(1.4) if the function $v(t,x)$ belonging to $C(\Omega_\tau, R^n)$ has a continuous derivative with respect to t on $\Omega_\tau \backslash \Omega_0$ and satisfies the one-parameter family of periodic boundary value problems for ordinary differential equations with delayed argument (1.1)–(1.3), where the functions $u(t,x)$ and $w(t,x)$ are connected with $v(t,x)$ and $\frac{\partial v(t,x)}{\partial t}$ by the integral relations (1.4).

Let $u^*(t,x)$ be a classical solution of problem (1)–(4). Then the triple $\{v^*(t,x),\ u^*(t,x),\ w^*(t,x)\}$, where $v^*(t,x) = \frac{\partial u^*(t,x)}{\partial x}$, $w^*(t,x) = \frac{\partial u^*(t,x)}{\partial t}$, is a solution to problem (1.1)–(1.4). Conversely, if a triple $\{\widetilde{v}(t,x), \widetilde{u}(t,x), \widetilde{w}(t,x)\}$ is a solution to problem (1.1)–(1.4), then $\widetilde{u}(t,x)$ is a classical solution to problem (1)–(4).

For fixed $w(t,x)$, $u(t,x)$ in problem (1.1)–(1.4) it is necessary to find a solution to a one-parameter family of periodic boundary value problems for system of ordinary differential equations with delayed argument.

Consider the family of periodic boundary value problems for system of ordinary differential equations with delayed argument

$$\frac{\partial v(t,x)}{\partial t} = A(t,x)v(t,x) + A_0(t,x)v(t-\tau,x) + F(t,x), \qquad t \in [0,T], \quad x \in [0,\omega], \quad v \in R^n; \tag{1.5}$$

$$v(z,x) = diag[v(0,x)] \cdot \varphi(z), \quad z \in [-\tau,0], \qquad x \in [0,\omega]; \tag{1.6}$$

$$v(0,x) = v(T,x), \qquad x \in [0,\omega], \tag{1.7}$$

where the n vector function $F(t,x)$ is continuous on Ω_τ.

Continuous function $v : \Omega_\tau \to R^n$ that has a continuous derivative with respect to t on $\Omega_\tau \backslash \Omega_0$ is called a solution to the family periodic boundary value problems with delayed argument (1.5)–(1.7) if it satisfies system (1.5) for all $(t,x) \in \Omega$ and has the values $v(0,x)$, $v(T,x)$ on the lines $t = 0$, $t = T$ and the equalities (1.6), (1.7) are valid for all $x \in [0,\omega]$, respectively.

For fixed $x \in [0,\omega]$ problem (1.5)–(1.7) is a linear periodic boundary value problem for the system of ordinary differential equations with delayed argument [5]. Suppose a variable x is changed on $[0,\omega]$; then we obtain a family of periodic boundary value problems for ordinary differential equations with delayed argument. In the [6] was investigated the family of periodical boundary value problems for the system of differential equations with delayed argument. Algorithms for finding solutions of the considered problem are constructed and their convergence is proved. Conditions of the unique solvability to family of periodical boundary value problems for the system of differential equations with delayed argument are established in the terms of the initial data. These conditions are also given the conditions of well-posed solvability to problem (1.5)–(1.7).

Definition 1. Problem (1.5)–(1.7) is called well-posed if for arbitrary $F(t,x) \in C(\Omega, R^n)$ it has a unique solution $v(t,x)$ and for it the estimate holds

$$\max_{t\in[0,T]} ||v(t,x)|| \le K \max_{t\in[0,T]} ||F(t,x)||, \tag{1.8}$$

where the constant K independent of $F(t,x)$ and $x \in [0,\omega]$.

Denote by $\Omega^\eta = [0,T] \times [0,\eta]$ and $||u||_{0,\eta} = \max_{(t,x)\in\Omega^\eta} ||u(t,x)||$.

