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Bounder solution on a strip to a system of nonlinear hyperbolic equations with mixed derivatives

The system of nonlinear hyperbolic equations with mixed derivatives is considered on the strip. Time variable of the unknown function changes on the whole axis, and the spatial variable belongs to a finite interval. A function, the partial derivative with respect to the spatial variable, is denoted as unknown function, and problem of finding a bounded on the strip solution to the origin system is reduced to the problem of finding a bounded on the strip solution to a system of integro - partial differential equations. The whole axes is divided into parts, and additional functional parameters are introduced as the values of unknown function on the initial lines of sub-domains. For the fixed values of functional parameters, the new unknown functions in the sub-domains are defined as the solutions to the Cauchy problems for integro-partial differential equations of the first order. Using the continuity conditions of the solution on the partition lines, the two-sided infinite system of nonlinear Volterra integral equations of the second kind with respect to introduced functional parameters is obtained. Algorithms for finding solutions of problem with functional parameters are proposed. Conditions for the convergence of algorithms, and existence of bounded on the strip solution of the system of nonlinear hyperbolic equations with mixed derivatives are obtained.

Key words: bounded on the strip solution, the system of nonlinear hyperbolic equations, functional parameters, algorithm.

We consider the system of nonlinear hyperbolic equations

$$\frac{\partial^2 u}{\partial x \partial t} = f\Big(x, t, u, \frac{\partial u}{\partial x}\Big), \qquad (x,t) \in \Omega = [0,\omega] \times \mathbb{R}, \qquad u \in \mathbb{R}^n; \tag{1}$$

where $f : \Omega \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a bounded function, continuous by $x \in [0,\omega]$ uniformly with respect to $t \in \mathbb{R}$ and continuous by $t \in \mathbb{R}$ at the fixed $x \in [0,\omega]$, $||u|| = \max\limits_{i=1:n} |u_i|$.

Denote by $C^*(\Omega, \mathbb{R}^n)$ is a space of bounded functions $u : \Omega \to \mathbb{R}^n$, continuous by $x \in [0,\omega]$ uniformly with respect to $t \in \mathbb{R}$ and continuous by $t \in \mathbb{R}$ at the fixed $x \in [0,\omega]$ with the norm $||u||_* = \sup\limits_{(x,t)\in\Omega} ||u(x,t)||$.

Bounded on the strip solutions to the system of hyperbolic equations with mixed derivatives are considered in [1–5].

The paper is devoted to the solutions of system (1) satisfying the conditions

$$u(0,t) = 0, \qquad t \in \mathbb{R}; \tag{2}$$

$$\frac{\partial u(x,t)}{\partial x} \in C^*(\Omega, \mathbb{R}^n). \tag{3}$$

A solution to problem (1)-(3) is a function $u^*(x,t) \in C^*(\Omega, \mathbb{R}^n)$, which has the partial derivatives $\frac{\partial}{\partial x} u^*(x,t) \in C^*(\Omega, \mathbb{R}^n)$, $\frac{\partial^2}{\partial x \partial t} u^*(x,t) \in C^*(\Omega, \mathbb{R}^n)$, satisfying the system of differential equations (1) for all $(x,t) \in \Omega$ and condition (2).

We set $v(x,t) = \frac{\partial u(x,t)}{\partial x}$, $(x,t) \in \Omega$ and reduce the problem (1)-(3) to the following problem for the system of integro-partial differential equations

$$\frac{\partial v}{\partial t} = f\Big(x, t, \int_0^x v(\xi,t) d\xi, v\Big), \qquad (x,t) \in \Omega, \tag{4}$$

$$v(x,t) \in C^*(\Omega,\mathbb{R}^n). \tag{5}$$

If a function $u^*(x,t)$ is a solution to problem (1)-(3), then the function $v^*(x,t) = \dfrac{\partial u^*(x,t)}{\partial t}$ is a solution to problem (4), (5). Conversely, if $\widetilde{v}(x,t)$ is a solution to problem (4), (5), then the function

$$\widetilde{u}(x,t) = \int_0^x \widetilde{v}(\xi,t)d\xi, \qquad (x,t) \in \Omega$$

is a solution to problem (1)-(3).

Take $h > 0$ and divide Ω on sub-domains

$$\Omega = \bigcup_{r=-\infty}^{\infty} \Omega_r, \qquad \Omega_r = [0,\omega] \times [(r-1)h, rh), \quad r \in \mathbb{Z}.$$

Let $v_r(x,t)$ be the restriction of function $v(x,t)$ on sub-domain Ω_r, $r \in \mathbb{Z}$.

Introduced the following spaces:

m_n is a space of bounded two sided infinite sequences $\mu = (\ldots,\mu_r,\mu_{r+1},\ldots)$ of vectors $\mu_r \in \mathbb{R}^n$, $r \in \mathbb{Z}$, with the norm

$$||\mu||_{m_n} = ||(\ldots,\mu_r,\mu_{r+1},\ldots)||_{m_n} = \sup_{r\in\mathbb{Z}} ||\mu_r||;$$

$C^*([0,\omega],m_n)$ is a space of uniforml; bounded and equicontinuous two sided infinite sequences $\lambda(x) = (\ldots,\lambda_r(x),\lambda_{r+1}(x),\ldots)$ of continuous functions $\lambda_r : [0,\omega] \to \mathbb{R}^n$, $r \in \mathbb{Z}$, with the norm

$$||\lambda||_1 = \max_{x\in[0,\omega]} ||\lambda(x)||_{m_n} = \max_{x\in[0,\omega]} ||(\ldots,\lambda_r(x),\lambda_{r+1}(x),\ldots)||_{m_n} = \max_{x\in[0,\omega]} \sup_{r\in\mathbb{Z}} ||\lambda_r(x)||;$$

$C^*(\mathbb{R},h,\mathbb{R}^n)$ is a space of uniformly bounded two sided infinite sequences $u[t] = (\ldots,u_r(t),u_{r+1}(t),\ldots)$ of continuous functions $u_r : [(r-1)h,rh) \to \mathbb{R}^n$, $r \in \mathbb{Z}$, with the norm

$$||u||_2 = \sup_{r\in\mathbb{Z}} \sup_{t\in[(r-1)h,rh)} ||u_r(t)||;$$

$L(X)$ is a space of linear bounded operators $\Lambda : X \to X$ with induced norm, where X is a Banach space; $C^*([0,\omega],C^*(\mathbb{R},h,\mathbb{R}^n))$ is a space of bounded two sided infinite sequences of functions $v(x,[t]) = s(\ldots,v_r(x,t),v_{r+1}(x,t),\ldots)$ with the norm

$$||v||_3 = \max_{x\in[0,\omega]} \sup_{r\in\mathbb{Z}} \sup_{t\in[(r-1)h,rh)} ||v_r(x,t)||,$$

where $v_r : \Omega_r \to \mathbb{R}^n$ is continuous and has finite limit as $t \to rh-0$, $r \in \mathbb{Z}$, uniform on $x \in [0,\omega]$.