Definition 2. Boundary value problem (1)-(4) is called well-posed if for arbitrary $f(t,x) \in C(\Omega, R^n)$ and $\psi(t) \in C^1([-\tau, T], R^n)$ it has a unique classical solution $u(t,x)$ and this solution satisfies the following estimate

$$\max\Big(||u||_{0,\eta}, \Big|\Big|\frac{\partial u}{\partial x}\Big|\Big|_{0,\eta}, \Big|\Big|\frac{\partial u}{\partial t}\Big|\Big|_{0,\eta}\Big) \leq \tilde{K}\max\Big(||f||_{0,\eta}, ||\psi||_{1,0}, \max_{x\in[0,\eta]}||\varphi(x)||\Big),$$

where constant $\tilde{K}$ independent of $f(t,x)$ and $\psi(t)$ and $\eta \in [0,\omega]$.

Theorem 1. The boundary value problem (1)–(4) is well-posed if and only if so is problem (1.5)–(1.7). Proof of Theorem 1 is similar to the proof of Theorem 1 in [4] taking into account the properties of the delayed argument.

From Theorem 1 it follows that the well-posedness of problem (1)–(4) are equivalent to the well-posedness of problem (1.5)–(1.7).

Hereby, the problem (1)–(4) reduce to an equivalent problem, consisting the family of periodic boundary value problem for system of differential equations with delayed argument and integral relations. For constructing of algorithms of finding approximate solutions to the equivalent problem (1.1)–(1.4) are used results of paper [7].

2 Algorithm for finding approximate solutions of problem (1)–(4) and its convergence

We suppose that problem (1.5)–(1.7) is well-posed. By virtue of the equivalence of problems (1)–(4) and (1.1)–(1.4), it suffices to justify the well-posedness of problem (1.1)–(1.4). We find a solution $\{v(t,x), u(t,x), w(t,x)\}$ of problem (1.1)–(1.4) by the successive approximation method. As the initial approximation $u(t,x)$ and $w(t,x)$ we take $\psi(t)$ and $\dot{\psi}(t)$, respectively, and then find $v^{(0)}(t,x)$ from the problem

$$\frac{\partial v^{(0)}(t,x)}{\partial t} = A(t,x)v^{(0)}(t,x) + A_0(t,x)v^{(0)}(t-\tau,x)+$$

$$+B(t,x)\dot{\psi}(t) + C(t,x)\psi(t) + f(t,x), \qquad (t,x)\in\Omega; \tag{2.1}$$

$$v^{(0)}(z,x) = diag[v^{(0)}(0,x)]\cdot\varphi(z), \quad z\in[-\tau,0], \qquad x\in[0,\omega]; \tag{2.2}$$

$$v^{(0)}(0,x) = v^{(0)}(T,x), \qquad x\in[0,\omega]. \tag{2.3}$$

By assumption, problem (2.1)–(2.3) has a unique solution $v^{(0)}(t,x)$.

Then $u^{(0)}(t,x)$ and $w^{(0)}(t,x)$ are determined from integral relations:

$$u^{(0)}(t,x) = \psi(t) + \int_0^x v^{(0)}(t,\xi)d\xi, \qquad w^{(0)}(t,x) = \dot{\psi}(t) + \int_0^x \frac{\partial v^{(0)}(t,\xi)}{\partial t}d\xi. \tag{2.4}$$

Suppose $u^{(k-1)}(t,x)$ and $w^{(k-1)}(t,x)$ are known. Then $v^{(k)}(t,x)$ can be found from the problem (1.1)–(1.3), where $w(t,x) = w^{(k-1)}(t,x)$, $u(t,x) = u^{(k-1)}(t,x)$:

$$\frac{\partial v^{(k)}(t,x)}{\partial t} = A(t,x)v^{(k)}(t,x) + A_0(t,x)v^{(k)}(t-\tau,x)+$$

$$+B(t,x)w^{(k-1)}(t,x) + C(t,x)u^{(k-1)}(t,x) + f(t,x), \qquad (t,x)\in\Omega; \tag{2.5}$$

$$v^{(k)}(z,x) = diag[v^{(k)}(0,x)]\cdot\varphi(z), \quad z\in[-\tau,0], \qquad x\in[0,\omega]; \tag{2.6}$$

$$v^{(k)}(0,x) = v^{(k)}(T,x), \qquad x\in[0,\omega]. \tag{2.7}$$

Once $v^{(k)}(t,x)$ is found the successive approximations for $u(t,x)$ and $w(t,x)$ are found from relations (1.4):

$$u^{(k)}(t,x) = \psi(t) + \int_0^x v^{(k)}(t,\xi)d\xi, \qquad w^{(k)}(t,x) = \dot{\psi}(t) + \int_0^x \frac{\partial v^{(k)}(t,\xi)}{\partial t}d\xi, \quad k=1,2,... \tag{2.8}$$