All spaces are complete.

Introduce functional parameters

$$\lambda_r(x) := v_r(x,(r-1)h), \qquad x \in [0,\omega], \qquad r \in \mathbb{Z}$$

and functions

$$z_r(x,t) := v_r(x,t) - \lambda_r(x), \qquad (x,t) \in \Omega_r, \qquad r \in \mathbb{Z}.$$

Now, problem (4), (5) converts to the problem with functional parameters

$$\frac{\partial z_r}{\partial t} = f\Big(x,t,\int_0^x \lambda_r(\xi)d\xi + \int_0^x z_r(\xi,t)d\xi, \lambda_r(x) + z_r\Big), \quad (x,t) \in \Omega_r, \quad r \in \mathbb{Z}; \tag{6}$$

$$z_r(x,(r-1)h) = 0, \qquad x \in [0,\omega], \quad r \in \mathbb{Z}; \tag{7}$$

$$\lambda_r(x) + \lim_{t\to rh-0} z_r(x,t) - \lambda_{r+1}(x) = 0, \qquad x \in [0,\omega], \quad r \in \mathbb{Z}; \tag{8}$$

$$(\lambda(x), z(x,[t])) \in C^*([0,\omega],m_n) \times C^*([0,\omega],C^*(\mathbb{R},h,\mathbb{R}^n)). \tag{9}$$

Here (8) are the continuity conditions on the lines $t = (r-1)h$, $r \in \mathbb{Z}$.

A solution to problem (6)-(9) is a pair $(\lambda^*(x), z^*(x,[t]))$ with elements

$$\lambda^*(x) = \big(\ldots, \lambda_r^*(x), \lambda_{r+1}^*(x), \ldots\big) \in C^*([0,\omega], m_n),$$

and

$$z^*(x,[t]) = \big(\ldots, z_r^*(x,t), z_{r+1}^*(x,t), \ldots\big) \in C^*([0,\omega], C^*(\mathbb{R}, h, \mathbb{R}^n)).$$

If a function $v^*(x,t)$ is a solution to problem (4), (5), then the pair $(\lambda^*(x), z^*(x,[t])$ with elements

$$\lambda^*(x) = \big(\ldots, \lambda_r^*(x), \lambda_{r+1}^*(x), \ldots\big), \quad z^*(x,[t]) = \big(\ldots, z_r^*(x,t), z_{r+1}^*(x,t), \ldots\big),$$

where $\lambda_r^*(x) = v^*(x,(r-1)h)$, $z_r^*(x,t) = v^*(x,t) - v^*(x,(r-1)h)$, $(x,t) \in \Omega_r$, $r \in \mathbb{Z}$, is a solution to problem (6)-(9).

And, vice versa, if a pair $(\lambda^*(x), z^*(x,[t])$ with elements

$$\lambda^*(x) = \big(\ldots, \lambda_r^*(x), \lambda_{r+1}^*(x), \ldots\big) \in C^*([0,\omega], m_n);$$

$$z^*(x,[t]) = \big(\ldots, z_r^*(x,t), z_{r+1}^*(x,t), \ldots\big) \in C^*([0,\omega], C^*(\mathbb{R}, h, \mathbb{R}^n)),$$

is a solution to problem (7)-(10), then the function $v^*(x,t)$ defined on Ω by the equalities

$$v^*(x,t) = \lambda_r^*(x) + z_r^*(x,t), \qquad (x,t) \in \Omega_r, \quad r \in \mathbb{Z},$$

is a solution to problem (4), (5).

At the given $\lambda_r(x)$, $x \in [0,\omega]$, the Cauchy problem for integro-partial differential equation (6), (7) is equivalent to the system of integral equations

$$z_r(x,t) = \int_{(r-1)h}^{t} f\Big(x, \tau, \int_0^x \lambda_r(\xi)d\xi + \int_0^x z_r(\xi,\tau)d\xi, \lambda_r(x) + z_r(x,\tau)\Big)d\tau, \tag{10}$$

$(x,t) \in \Omega_r$, $\quad r \in \mathbb{Z}$. Substituting the corresponding expressions from (10) into (8), we obtain the two-sided infinite system of nonlinear integral equations with respect to functional parameters

$$\lambda_r(x) + \int_{(r-1)h}^{rh} f\Big(x, \tau, \int_0^x \lambda_r(\xi)d\xi + \int_0^x z_r(\xi,\tau)d\xi, \lambda_r(x) + z_r(x,\tau)\Big)d\tau - \lambda_{r+1}(x) = 0, \tag{11}$$

$$x \in [0,\omega], \qquad r \in \mathbb{Z}.$$

Write down system (11) in the form

$$Q_{1,h}\Big(x, \int_0^x \lambda(\xi)d\xi, \lambda(x), z\Big) = 0, \qquad x \in [0,\omega], \qquad \lambda(x) \in C^*([0,\omega], m_n).$$

Condition A. There exists $h > 0$ such that the implicit system of nonlinear Volterra integral equations

$$Q_{1,h}\Big(x, \int_0^x \lambda(\xi)d\xi, \lambda(x), 0\Big) = 0, \qquad x \in [0,\omega],$$

has a solution $\lambda^{(0)}(x) = (\ldots, \lambda_r^{(0)}(x), \lambda_{r+1}^{(0)}(x), \ldots) \in C^*([0,\omega], m_n)$, and the Cauchy problems for integro-partial differential equations

$$\frac{\partial z_r}{\partial t} = f\Big(x, t, \int_0^x \lambda_r^{(0)}(\xi)d\xi + \int_0^x z_r(\xi,t)d\xi, \lambda_r^{(0)}(x) + z_r\Big), \quad (x,t) \in \Omega_r, \quad r \in \mathbb{Z},$$

$$z_r(x,(r-1)h) = 0, \qquad x \in [0,\omega], \quad r \in \mathbb{Z},$$

has a solution $z^{(0)}(x,[t]) \in C^*([0,\omega], C^*(\mathbb{R}, h, \mathbb{R}^n))$.

Under condition A, we define the functions

$$v^{(0)}(x,t) = \lambda_r^{(0)}(x) + z_r^{(0)}(x,t), \quad (x,t) \in \Omega_r, \quad r \in \mathbb{Z}$$

and

$$u^{(0)}(x,t)=\int_0^x v^{(0)}(\xi,t)d\xi, \quad (x,t)\in\bar{\Omega}.$$

Take numbers $\rho_\lambda>0$, $\rho_z>0$, $\rho_v>0$, $\rho_u>0$ and determine the sets:

$$S(\lambda^{(0)}(x),\rho_\lambda)=\{\lambda(x)\in C^*([0,\omega],m_n):||\lambda-\lambda^{(0)}||_1<\rho_\lambda\};$$

$$S_h(z^{(0)}(x,[t]),\rho_z)=\{z(x,[t])\in C^*([0,\omega],C^*(\mathbb{R},h,\mathbb{R}^n)):||z-z^{(0)}||_3<\rho_z\};$$

$$S(v^{(0)}(x,t),\rho_v)=\{v(x,t)\in C^*(\Omega,\mathbb{R}^n):||v-v^{(0)}||_*<\rho_v\};$$

$$S(u^{(0)}(x,t),\rho_u)=\{u(x,t)\in C^*(\Omega,\mathbb{R}^n):||u-u^{(0)}||_*<\rho_u\};$$

$$G^0(x,t,\rho_u,\rho_v)=\{(x,t,u,v)\in\Omega\times\mathbb{R}^{2n}:(x,t)\in\Omega,||u-u^{(0)}(x,t)||<\rho_u,||v-v^{(0)}(x,t)||<\rho_v\}.$$

Condition B. The function $f(x,t,u,v)$ has uniformly continuous partial derivatives $\dfrac{\partial f(x,t,u,v)}{\partial u}$; $\dfrac{\partial f(x,t,u,v)}{\partial v}$ on $G^0(x,t,\rho_u,\rho_v)$ and the inequalities

$$\Big|\Big|\frac{\partial f(x,t,u,v)}{\partial u}\Big|\Big|\le L_1, \qquad \Big|\Big|\frac{\partial f(x,t,u,v)}{\partial v}\Big|\Big|\le L_2,$$

where L_1, L_2 are constants, hold.

Take the pair $(\lambda^{(0)}(x),z^{(0)}(x,[t]))$, and construct the sequence of pairs $(\lambda^{(k)}(x),z^{(k)}(x,[t]))$, $k\in\mathbb{N}$, by the following algorithm.

Step 1. a) Solving the implicit system of nonlinear Volterra integral equations of the second kind

$$Q_{1,h}\Big(x,\int_0^x\lambda(\xi)d\xi,\lambda(x),z^{(0)}\Big)=0, \quad x\in[0,\omega],$$

we find $\lambda^{(1)}(x)=(\ldots,\lambda_r^{(1)}(x),\lambda_{r+1}^{(1)}(x),\ldots)\in C^*([0,\omega],m_n)$.

b) Solving the Cauchy problems for integro-partial differential equation (6), (7) with $\lambda_r(x)=\lambda_r^{(1)}(x)$, $x\in[0,\omega]$, $r\in\mathbb{Z}$, we find the function system $z^{(1)}(x,[t])=(\ldots,z_r^{(1)}(x,t),z_{r+1}^{(1)}(x,t),\ldots)\in C^*([0,\omega],C^*(\mathbb{R},h,\mathbb{R}^n))$.

Step 2. a) Solving the implicit system of nonlinear Volterra integral equations of the second kind

$$Q_{1,h}\Big(x,\int_0^x\lambda(\xi)d\xi,\lambda(x),z^{(1)}\Big)=0, \quad x\in[0,\omega],$$

we find $\lambda^{(2)}(x)=(\ldots,\lambda_r^{(2)}(x),\lambda_{r+1}^{(2)}(x),\ldots)\in C^*([0,\omega],m_n)$.

b) Solving the Cauchy problems for integro-partial differential equation (6), (7) with $\lambda_r(x)=\lambda_r^{(2)}(x)$, $x\in[0,\omega]$, $r\in\mathbb{Z}$, we find the function system $z^{(2)}(x,[t])=(\ldots,z_r^{(2)}(x,t),z_{r+1}^{(2)}(x,t),\ldots)\in C^*([0,\omega],C^*(\mathbb{R},h,\mathbb{R}^n))$. And so on.

Sufficient conditions of feasibility and convergence of the algorithm are established by the next statement.

Theorem. Suppose there are $h>0$, $\rho_\lambda>0$, $\rho_z>0$, $\rho_v>0$, $\rho_u>0$, such that conditions A, B are valid, the two-sided infinite Jacob matrix $\dfrac{\partial}{\partial w_2}Q_{1,h}(x,w_1,w_2,z):C^*([0,\omega],m_n)\to C^*([0,\omega],m_n)$, is invertible for all $(x,w_1,w_2,z)\in[0,\omega]\times S(\int_0^x\lambda^{(0)}(\xi)d\xi,\omega\rho_\lambda)\times S(\lambda^{(0)}(x),\rho_\lambda)\times S_h(z^{(0)}(x,[t]),\rho_z)$ and the following inequalities are true:

1) $||\Big(\dfrac{\partial}{\partial w_2}Q_{1,h}(x,w_1,w_2,z)\Big)^{-1}||_{L(m_n)}\le\gamma_1(h,x)\le\gamma_{1,0}(h)$;

2) $q_{1,0}(h)=\gamma_{1,0}(h)e^{\gamma_{1,0}(h)hL_1\omega}\Big(e^{(L_1\omega+L_2)h}-1-(L_1\omega+L_2)h\Big)<1$;

3) $\dfrac{\gamma_{1,0}(h)}{1-q_{1,0}(h)}e^{\gamma_{1,0}(h)hL_1\omega}\max\limits_{x\in[0,\omega]}||Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda^{(0)}(x),z^{(0)}\Big)||_{m_n}<\rho_\lambda$;

4) $\Big(e^{(L_1\omega+L_2)h}-1\Big)\cdot\dfrac{\gamma_{1,0}(h)}{1-q_{1,0}(h)}e^{\gamma_{1,0}(h)hL_1\omega}\times\max\limits_{x\in[0,\omega]}||Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda^{(0)}(x),z^{(0)}\Big)||_{m_n}<\rho_z$;

5) $\rho_\lambda+\rho_z<\rho_v$, $\omega(\rho_\lambda+\rho_z)<\rho_u$.