We construct the differences $\Delta v^{(k)}(t,x) = v^{(k)}(t,x) - v^{(k-1)}(t,x)$, $\Delta u^{(k)}(t,x) = u^{(k)}(t,x) - u^{(k-1)}(t,x)$, $\Delta w^{(k)}(t,x) = w^{(k)}(t,x) - w^{(k-1)}(t,x)$, and by using the well-posedness of problem (1.5)–(1.7) establish estimates

$$\max\Big\{\max_{t\in[0,T]}||\Delta v^{(k+1)}(t,x)||, \max_{t\in[0,T]}\Big|\Big|\frac{\partial\Delta v^{(k+1)}(t,x)}{\partial t}\Big|\Big|\Big\} \leq \max\Big\{K, \max_{t\in[0,T]}||A(t,x)||K+1\Big\}\times$$

$$\times K_1(x)\max\Big\{\max_{t\in[0,T]}||\Delta w^{(k)}(t,x)||, \max_{t\in[0,T]}||\Delta u^{(k)}(t,x)||\Big\}; \tag{2.9}$$

$$\max\Big\{\max_{t\in[0,T]}||\Delta w^{(k)}(t,x)||,\max_{t\in[0,T]}||\Delta u^{(k)}(t,x)||\Big\}\le$$

$$\le\int_0^x \max\Big\{\max_{t\in[0,T]}||\Delta v^{(k)}(t,\xi)||,\max_{t\in[0,T]}\Big|\Big|\frac{\partial\Delta v^{(k)}(t,\xi)}{\partial t}\Big|\Big|\Big\}d\xi, \qquad (2.10)$$

where $K_1(x)=\max_{t\in[0,T]}||B(t,x)||+\max_{t\in[0,T]}||C(t,x)||$.
This implies the main inequality

$$\max\Big\{\max_{t\in[0,T]}||\Delta v^{(k+1)}(t,x)||,\max_{t\in[0,T]}\Big|\Big|\frac{\partial\Delta v^{(k+1)}(t,x)}{\partial t}\Big|\Big|\Big\}\le\max\Big\{K,\max_{t\in[0,T]}||A(t,x)||K+1\Big\}\times$$

$$\times K_1(x)\int_0^x \max\Big\{\max_{t\in[0,T]}||\Delta v^{(k)}(t,\xi)||,\max_{t\in[0,T]}\Big|\Big|\frac{\partial\Delta v^{(k)}(t,\xi)}{\partial t}\Big|\Big|\Big\}d\xi. \qquad (2.11)$$

From (2.11) it follows that the sequences $\{v^{(k)}(t,x)\}$ and $\{\frac{\partial v^{(k)}(t,x)}{\partial t}\}$ are convergent in the space $C(\Omega,R^n)$ as $k\to\infty$. Then the uniform convergence on Ω of the sequences $\{u^{(k)}(t,x)\}$ and $\{w^{(k)}(t,x)\}$ follows from the estimate (2.10).

In this case, the limit functions $v^*(t,x)$, $\frac{\partial v^*(t,x)}{\partial t}$, $u^*(t,x)$ and $w^*(t,x)$ are continuous on Ω, and the triple $\{v^*(t,x),u^*(t,x),w^*(t,x)\}$ is a solution to problem (1.1)–(1.4). For $\eta\in[0,\omega]$ by using the estimates (2.9)–(2.11), we obtain

$$\max\Big(||v^*||_{0,\eta},||u^*||_{0,\eta},||w^*||_{0,\eta}\Big)\le\hat{K}\cdot\max\Big(||f||_{0,\eta},||\psi||_{1,0},\max_{x\in[0,\eta]}||\varphi(x)||\Big), \qquad (2.12)$$

where $\hat{K}=\max\Big(e^{K_0K_1\omega}[1+\omega K_0],K[K_1(1+\omega K_0)+1]\Big)$, $K_0=\max(K,||A||_0K+1)$, $K_1=\max_{x\in[0,\omega]}K_1(x)$ and they independent of f and ψ.