Then the sequence of pairs $(\lambda^{(k)}(x),z^{(k)}(x,[t]))$, $k\in\mathbb{N}$, belongs to $S(\lambda^{(0)}(x),\rho_\lambda)\times S_h(z^{(0)}(x,[t]),\rho_z)$, converges to $(\lambda^*(x),z^*(x,[t]))$, the solution to the problem with functional parameters (6)-(9), and the estimates

$$||\lambda^*-\lambda^{(0)}||_1\le\frac{\gamma_{1,0}(h)h}{1-q_{1,0}(h)}e^{\gamma_{1,0}(h)hL_1\omega}e^{h(L_1\omega+L_2)}\sup_{r\in\mathbb{Z}}\sup_{(x,t)\in\Omega_r}||f\Big(x,t,\int_0^x\lambda_r^{(0)}(\xi)d\xi,\lambda_r^{(0)}(x)\Big)||,\qquad(12)$$

$$||z^*-z^{(0)}||_3\le\Big(e^{(L_1\omega+L_2)h}-1\Big)||\lambda^*-\lambda^{(0)}||_1\qquad(13)$$

hold.

Proof. For any pair $(\lambda(x),z(x,[t]))\in S(\lambda^{(0)}(x),\rho_\lambda)\times S_h(z^{(0)}(x,[t]),\rho_z)$, the inequalities

$$||\lambda_r(x)-\lambda_r^{(0)}(x)+z_r(x,t)-z_r^{(0)}(x,t)||\le||\lambda_r-\lambda_r^{(0)}||_1+||z-z^{(0)}||_3<\rho_\lambda+\rho_z<\rho_v,$$

$$(x,t)\in\Omega_r,\ r\in\mathbb{Z},\qquad(14)$$

$$||\int_0^x\lambda_r(\xi)d\xi-\int_0^x\lambda_r^{(0)}(\xi)d\xi+\int_0^x z_r(\xi,t)d\xi-\int_0^x z_r^{(0)}(\xi,t)d\xi||\le$$
$$\le\int_0^x||\lambda-\lambda^{(0)}||_1d\xi+\int_0^x||z-z^{(0)}||_3d\xi<\omega(\rho_\lambda+\rho_z)<\rho_u,\ (x,t)\in\Omega_r,\quad r\in\mathbb{Z},\qquad(15)$$

are true.

In view of (14), (15) and inequality 5) of Theorem the fours

$$\Big(x,t,\int_0^x\lambda_r(\xi)d\xi+\int_0^x z_r(\xi,t)d\xi,\lambda_r+z_r(x,t)\Big),\quad(x,t)\in\Omega_r,\quad r\in\mathbb{Z},$$

belong to the set $G_1^0(x,\rho_u,\rho_v)$. Take the pair $(\lambda^{(0)}(x),z^{(0)}(x,[t]))$ from Condition A.

Since the components of two-sided infinite sequences of functions $z^{(0)}(x,[t])$ are the solutions of Cauchy problem for integro-partial differential equations (6), (7) with $\lambda_r(x)=\lambda_r^{(0)}(x)$, the estimate

$$||z_r^{(0)}(x,t)||\le||\int_{(r-1)h}^t f\Big(x,\tau,\int_0^x\lambda_r^{(0)}(\xi)d\xi+\int_0^x z_r^{(0)}(\xi,\tau)d\xi,\lambda_r^{(0)}(x)+z_r^{(0)}(x,\tau)\Big)d\tau-$$
$$-\int_{(r-1)h}^t f\Big(x,\tau,\int_0^x\lambda_r^{(0)}(\xi)d\xi,\lambda_r^{(0)}(x)\Big)d\tau||+||\int_{(r-1)h}^t f\Big(x,\tau,\int_0^x\lambda_r^{(0)}(\xi)d\xi,\lambda_r^{(0)}(x)\Big)d\tau||\qquad(16)$$

holds. Condition B and inequality (16) imply

$$\max_{x\in[0,\omega]}||z_r^{(0)}(x,t)||\le\max_{x\in[0,\omega]}\int_{(r-1)h}^t||f\Big(x,\tau,\int_0^x\lambda_r^{(0)}(\xi)d\xi,\lambda_r^{(0)}(x)\Big)||d\tau e^{(L_1\omega+L_2)(t-(r-1)h)},\qquad(17)$$

$$(x,t)\in\Omega_r,\quad r\in\mathbb{Z}.$$

Find the solution $\lambda^{(1)}(x)=(\ldots,\lambda_r^{(1)}(x),\lambda_{r+1}^{(1)}(x),\ldots)$ of the equation

$$Q_{1,h}\Big(x,\int_0^x\lambda(\xi)d\xi,\lambda(x),z^{(0)}\Big)=0,\quad\lambda(x)\in C^*([0,\omega],m_n).$$

Consider the system of equations

$$Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda(x),z^{(0)}\Big)=0,\quad\lambda(x)\in C^*([0,\omega],m_n).\qquad(18)$$

In accordance with the conditions of Theorem, operator $Q_{1,h}\Big(x,\int_0^x \lambda(\xi)d\xi,\lambda(x),z^{(0)}\Big)$ at the fixed $x\in[0,\omega]$ satisfies the conditions of Theorem 1 [6; 39] in $S(\lambda^{(0)}(x),\rho_\lambda)$. Take a number $\varepsilon_0>0$, satisfying the inequalities

$$\varepsilon_0\gamma_{1,0}(h)e^{\gamma_{1,0}(h)hL_1\omega}\le\frac{1}{2};$$

$$\frac{\gamma_{1,0}(h)e^{\gamma_{1,0}(h)hL_1\omega}}{1-\varepsilon_0\gamma_{1,0}(h)e^{\gamma_{1,0}(h)hL_1\omega}}\max_{x\in[0,\omega]}||Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda^{(0)}(x),z^{(0)}\Big)||_{m_n}<\rho_\lambda.$$

Using uniform continuity of Jacobi's matrix

$$\frac{\partial}{\partial w_2}Q_{1,h}(x,w_1,w_2,z)$$

we find $\delta_0\in(0,0.5\rho_\lambda]$ such that

$$||\frac{\partial}{\partial w_2}Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda(x),z\Big)-\frac{\partial}{\partial w_2}Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\bar\lambda(x),z\Big)||_{L(m_n)}<\varepsilon_0$$

for all $\lambda(x)$, $\bar\lambda(x)\in S(\lambda^{(0)}(x),\rho_\lambda)$ satisfying $||\lambda-\bar\lambda||_1<\delta_0$.

Choose

$$\alpha\geqslant\alpha_0=\max\Big\{1,\frac{\gamma_{1,0}(h)}{\delta_0}e^{\gamma_{1,0}(h)hL_1\omega}\max_{x\in[0,\omega]}||Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda^{(0)}(x),z^{(0)}\Big)||_{m_n}\Big\}$$

and construct iterative processes: $\lambda^{(1,1,0)}(x)=\lambda^{(0)}(x)$;

$$\lambda^{(1,1,m+1)}(x)=\lambda^{(1,1,m)}(x)-\frac{1}{\alpha}\Big(\frac{\partial}{\partial w_2}Q_{1,h}\Big(x,\int_0^x\lambda^{(1,1,m)}(\xi)d\xi,\lambda^{(1,1,m)}(x),z^{(0)}\Big)\Big)^{-1}\times$$

$$\times Q_{1,h}\Big(x,\int_0^x\lambda^{(1,1,m)}(\xi)d\xi,\lambda^{(1,1,m)}(x),z^{(0)}\Big),\quad m=0,1,2,\ldots.\tag{19}$$

By Theorem 1 [6; 39] the iterative processes (19) converges to $\lambda^{(1,1)}(x)$, isolated solution of equation (18) at each $x\in[0,\omega]$, and the inequality

$$||\lambda^{(1,1)}(x)-\lambda^{(0)}(x)||_{m_n}\leqslant\gamma_1(h,x)||Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda^{(0)}(x),z^{(0)}\Big)||_{m_n}<\rho_\lambda\tag{20}$$

holds.

Taking into account that $Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda^{(0)}(x),0\Big)=0$, we have

$$||Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda^{(0)}(x),z^{(0)}\Big)||_{m_n}=$$

$$=||Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda^{(0)}(x),z^{(0)}\Big)-Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda^{(0)}(x),0\Big)||_{m_n}\le$$

$$\le h(L_1x+L_2)||z^{(0)}||_3.\tag{21}$$

Inequalities (20) and (21) imply the estimate

$$||\lambda^{(1,1)}(x)-\lambda^{(0)}(x)||_{m_n}\leqslant\gamma_1(h,x)h(L_1x+L_2)||z^{(0)}||_3.$$

Find the solution of system

$$Q_{1,h}\Big(x,\int_0^x\lambda^{(1,1)}(\xi)d\xi,\lambda(x),z^{(0)}\Big)=0,\quad\lambda(x)\in C^*([0,\omega],m_n),\tag{22}$$

by the iterative processes $\lambda^{(1,2,0)}(x) = \lambda^{(1,1)}(x)$,

$$\lambda^{(1,2,m+1)}(x) = \lambda^{(1,2,m)}(x) - \frac{1}{\alpha}\Big(\frac{\partial}{\partial w_2} Q_{1,h}\Big(x, \int_0^x \lambda^{(1,2,m)}(\xi)d\xi, \lambda^{(1,2,m)}(x), z^{(0)}\Big)\Big)^{-1} \times$$

$$\times Q_{1,h}\Big(x, \int_0^x \lambda^{(1,2,m)}(\xi)d\xi, \lambda^{(1,2,m)}(x), z^{(0)}\Big), \quad m = 0, 1, 2, \dots \tag{23}$$

The iterative processes (23) converges to $\lambda^{(1,2)}(\bar{x}) \in S(\lambda^{(0)}(\bar{x}), \rho_\lambda)$, isolated solution of equation (22) at $x = \bar{x}$, $\bar{x} \in [0, \omega]$ and the estimate

$$||\lambda^{(1,2)}(\bar{x}) - \lambda^{(1,1)}(\bar{x})||_{m_n} \leqslant \gamma_1(h, \bar{x})||Q_{1,h}\Big(\bar{x}, \int_0^{\bar{x}} \lambda^{(1,1)}(\xi)d\xi, \lambda^{(1,1)}(\bar{x}), z^{(0)}\Big)||_{m_n}$$

is valid. It easily seen

$$||Q_{1,h}\Big(\bar{x}, \int_0^{\bar{x}} \lambda^{(1,1)}(\xi)d\xi, \lambda^{(1,1)}(\bar{x}), z^{(0)}\Big) - Q_{1,h}\Big(\bar{x}, \int_0^{\bar{x}} \lambda^{(0)}(\xi)d\xi, \lambda^{(1,1)}(\bar{x}), z^{(0)}\Big)||_{m_n} \leq$$

$$\leq hL_1\bar{x} \sup_{\eta\in[0,\bar{x}]} ||\lambda^{(1,1)}(\eta) - \lambda^{(0)}(\eta)||_{m_n}.$$

Therefore, the following inequality is true

$$\sup_{\eta\in[0,\bar{x}]} ||\lambda^{(1,2)}(\eta) - \lambda^{(1,1)}(\eta)||_{m_n} \leqslant \gamma_1(h, \bar{x})hL_1\bar{x} \sup_{\eta\in[0,\bar{x}]} ||\lambda^{(1,1)}(\eta) - \lambda^{(0)}(\eta)||_{m_n}.$$

Find the solution $\lambda^{(1,3)}(x)$ of the system of equations

$$Q_{1,h}\Big(x, \int_0^x \lambda^{(1,2)}(\xi)d\xi, \lambda(x), z^{(0)}\Big) = 0, \quad \lambda(x) \in C^*([0, \omega], m_n). \tag{24}$$

For this construct the iterative processes $\lambda^{(1,3,0)}(x) = \lambda^{(1,2)}(x)$;

$$\lambda^{(1,3,m+1)}(x) = \lambda^{(1,3,m)}(x) - \frac{1}{\alpha}\Big(\frac{\partial}{\partial w_2} Q_{1,h}\Big(x, \int_0^x \lambda^{(1,3,m)}(\xi)d\xi, \lambda^{(1,3,m)}(x), z^{(0)}\Big)\Big)^{-1} \times$$

$$\times Q_{1,h}\Big(x, \int_0^x \lambda^{(1,3,m)}(\xi)d\xi, \lambda^{(1,3,m)}(x), z^{(0)}\Big), \quad m = 0, 1, 2, \dots \tag{25}$$