Now let $\{\widetilde{v}(t,x),\widetilde{u}(t,x),\widetilde{w}(t,x)\}$ be a solution to problem (1.1)–(1.4), where $f(t,x)=0$ and $\psi(t)=0$ for all $(t,x)\in\Omega$. Then the well-posedness of problem (1.5)–(1.7) together with (1.4) imply that $\widetilde{v}(t,x)=0$, $\widetilde{u}(t,x)=0$, and $\widetilde{w}(t,x)=0$ for all $(t,x)\in\Omega$. Thus it follows from the estimate (2.12) that problem (1)–(4) is well-posedness.

So, the problem (1)–(4) reduce to an equivalent problem, consisting the family of periodic boundary value problem for system of differential equations with delayed argument and integral relations. For constructing of algorithms of finding approximate solutions to the equivalent problem are used results of paper [5]. For solve of the family to the periodic boundary value problems for system of differential equations with delayed argument are used results of articles [6-7]. Algorithms of finding solutions to the families of periodic boundary value problems for differential equations with delayed argument are constructed and their convergence proved. The conditions of the solvability to the periodic boundary value problems for hyperbolic equations with delayed argument are established. These results are partially announced in the [8].

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Кешігулі аргументі бар гиперболалық теңдеулер жүйесі үшін периодты шеттік есептің корректілі шешілімділігі

Кешігулі аргументі бар гиперболалық теңдеулер жүйесі үшін периодты шеттік есеп қарастырылды. Жаңа функциялар енгізу әдісі арқылы зерттеліп отырған есеп кешігулі аргументі бар дифференциалдық теңдеулер жүйесі үшін периодты шеттік есептер әулеті мен интегралдық қатынастар пара-пар есепке келтірілді. Кешігулі аргументі бар гиперболалық теңдеулер жүйесі үшін периодты шеттік есеп пен кешігулі аргументі бар жай дифференциалдық теңдеулер жүйесі үшін периодты шеттік есептер әулетінің өзара байланысы орнатылған. Пара-пар есептің шешімін табу алгоритмдері құрылған және олардың жинақтылығы дәлелденген. Кешігулі аргументі бар гиперболалық теңдеулер жүйесі үшін периодты шеттік есептің корректілі шешілімділігінің жеткілікті және қажетті шарттары алынған.

Кілт сөздер: периодты шеттік есеп, гиперболалық теңдеулер жүйесі, кешігулі аргумент, периодты шеттік есептер әулеті, кешігулі аргументі бар дифференциалдық теңдеулер жүйесі, алгоритм, бірмәнді шешілімділік, корректілі шешілімділік.

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Корректная разрешимость периодической краевой задачи для системы гиперболических уравнений с запаздывающим аргументом

Рассмотрена периодическая краевая задача для системы гиперболических уравнений с запаздывающим аргументом. Методом введения новых функций исследуемая задача сводится к эквивалентной задаче, состоящей из семейства периодических краевых задач для системы дифференциальных уравнений с запаздывающим аргументом и интегральных соотношений. Установлена взаимосвязь периодической краевой задачи для системы гиперболических уравнений с запаздывающим аргументом и семейства периодических краевых задач для системы дифференциальных уравнений с запаздывающим аргументом. Построены алгоритмы нахождения решений эквивалентной задачи и доказана их сходимость. Получены достаточные и необходимые условия корректной разрешимости периодической краевой задачи для системы гиперболических уравнений с запаздывающим аргументом.

Ключевые слова: периодическая краевая задача, система гиперболических уравнений, запаздывающий аргумент, семейство периодических краевых задач, система дифференциальных уравнений с запаздывающим аргументом, алгоритм, однозначная разрешимость, корректная разрешимость.

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