Iterative processes (25) converges to $\lambda^{(1,3)}(x) \in S(\lambda^{(0)}(x), \rho_\lambda)$, isolated solution of equation (24) at fixed $x \in$ $\in [0, \omega]$ and the estimate

$$||\lambda^{(1,3)}(x) - \lambda^{(1,2)}(x)||_{m_n} \leqslant \gamma_{1,0}(h)||Q_{1,h}\Big(x, \int_0^x \lambda^{(1,2)}(\xi)d\xi, \lambda^{(1,2)}(x), z^{(0)}\Big)||_{m_n}$$

holds. It easily seen

$$||Q_{1,h}\Big(x, \int_0^x \lambda^{(1,2)}(\xi)d\xi, \lambda^{(1,2)}(x), z^{(0)}\Big) - Q_{1,h}\Big(x, \int_0^x \lambda^{(1,1)}(\xi)d\xi, \lambda^{(1,2)}(x), z^{(0)}\Big)||_{m_n} \leqslant$$

$$\leqslant \sup_{r\in\mathbb{Z}} \Big\| \int_{(r-1)h}^{rh} f\Big(x, \tau, \int_0^x \lambda_r^{(1,2)}(\xi)d\xi + \int_0^x z_r^{(0)}(\xi, \tau)d\xi, \lambda_r^{(1,2)}(x) + z_r^{(0)}(x, \tau)\Big)d\tau -$$

$$- \int_{(r-1)h}^{rh} f\Big(x, \tau, \int_0^x \lambda_r^{(1,1)}(\xi)d\xi + \int_0^x z_r^{(0)}(\xi, \tau)d\xi, \lambda_r^{(1,2)}(x) + z_r^{(0)}(x, \tau)\Big)d\tau\Big\| \leqslant$$

$$\leqslant \sup_{r\in\mathbb{Z}} \int_{(r-1)h}^{rh} L_1 \int_0^x \|\lambda^{(1,2)}(\xi) - \lambda^{(1,1)}(\xi)\|_{m_n} d\xi d\tau \leqslant$$

$$\leqslant \sup_{r\in\mathbb{Z}} \int_{(r-1)h}^{rh} L_1 \int_0^x \sup_{\xi\in[0,x]} \|\lambda^{(1,2)}(\xi) - \lambda^{(1,1)}(\xi)\|_{m_n} d\xi d\tau \leqslant$$

$$\leqslant \sup_{r\in\mathbb{Z}} \int_{(r-1)h}^{rh} L_1 \int_0^x \gamma_{1,0}(h) h L_1 \xi \sup_{\xi\in[0,x]} ||\lambda^{(1,1)}(\xi) - \lambda^{(0)}(\xi)||_{m_n} d\xi d\tau \leqslant$$

$$\leqslant \gamma_{1,0}(h) \frac{(hL_1 x)^2}{2!} \sup_{\xi\in[0,x]} ||\lambda^{(1,1)}(\xi) - \lambda^{(0)}(\xi)||_{m_n}.$$

Therefore, the following inequalities are true

$$||\lambda^{(1,3)}(x) - \lambda^{(1,2)}(x)||_{m_n} \leqslant \frac{(\gamma_{1,0}(h) h L_1 x)^2}{2!} \sup_{x\in[0,\omega]} ||\lambda^{(1,1)}(x) - \lambda^{(0)}(x)||_{m_n};$$

$$||\lambda^{(1,3)}(x) - \lambda^{(0)}(x)||_{m_n} \leqslant ||\lambda^{(1,3)}(x) - \lambda^{(1,2)}(x)||_{m_n} + ||\lambda^{(1,2)}(x) - \lambda^{(1,1)}(x)||_{m_n} +$$

$$+ ||\lambda^{(1,1)}(x) - \lambda^{(0)}(x)||_{m_n} \leq \Big(\frac{(\gamma_{1,0}(h) h L_1 x)^2}{2!} + \gamma_{1,0}(h) h L_1 x + 1 \Big) ||\lambda^{(1,1)}(x) - \lambda^{(0)}(x)||_{m_n} <$$

$$< e^{\gamma_{1,0}(h) h L_1 x} ||\lambda^{(1,1)}(x) - \lambda^{(0)}(x)||_{m_n}.$$

Continuing the process, we find $\lambda^{(1,\ell+1)}(x)$, $x \in [0,\omega]$, and establish the inequalities:

$$||\lambda^{(1,\ell+1)}(x) - \lambda^{(1,\ell)}(x)||_{m_n} \leqslant \frac{1}{\ell!} (\gamma_{1,0}(h) h L_1 x)^{\ell} ||\lambda^{(1,1)}(x) - \lambda^{(0)}(x)||_{m_n};$$

$$||\lambda^{(1,\ell+1)}(x) - \lambda^{(0)}(x)||_{m_n} \leqslant \sum_{j=0}^{\ell} \frac{1}{j!} (\gamma_{1,0}(h) h L_1 x)^{j} ||\lambda^{(1,1)}(x) - \lambda^{(0)}(x)||_{m_n} <$$

$$< e^{\gamma_{1,0}(h) h L_1 x} ||\lambda^{(1,1)}(x) - \lambda^{(0)}(x)||_{m_n}. \quad (26)$$

The sequence $\{\lambda^{(1,\ell)}(x)\}$ converges to the solution of equation (18). In (26) tending to the limit as $\ell \to \infty$, we establish the estimate

$$||\lambda^{(1)} x - \lambda^{(0)}(x)||_{m_n} \leqslant \gamma_{1,0}(h) e^{\gamma_{1,0}(h) h L_1 x} ||Q_{1,h}\Big(x, \int_0^x \lambda^{(0)}(\xi) d\xi, \lambda^{(0)}(x), z^{(0)}\Big)||_{m_n}. \quad (27)$$

Hence, based on the inequalities (21), (17), we get

$$||\lambda^{(1)}(x) - \lambda^{(0)}(x)||_{m_n} \leqslant$$

$$\leqslant \gamma_{1,0}(h) h e^{\gamma_{1,0}(h) h L_1 x} e^{h(L_1\omega + L_2)} \sup_{r\in\mathbb{Z}} \sup_{(x,t)\in\Omega_r} ||f\Big(x, t, \int_0^x \lambda_r^{(0)}(\xi) d\xi, \lambda_r^{(0)}(x)\Big)||.$$

In view of the Condition B and inequality 2) of Theorem the Cauchy problem for integro-partial differential equation (6), (7) with $\lambda_r(x) = \lambda_r^{(1)}(x)$, $x \in [0,\omega]$, $r \in \mathbb{Z}$, has the unique solution $z_r^{(1)}(x,t)$, $(x,t) \in \Omega_r$, $r \in \mathbb{Z}$.

For any $\bar{x} \in [0,\omega]$, the inequality

$$\|z_r^{(1)}(\bar{x},t) - z_r^{(0)}(\bar{x},t)\| =$$

$$= \Big\| \int_{(r-1)h}^{t} f\Big(\bar{x}, \tau, \int_0^{\bar{x}} \lambda_r^{(1)}(\xi) d\xi + \int_0^{\bar{x}} z_r^{(1)}(\xi,\tau) d\xi, \lambda_r^{(1)}(\bar{x}) + z_r^{(1)}(\bar{x},\tau)\Big) d\tau -$$

$$- \int_{(r-1)h}^{t} f\Big(\bar{x}, \tau, \int_0^{\bar{x}} \lambda_r^{(0)}(\xi) d\xi + \int_0^{\bar{x}} z_r^{(0)}(\xi,\tau) d\xi, \lambda_r^{(0)}(\bar{x}) + z_r^{(0)}(\bar{x},\tau)\Big) d\tau \Big\| \leqslant$$

$$\leqslant \int_{(r-1)h}^{t} \Big(L_1 \int_0^{\bar{x}} \|\lambda_r^{(1)}(\xi) - \lambda_r^{(0)}(\xi)\| d\xi + L_1 \int_0^{\bar{x}} \|z_r^{(1)}(\xi,\tau) - z_r^{(0)}(\xi,\tau)\| d\xi +$$

$$+ L_2 \|\lambda_r^{(1)}(\bar{x}) - \lambda_r^{(0)}(\bar{x})\| + L_2 \|z_r^{(1)}(\bar{x},\tau) - z_r^{(0)}(\bar{x},\tau)\| \Big) d\tau$$

holds, and for every $\eta \in [0,\bar{x}]$ it is valid the relation

$$\|z_r^{(1)}(\eta,t) - z_r^{(0)}(\eta,t)\| \leqslant$$

$$\leqslant \int_{(r-1)h}^{t}\Big(L_1\int_0^{\bar{x}}\|\lambda_r^{(1)}(\xi)-\lambda_r^{(0)}(\xi)\|d\xi+L_1\int_0^{\bar{x}}\|z_r^{(1)}(\xi,\tau)-z_r^{(0)}(\xi,\tau)\|d\xi+$$

$$+L_2\sup_{\eta\in[0,\bar{x}]}\|\lambda_r^{(1)}(\eta)-\lambda_r^{(0)}(\eta)\|+L_2\sup_{\eta\in[0,\bar{x}]}\|z_r^{(1)}(\eta,\tau)-z_r^{(0)}(\eta,\tau)\|\Big)d\tau.$$

Then

$$\sup_{\eta\in[0,\bar{x}]}\|z_r^{(1)}(\eta,t)-z_r^{(0)}(\eta,t)\|\leqslant$$

$$\leqslant\int_{(r-1)h}^{t}(L_1\bar{x}+L_2)\Big(\sup_{\eta\in[0,\bar{x}]}\|\lambda_r^{(1)}(\eta)-\lambda_r^{(0)}(\eta)\|+\sup_{\eta\in[0,\bar{x}]}\|z_r^{(1)}(\eta,\tau)-z_r^{(0)}(\eta,\tau)\|\Big)d\tau$$

and using the Gronwall-Bellman inequality, we obtain

$$\sup_{\eta\in[0,\bar{x}]}\|z_r^{(1)}(\eta,t)-z_r^{(0)}(\eta,t)\|\leq\Big(e^{(L_1\bar{x}+L_2)(t-(r-1)h)}-1\Big)\sup_{\eta\in[0,\bar{x}]}\|\lambda_r^{(1)}(\eta)-\lambda_r^{(0)}(\eta)\|.$$

For the components of system of functions $z^{(1)}(x,[t])=(\ldots,z_r^{(1)}(x,t),z_{r+1}^{(1)}(x,t),\ldots)$ the inequality

$$||z^{(1)}-z^{(0)}||_3\leqslant\Big(e^{(L_1\omega+L_2)h}-1\Big)||\lambda^{(1)}-\lambda^{(0)}||_1$$

are valid. It easily established that

$$||Q_{1,h}\Big(x,\int_0^x\lambda^{(1)}(\xi)d\xi,\lambda^{(1)}(x),z^{(1)}\Big)||_{m_n}=$$

$$=||Q_{1,h}\Big(x,\lambda^{(1)}(x),\int_0^x\lambda^{(1)}(\xi)d\xi,z^{(1)}\Big)-Q_{1,h}\Big(x,\int_0^x\lambda^{(1)}(\xi)d\xi,\lambda^{(1)}(x),z^{(0)}\Big)||_{m_n}\leqslant$$

$$\leqslant\sup_{r\in\mathbb{Z}}\Big(e^{(L_1x+L_2)h}-1-(L_1x+L_2)h\Big)\sup_{\eta\in[0,x]}\|\lambda_r^{(1)}(\eta)-\lambda_r^{(1)}(\eta)\|;$$

$$\gamma_{1,0}(h)e^{\gamma_{1,0}(h)hL_1\omega}\max_{x\in[0,\omega]}||Q_{1,h}\Big(x,\int_0^x\lambda^{(1)}(\xi)d\xi,\lambda^{(1)}(x),z^{(1)}\Big)||_{m_n}\leqslant q_{1,0}(h)||\lambda^{(1)}-\lambda^{(0)}||_1. \tag{28}$$

If $\lambda(x)\in S(\lambda^{(1)}(x),\rho_1+\widetilde{\varepsilon})$, where

$$\rho_1=\gamma_{1,0}(h)e^{\gamma_{1,0}(h)hL_1\omega}\max_{x\in[0,\omega]}||Q_{1,h}\Big(x,\int_0^x\lambda^{(1)}(\xi)d\xi,\lambda^{(1)}(x),z^{(1)}\Big)||_{m_n},$$

then, in view of inequalities 3), 4) Theorem and inequalities (27), (28), the estimate

$$||\lambda-\lambda^{(0)}||_1\leqslant||\lambda-\lambda^{(1)}||_1+||\lambda^{(1)}-\lambda^{(0)}||_1<\rho_1+\widetilde{\varepsilon}+||\lambda^{(1)}-\lambda^{(0)}||_1\leqslant$$

$$\leqslant(q_{1,0}(h)+1)||\lambda^{(1)}-\lambda^{(0)}||_1<\frac{\gamma_{1,0}(h)}{1-q_{1,0}(h)}e^{\gamma_{1,0}(h)hL_1\omega}\times$$

$$\times\max_{x\in[0,\omega]}||Q_{1,h}\Big(x,\int_0^x\lambda^{(0)}(\xi)d\xi,\lambda^{(0)}(x),z^{(0)}\Big)||_{m_n}<\rho_\lambda$$

holds, i.e. $S(\lambda^{(1)}(x),\rho_1)\subset S(\lambda^{(0)}(x),\rho_\lambda)$.

Continuing the process, in similar way, on kth step of algorithm, we find the pair $(\lambda^{(k)}(x),z^{(k)}(x,[t]))$ and establish the estimates

$$||\lambda^{(k)}-\lambda^{(k-1)}||_1\leq q_{1,0}(h)||\lambda^{(k-1)}-\lambda^{(k-2)}||_1; \tag{29}$$

$$||z^{(k)}-z^{(k-1)}||_3\leq\Big(e^{(L_1\omega+L_2)h}-1\Big)||\lambda^{(k)}-\lambda^{(k-1)}||_1. \tag{30}$$

Inequalities (29), (30) and condition 2) of Theorem imply that the sequence of pairs $(\lambda^{(k)}(x),z^{(k)}(x,[t]))$ converges to $(\lambda^*(x),z^*(x,[t]))$, the solution of problem (6)-(9), as $k\to\infty$. Based on inequalities 4) and 5) of Theorem the pairs $(\lambda^{(k)}(x), z^{(k)}(x,[t]))$, $k \in \mathbb{N}$, and $(\lambda^*(x), z^*(x,[t]))$ belongs to $S(\lambda^{(0)}(x), \rho_\lambda) \times S_h(z^{(0)}(x,[t]), \rho_z)$. In the inequalities

$$||\lambda^{(k+p)} - \lambda^{(0)}||_1 < \frac{1}{1 - q_{1,0}(h)} ||\lambda^{(1)} - \lambda^{(0)}||_1;$$

$$||z^{(k+p)} - z^{(0)}||_{m_n(x,h)} \leq \left(e^{(L_1\omega + L_2)h} - 1\right) ||\lambda^{(k+p)} - \lambda^{(0)}||_1,$$

passing to limit as $p \to \infty$, we obtain estimates (12), (13).

Theorem is proved.

References

1 *Кигурадзе Т.И.* Об ограниченных и периодических в полосе решениях квазилинейных гиперболических систем // Диф. уравнения. — 1994. — Т. 30. — № 10. — С. 1760–1773.

2 *Кигурадзе Т.И.* Об ограниченных в полосе решениях квазилинейных дифференциальных уравнений гиперболического типа // Применяемый анализ. — 1995. — Т. 58. — № 3, 4. — С. 199–214.

3 *Асанова А.Т., Джумабаев Д.С.* Об ограниченных решениях системы гиперболических уравнений и их аппроксимация // Вычисл. математика и матем. физика. — 2003. — Т. 42. — № 8. — С. 1132–1148.

4 *Кигурадзе Т., Кусано Т.* Об ограниченных и периодических в полосе решениях нелинейных гиперболических систем с двумя независимыми переменными // Вычисл. математика и матем. физика. — 2005. —Т. 49. — № 2, 3. — С. 335–364.

5 *Кигурадзе Т., Лакшмиканхан В.* О начально-краевой проблеме в ограниченных и неограниченных областях для одного класса нелинейных гиперболических уравнений третьего порядка // Журн. мат. анализа и приложения. — 2006. — Т. 324. — № 2. — С. 1242–1261.

6 *Джумабаев Д.С., Темешева С.М.* Метод параметризации для решения нелинейных двухточечных краевых задач // Вычисл. математика и матем. физика. — 2007. — Т. 47. — № 1. — С. 37–61.

Д.С. Жұмабаев, С.М. Темешева

Сызықтық емес аралас туындылы гиперболалық теңдеулер жүйесінің жолақта шектелген шешімі

Мақалада сызықтық емес аралас туындылы гиперболалық теңдеулер жүйесінің жолақта шектелген шешімі қарастырылды. Белгісіз функцияның уақыт бойынша айнымалысы бүкіл осьте өзгереді, ал кеңістіктік айнымалы ақырлы аралыққа тиесілі. Функция — кеңістіктік айнымалы бойынша дербес туынды, белгісіз функция ретінде белгіленеді және бастапқы жүйенің жолақта шектелген шешімін табу есебі дербес туындылы интегралдық-дифференциалдық теңдеулер жүйесінің жолақта шектелген шешімін табу есебіне келтірілді. Бүкіл ось бөліктерге бөлінеді және қосымша функционалдық параметрлер белгісіз функцияның ішкі облыстардың бастапқы сызықтарындағы мәндері ретінде енгізілді. Функционалдық параметрлер бекітілген мәндерінде ішкі облыстардағы жаңа белгісіз функциялар бірінші ретті дербес туындылы интегралдық-дифференциалдық теңдеулер үшін Коши есептерінің шешімдері ретінде анықталды. Бөліктеудің ішкі сызықтарындағы шешімнің үзіліссіздік шарттарын пайдалана отырып енгізілген функционалдық параметрлерге қатысты сызықты емес Вольтерра интегралдық теңдеулерінің екі жақты шексіз жүйесі алынған. Сондай-ақ функционалдық параметрлері бар есептің шешімдерін табу алгоритмдері ұсынылған. Алгоритмдердің жинақтылығы мен сызықтық емес аралас туындылы гиперболалық теңдеулер жүйесінің жолақта шектелген шешімінің бар болуының шарттары алынған.

Д.С. Джумабаев, С.М. Темешева

Ограниченное на полосе решение системы нелинейных гиперболических уравнений со смешанными производными

В статье рассмотрена система нелинейных гиперболических уравнений со смешанными производными на полосе. Временная переменная неизвестной функции меняется на всей оси, а пространственная переменная принадлежит конечному интервалу. Функция — частная производная относительно пространственной переменной — обозначается как неизвестная функция, и задача нахождения ограниченного на полосе решения исходной системы сводится к задаче нахождения ограниченного на полосе решения системы интегро-дифференциальных уравнений в частных производных. Вся ось делится на части, и дополнительные функциональные параметры вводятся как значения неизвестной функции на начальных линиях подобластей. Для фиксированных значений функциональных параметров новые неизвестные функции в подобластях определяются как решения задач Коши для интегро-дифференциальных уравнений в частных производных первого порядка. Используя условия непрерывности решения на внутренних линиях разбиения, получена двусторонне-бесконечная система нелинейных интегральных уравнений Вольтерра второго рода относительно введенных функциональных параметров. Предложены алгоритмы нахождения решений задачи с функциональными параметрами. Получены условия сходимости алгоритмов и существования ограниченного на полосе решения системы нелинейных гиперболических уравнений со смешанными производными.

References

1 Kiguradze T.I. *Differential Equations,* 1994, 30, 10, p. 1760–1773.

2 Kiguradze T. *Applicable Analysis,* 1995, 58, 3, 4, p. 199–214.

3 Asanova A.T., Dzhumabaev D.S. *Computational Mathematics and Mathematical Physics,* 2003, 42, 8, p. 1132–1148.

4 Kiguradze T., Kusano T. *Computational Mathematics and Mathematical Physics,* 2005, 49, 2, 3, p. 335–364.

5 Kiguradze T., Lakshmikanthan V. *Journal of Mathematical Analysis and Applications,* 2006, 324, 2, p. 1242–1261.

6 Dzhumabaev D.S., Temesheva S.M. *Computational Mathematics and Mathematical Physics,* 2007, 47, 1, p. 37–61.